CUET UG Math Practice Test 2 with Answers PDF

1. If $f:[1,\infty)\to[2,\infty)$ is given by $f(x)=x+\frac{1}{x}$ then $f^{-1}(x)$ equals:

a. $\frac{x+\sqrt{x^2-4}}{2}$ b. $\frac{x}{1+x^2}$

c. $\frac{x-\sqrt{x^2-4}}{2}$ d. $1+\sqrt{x^2-4}$

2. Let α and β be the roots of the equation $x^2+x+1=0$. The equation whose roots are α^{19},β^{7} is

a. $x^2-x-1=0$ b. $x^2-x+1=0$

c. $x^2+x-1=0$ d. $x^2+x+1=0$

3. Let $\omega_n=\cos\left(\frac{2\pi}{n}\right)+i\sin\left(\frac{2\pi}{n}\right), i^2=-1$, then $(x+y\omega_3+z\omega_3^2)(x+y\omega_3^2+z\omega_3)$ is equal to

a. 0

b. $x^2+y^2+z^2$

c. $x^2+y^2+z^2-yz-zx-xy$

d. $x^2+y^2+z^2+yz+zx+xy$

4. The value of $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7}-i\cos\frac{2\pi k}{7}\right)$ is

a. -1 b. 0

c. $-i$ d. i

5. The inverse of the matrix $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ is

a. $\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$ b. $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

c. $\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$ d. $\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$

6. $1+\frac{1+2}{2!}+\frac{1+2+3}{3!}+\frac{1+2+3+4}{4!}+\ldots\infty=$

a. e b. 3e

c. e/2 d. 3e/2

7. $\log_e 2+\log_e\left(1+\frac{1}{2}\right)+\log_e\left(1+\frac{1}{3}\right)+\ldots+\log_e\left(1+\frac{1}{n-1}\right)$ is equal to

a. $\log_e 1$ b. $\log_e n$

c. $\log_e(1+n)$ d. $\log_e(1-n)$

8. If $x\sin 45^\circ\cos^2 60^\circ=\frac{\tan^2 60^\circ\operatorname{cosec} 30^\circ}{\sec 45^\circ\cot^2 30^\circ}$, then $x=$

a. 2 b. 4

c. 8 d. 16

9. If $\sin\theta+\sin 2\theta+\sin 3\theta=\sin\alpha$ and $\cos\theta+\cos 2\theta+\cos 3\theta=\cos\alpha$, then θ is equal to

a. $\alpha/2$ b. α

c. 2α d. $\alpha/6$

10. Find imaginary part of $\sin^{-1}\left(\frac{5\sqrt{7}-9i}{16}\right)$

a. log2 b. $-\log 2$

c. 0 d. None of these

11. Let $f(x)=\begin{cases}\frac{x^3+x^2-16x+20}{(x-2)^2}, & \text{if } x\neq 2\\ k, & \text{if } x=2\end{cases}$.

If $f(x)$ be continuous for all x, then $k=$

a. 7 b. -7

c. ± 7 d. None of these

12. If $y=e^{x+e^{x+e^{x+\ldots\infty}}}$, then $\frac{dy}{dx}=$

a. $\frac{y}{1-y}$ b. $\frac{1}{1-y}$

c. $\frac{y}{1+y}$ d. $\frac{y}{y-1}$

13. One maximum point of $\sin^p x\cos^q x$ is

a. $x=\tan^{-1}\sqrt{(p/q)}$ b. $x=\tan^{-1}\sqrt{(q/p)}$

c. $x=\tan^{-1}(p/q)$ d. $x=\tan^{-1}(q/p)$

14. $\int\frac{1}{x(\log x)^2}dx=$

a. $\frac{1}{\log x}+c$ b. $-\frac{1}{\log x}+c$

c. $\log\log x+c$ d. $-\log\log x+c$

15. Area bounded by the parabola $y^2 = 2x$ and the ordinates $x = 1, x = 4$ is

a. $\frac{4\sqrt{2}}{3}$ sq. unit
b. $\frac{28\sqrt{2}}{3}$ sq. unit
c. $\frac{56}{3}$ sq. unit
d. None of these

16. The solution of the differential equation $\frac{dy}{dx} + \frac{1+x^2}{x} = 0$ is

a. $y = -\frac{1}{2}\tan^{-1}x + c$
b. $y + \log x + \frac{x^2}{2} + c = 0$
c. $y = \frac{1}{2}\tan^{-1}x + c$
d. $y - \log x - \frac{x^2}{2} = c$

17. The solution of the differential equation $\cos y \log(\sec x + \tan x)dx = \cos x \log(\sec y + \tan y)dy$ is

a. $\sec^2 x + \sec^2 y = c$
b. $\sec x + \sec y = c$
c. $\sec x - \sec y = c$
d. None of these

18. The vertex of an equilateral triangle is $(2, -1)$ and the equation of its base in $x + 2y = 1$. The length of its sides is

a. $4/\sqrt{15}$
b. $2/\sqrt{15}$
c. $4/3\sqrt{3}$
d. $1/\sqrt{5}$

19. If the equation $\frac{k(x+1)^2}{3} + \frac{(y+2)^2}{4} = 1$ represents a circle, then k =

a. 3/4
b. 1
c. 4/3
d. 12

20. The equation of the common tangent to the curves $y^2 = 8x$ and $xy = -1$ is

a. $3y = 9x + 2$
b. $y = 2x + 1$
c. $2y = x + 8$
d. $y = x + 2$

21. The sum of the first five terms of the series $3 + 4\frac{1}{2} + 6\frac{3}{4} + \ldots$ will be

a. $39\frac{9}{16}$
b. $18\frac{3}{16}$
c. $39\frac{7}{16}$
d. $13\frac{9}{16}$

22. If $(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^2 + \ldots + a_{12}x^{12}$, then the expression $a_2 + a_4 + a_6 + \ldots + a_{12}$ has the value

a. 32
b. 63
c. 64
d. None of these

23. There are 10 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated is.

a. 10^2
b. 1023
c. 2^{10}
d. 10 !

24. A fair coin is tossed repeatedly. If tail appears on first four tosses, then the probability of head appearing on fifth toss equals

a. $\frac{1}{2}$
b. $\frac{1}{32}$
c. $\frac{31}{32}$
d. $\frac{1}{5}$

25. An observer on the top of a tree, finds the angle of depression of a car moving towards the tree to be 30° . After 3 minutes this angle becomes 60°. After how much more time, the car will reach the tree

a. 4 min
b. 4.5 min
c. 1.5 min
d. 2 min

26. The variance of first 50 even natural numbers is:

a. $\frac{833}{4}$
b. 833
c. 437
d. $\frac{437}{4}$

27. The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$ can be written as

a. $\sin A \cos A + 1$
b. $\sec A$ coeec $A + 1$
c. $\tan A + \cot A$
d. $\sec A + \text{cosec } A$

28. The value of $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1 + \sum_{k=1}^{n}2k\right)\right)$ is

a. $\frac{23}{25}$
b. $\frac{25}{23}$
c. $\frac{23}{24}$
d. $\frac{24}{23}$

29. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point R, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is:

a. 6
b. 10
c. 20
d. 5

30. $\lim\limits_{x\to 0}\dfrac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to

a. $-\dfrac{1}{4}$ b. $\dfrac{1}{2}$

c. 1 d. 2

31. Let the function $g:(-\infty,\infty)\to\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$ be given by

$g(u)=2\tan^{-1}(e^{u})-\dfrac{\pi}{2}$.

Then, g is

a. even and is strictly increasing in $(0,\infty)$

b. odd and is strictly decreasing in $(-\infty,\infty)$

c. odd and is strictly increasing in $(-\infty,\infty)$

d. neither even nor odd, but is strictly increasing in $(-\infty,\infty)$

32. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is:

a. 175 b. 162

c. 160 d. 180

33. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0,1,2,3,4,5 (repetition of digits is allowed) is:

a. 288 b. 306

c. 360 d. 310

34. If the fractional part of the number $\dfrac{2^{403}}{15}$ is $\dfrac{k}{15}$, then k is equal to

a. 14 b. 6 c. 4 d. 8

35. The number of natural numbers less than 7,000 which can be formed by using the digits 0,1,3,7,9 (repetition of digits allowed) is equal to:

a. 250 b. 374 c. 372 d. 375

36. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is:

a. 1365 b. 1256

c. 1465 d. 1356

37. $\sum\limits_{i=1}^{20}\left(\dfrac{{}^{20}C_{i-1}}{{}^{20}C_{i}+{}^{20}C_{i-1}}\right)=\dfrac{k}{21}$, then k equals:

a. 200 b. 50

c. 100 d. 400

38. The number of functions f from {1, 2, 3,..., 20} onto {1, 2, 3,....., 20} such that f(k) is a multiple of 3, whenever k is a multiple of 4, is:

a. $(15)!\times 6!$ b. 56×15

c. $5!\times 6!$ d. $65\times(15)!$

39. Consider three boxes, each containing 10 balls labelled 1,2,....,10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the i^{th} box, (i = 1, 2, 3). Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is:

a. 82 b. 240

c. 164 d. 120

40. Let $z_k=\cos\left(\dfrac{2k\pi}{10}\right)+i\sin\left(\dfrac{2k\pi}{10}\right)$; $k=1,2,\ldots,9$.

Column I	Column II
(A) For each z_k there exists a z_j such $z_k.z_j=1$	1. True
(B) There exists a $k\in\{1,2,\ldots,9\}$ such that $z_1.z=z_k$ has no solution z in the set of complex numbers	2. False
(C) $\dfrac{\lvert 1-z_1\rvert\lvert 1-z_2\rvert\ldots\lvert 1-z_9\rvert}{10}$ equals	3. 1
(D) $1-\sum\limits_{k=1}^{9}\cos\left(\dfrac{2k\pi}{10}\right)$ equals	4. 2

a. A→ 2; B→ 1; C→ 3; D→ 4

b. A→ 3; B→ 2; C→ 1; D→ 4

c. A→ 1; B→ 2; C→ 3; D→ 4

d. A→ 1; B→ 3; C→ 4; D→ 2

41. Let (x, y) be such that $\sin^{-1}(ax)+\cos^{-1}(y)+\cos^{-1}(bxy)=\dfrac{\pi}{2}$ Match the statements in Column I with the statements in Column II.

Column I	Column II
(A) If $a=1$ and $b=0$, then (x, y)	1. lies on the circle $x^2+y^2=0$
(B) If $a=1$ and $b=1$, then (x, y)	2. lies on $(x^2-1)(y^2-1)=0$
(C) If $a=1$ and $b=2$, then (x, y)	3. lies on $y=x$
(D) If $a=2$ and $b=2$, then (x, y)	4. lies on $(4x^2-1)(y^2-1)=0$

a. A→ 1; B→ 2; C→ 1; D→ 4

b. A→ 4; B→ 2; C→ 1; D→ 3

c. A→ 1; B→ 2; C→ 3; D→ 4

d. A→ 1; B→ 3; C→ 4; D→ 2

42. Match the Statements/Expressions in Column I with the Statements/Expressions in Column II.

Column I	Column II
(A) The minimum value of $\dfrac{x^2+2x+4}{x+2}$ is	1. 0
(B) Let A and B be 3×3 matrices of real numbers, where A is symmetric, B is skew symmetric, and $(A+B)(A-B) = (A-B)(A+B)$. If $(AB)^t = (-1)^k AB$, where $(AB)^t$ is the transpose of the matrix AB, then the possible values of k are	2. 1
(C) Let $a = \log_3 \log_3 2$. An integer k satisfying $1 < 2^{(-k+3^{-a})} < 2$, must be less than	3. 2
(D) If $\sin\theta + \cos\phi$, then the possible value of $\frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right)$ are	4. 3

a. A→3; B→2,4; C→3,4; D→1,3
b. A→2; B→2,3; C→2,4; D→1,2
c. A→3; B→2,3; C→1,2; D→1,3
d. A→2; B→2,4; C→1,4; D→1,2

43. In the following [x] denotes the greatest integer less than or equal to x. Match the functions in Column I with the properties Column II.

Column I	Column II
(A) $x\|x\|$	1. continuous in $(-1,1)$
(B) $\sqrt{\|x\|}$	2. differentiable in $(-1,1)$
(C) $x+[x]$	3. strictly increasing in $(-1,1)$
(D) $\|x-1\|+\|x+1\|$	4. not differentiable at least at one point in $(-1,1)$

a. A→1,2; B→2,4; C→3,4; D→1,3
b. A→1,2,3; B→1,4; C→3,4; D→1,2
c. A→3; B→2,3; C→1,2,3; D→1,3
d. A→2; B→2,4; C→1,4; D→1,2

44. Let $z = \cos\theta + i\sin\theta$. Then the value of $\sum_{m=1}^{15} \text{Im}(z^{2m-1})$ at $\theta = 2°$ is

a. $\frac{1}{\sin 2°}$
b. $\frac{1}{3\sin 2°}$
c. $\frac{1}{2\sin 2°}$
d. $\frac{1}{4\sin 2°}$

45. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then the ordered pair (a, b) is equal to:

a. $(2,-1)$
b. $(-2,1)$
c. $(2,1)$
d. $(-2,-1)$

46. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777,..... is

a. $\frac{7}{81}(179-10^{-20})$
b. $\frac{7}{9}(99-10^{-20})$
c. $\frac{7}{81}(179+10^{-20})$
d. $\frac{7}{9}(99+10^{-20})$

47. Coefficient of x^{11} in the expansion of $(1+x^2)^4 (1+^3)^7 (1+x^4)^{12}$ is

a. 1051
b. 1106
c. 1113
d. 1120

48. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is

a. 55
b. 66
c. 77
d. 88

49. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations values 3, 4 and 5 are added to the data, then the mean of the resultant data, is

a. 16.8
b. 16.0
c. 15.8
d. 14.0

50. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the numbe r of games played by the

men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is:

a. 9 b. 11
c. 12 d. 7

Answer and Solutions

1. (a) Use the identity $f(f^{-1}(x)) = x$

replace x by $f^{-1}(x)$, in the given function we get

$$f(f^{-1}(x)) = f^{-1}(x) + \frac{1}{f^{-1}(x)}$$

$\Rightarrow$ $x = f^{-1}(x) + \frac{1}{f^{-1}(x)}$, solve to find $f^{-1}x$

2. (d) Given $x^2 + x + 1 = 0$

$\therefore$ $x = \frac{1}{2}[-1 \pm i\sqrt{3}] = \frac{1}{2}(-1 + i\sqrt{3}), \frac{1}{2}(-1 - i\sqrt{3}) = \omega, \omega^2$

But $\alpha^{19} = \omega^{19} = \omega$ and $\beta^7 = \omega^{14} = \omega^2$.

Hence the equation will be same.

3. (c) $\omega_n = \cos\left(\frac{2\pi}{n}\right) + i\sin\left(\frac{2\pi}{n}\right)$

$\Rightarrow$ $\omega_3 = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = -\frac{1}{2} + \frac{i\sqrt{3}}{2} = \omega$

and $\omega_3^2 = \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^2 = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$

$= -\frac{1}{2} - \frac{i\sqrt{3}}{2} = \omega^2$.

$\therefore$ $(x + y\omega_3 + z\omega_3^2)(x + y\omega_3^2 + z\omega_3)$

$= (x + y\omega + z\omega^2)(x + y\omega^2 + z\omega)$

$= x^2 + y^2 + z^2 - xy - yz - zx$.

4. (d) $\sum_{k=1}^{6}\left(\sin\frac{2k\pi}{7} - i\cos\frac{2k\pi}{7}\right)$

$= \sum_{k=1}^{6} -i\left(\cos\frac{2k\pi}{7} + i\sin\frac{2k\pi}{7}\right) = -i\left\{\sum_{k=1}^{6} e^{\frac{i2k\pi}{7}}\right\}$

$= -i\{e^{i2\pi/7} + e^{i4\pi/7} + e^{i6\pi/7} + e^{i8\pi/7} + e^{i10\pi/7} + e^{i12\pi/7}\}$

$= -i\left\{e^{i2\pi/7}\frac{(1 - e^{i12\pi/7})}{1 - e^{i2\pi/7}}\right\} = -i\left\{\frac{e^{i2\pi/7} - e^{i14\pi/7}}{1 - e^{i2\pi/7}}\right\}$

$(\because e^{i14\pi/7} = 1)$ $= -i\left\{\frac{e^{i2\pi/7} - 1}{1 - e^{i2\pi/7}}\right\} = i$

5. (b) It is understandable.

6. (d) $T_n = \frac{\Sigma n}{n!} = \frac{n(n+1)}{2(n)!}$

$= \frac{1}{2}\left[\frac{(n+1)}{(n-1)!}\right] = \frac{1}{2}\left[\frac{n-1}{(n-1)!} + \frac{2}{(n-1)!}\right]$

$= \frac{1}{2}\left[\frac{1}{(n-2)!} + \frac{2}{(n-1)!}\right] = \frac{(e+2e)}{2} = \frac{3e}{2}$.

7. (b) The given series reduces to

$\log_e 2 + \log_e\left(\frac{3}{2}\right) + \log_e\left(\frac{4}{3}\right) + \ldots + \log_e\left(\frac{n}{n-1}\right)$

$= \log_e 2 + \log_e 3 - \log_e 2 + \log_e 4 - \log_e 3 + \ldots$

$+ \log_e(n) - \log_e(n-1) = \log_e n$.

8. (c) $x.\frac{1}{\sqrt{2}}.\frac{1}{4} = \frac{3.2}{\sqrt{2}.3} \Rightarrow \frac{x}{4\sqrt{2}} = \sqrt{2} \Rightarrow x = 8$.

9. (a) $\sin\theta + \sin 3\theta + \sin 2\theta = \sin\alpha$

$\Rightarrow$ $2\sin 2\theta\cos\theta + \sin 2\theta = \sin\alpha$

$\Rightarrow$ $\sin 2\theta(2\cos\theta + 1) = \sin\alpha$ ...(i)

Now, $\cos\theta + \cos 3\theta + \cos 2\theta = \cos\alpha$

$2\cos 2\theta\cos\theta + \cos 2\theta = \cos\alpha$

$\Rightarrow$ $\cos 2\theta(2\cos\theta + 1) = \cos\alpha$ ...(ii)

From (i) and (ii),

$\Rightarrow$ $\tan 2\theta = \tan\alpha \Rightarrow 2\theta = \alpha \Rightarrow \theta = \alpha/2$.

10. (b) Imaginary part of $\left[\sin^{-1}\left(\frac{5\sqrt{7}}{16} - \frac{9i}{16}\right)\right]$

$= -\log\left[\sqrt{\frac{9}{16}} + \sqrt{1 + \frac{9}{16}}\right] = -\log(2)$.

11. (a) For continuous $\lim_{x\to 2} f(x) = f(2) = k$

$\Rightarrow$ $k = \lim_{x\to 2}\frac{x^3 + x^2 - 16x + 20}{(x-2)^2}$

$= \lim_{x\to 2}\frac{(x^2 - 4x + 4)(x + 5)}{(x-2)^2} = 7$.

12. (a) $y = e^{x+y} \Rightarrow \log y = (x + y)\log e$

$\Rightarrow$ $\frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{y}{1-y}$.

13. (a) Let $y = \sin^p x \cos^q x$

$\frac{dy}{dx} = p\sin^{p-1}x\cos x\cos^q x + q\cos^{q-1}x(-\sin x)\sin^p x$

$\frac{dy}{dx} = p\sin^{p-1}x\cos^{q+1}x - q\cos^{q-1}x\sin^{p+1}x$

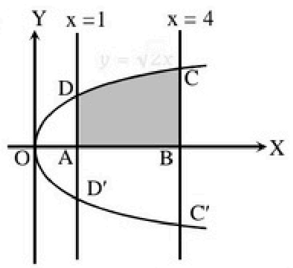
Put $\frac{dy}{dx}=0,\ \therefore \tan^2 x=\frac{p}{q} \Rightarrow \tan x=\pm\sqrt{\frac{p}{q}}$

$\therefore$ Point of maxima $x=\tan^{-1}\sqrt{\frac{p}{q}}$.

14. (b) Put $\log x=t \Rightarrow \frac{1}{x}dx=dt$, then

$$\int \frac{1}{x(\log x)^2}dx=\int \frac{1}{t^2}dt=-\frac{1}{t}+c=-\frac{1}{\log x}+c.$$

15. (b)



Required area $= CDD'C' = 2\times ABCD$

$$=2\int_1^4 \sqrt{2}\,x^{1/2}\,dx=\frac{28\sqrt{2}}{3} \text{ sq. unit.}$$

16. (b) $\frac{dy}{dx}+\frac{1+x^2}{x}=0 \Rightarrow dy+\left(\frac{1}{x}+x\right)dx=0$

On integrating, we get $y+\log x+\frac{x^2}{2}+c=0$.

17. (d) $\cos y\log(\sec x+\tan x)dx=\cos x\log(\sec y+\tan y)dy$

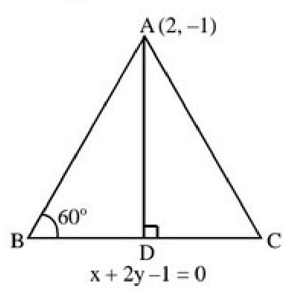
$\Rightarrow \int \sec y\log(\sec y+\tan y)dy$

$=\int \sec x\log(\sec x+\tan x)dx$

Put $\log(\sec x+\tan x)=t$ and $\log(\sec y+\tan y)=z$

$$\frac{[\log(\sec x+\tan x)]^2}{2}=\frac{[\log(\sec y+\tan y)]^2}{2}+c.$$

18. (b)



$$|AD|=\left|\frac{2-2-1}{\sqrt{1^2+2^2}}\right|=\frac{1}{\sqrt{5}}$$

$\Rightarrow \tan 60^\circ=\frac{AD}{BD} \qquad \Rightarrow \sqrt{3}=\frac{1/\sqrt{5}}{BD}$

$\Rightarrow BD=\frac{1}{\sqrt{15}} \qquad \Rightarrow BC=2BD=2/\sqrt{15}$.

19. (a) It represents a circle, if $a=b$

$\Rightarrow \frac{k}{3}=\frac{1}{4} \Rightarrow k=\frac{3}{4}$.

20. (d) Tangent to the curve $y^2=8x$ is $y=mx+2/m$

So, it must satisfy $xy=-1$.

$\Rightarrow x\left(mx+\frac{2}{m}\right)=-1 \Rightarrow mx^2+\frac{2}{m}x+1=0$

Since, it has equal roots. $\therefore D=0$

$\Rightarrow \frac{4}{m^2}-4m=0 \Rightarrow m^3=1$

$\Rightarrow m=1$ Hence, equation of common tangent is $y=x+2$.

21. (a) Given series is $3+4\frac{1}{2}+6\frac{3}{4}+\dots\dots = 3+\frac{9}{2}+\frac{27}{4}+\dots$

$=3+\frac{3^2}{2}+\frac{3^3}{4}+\frac{3^4}{8}+\frac{3^5}{16}+\dots\dots$ (in G.P.)

Here $a=3, r=\frac{3}{2}$, then sum of the five terms

$$S_5=\frac{a(r^n-1)}{r-1}=\frac{3\left[\left(\frac{3}{2}\right)^5-1\right]}{\frac{3}{2}-1}=\frac{1\left[\frac{3^5}{32}-1\right]}{\frac{1}{2}}$$

$$=6\left[\frac{243-32}{32}\right]=\frac{211\times 3}{16}=\frac{633}{16}=39\frac{9}{16}.$$

22. (d) $(1+x-2x^2)^6=1+a_1x+a_2x^2+\dots+a_{12}x^{12}$.

Putting $x=1$ and $x=-1$ and adding the results

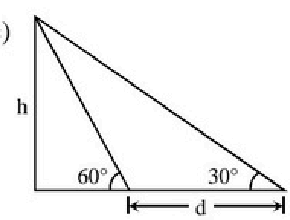
$64=2(1+a_2+a_4+\dots) \quad \therefore a_2+a_4+a_6+\dots+a_{12}=31$.

23. (b) $2^{10}-1=1023$, corresponds to none of the lamps is being switched ON.

24. (a) The event that the fifth toss results in a head is independent the event that the first four tosses result in tails.

$\therefore$ Probability of the required event $=\frac{1}{2}$

25. (c)



$d=h\cot 30^\circ-h\cot 60^\circ$ and time = 3 min.

$\therefore$ Speed $=\frac{h(\cot 30^\circ-\cot 60^\circ)}{3}$ per minute

It will travel distance $h\cot 60^\circ$ in

$$\frac{h\cot 60^\circ\times 3}{h(\cot 30^\circ-\cot 60^\circ)}=1.5 \text{ minute.}$$

26. (b) Variance $= \frac{\sum x_i^2}{n} - (\bar{x})^2$

$\Rightarrow \sigma^2 \left(\frac{2^2+4^2+6^2+\ldots+100^2}{50}\right) - \left(\frac{2+4+\ldots+100}{50}\right)^2$

$\Rightarrow \sigma^2 = 3434 - 2601 = 833$

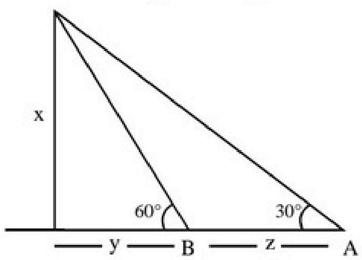
27. (b) Exp. $= \frac{\tan^2 A}{\tan A - 1} + \frac{1}{\tan A - \tan^2 A} = \frac{1}{\tan A - 1}\left[\tan^2 A - \frac{1}{\tan A}\right]$

$= \frac{\tan^2 A + \tan A + 1}{\tan A} = \tan A + \cot A + 1 = \sec A . \operatorname{cosec} A + 1$

28. (b) $\cot\left(\sum_{n=1}^{23} \cot^{-1}(n^2+n+1)\right)$ $\cot\left(\sum_{n=1}^{23} \tan^{-1}\left(\frac{n+1-n}{1+n(n+1)}\right)\right)$

$\Rightarrow \cot\left(\tan^{-1}\left(\frac{23}{325}\right)\right) = \frac{25}{23}$.

29. (d) $\tan 30° = \frac{x}{y+z} = \frac{1}{\sqrt{3}}$



$\Rightarrow \sqrt{3}x = y + z$

$\Rightarrow \tan 60° = \frac{x}{y} = \sqrt{3}$

$\Rightarrow x = \sqrt{3}y = y + z$

$3y = y + z$

$\Rightarrow 2y = z$

For 2y distance time = 10 min.

So For y distance time = 5 min.

30. (d) $\lim_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$

$= \lim_{x\to 0} \frac{(2\sin^2 x)(3+\cos x)}{x\left(\frac{\tan 4x}{4x}\right)\times 4x}$

$= \lim_{x\to 0} \frac{2\sin^2 x(3+\cos x)}{4x^2} = \frac{2}{4}(3+1) = 2$

31. (c) $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$

$= 2\tan^{-1} e^u - \tan^{-1} e^u - \cot^{-1} e^u = \tan^{-1} e^u - \cot^{-1} e^u$

$g(-x) = -g(x)$

$\Rightarrow g(x)$ is odd and $g'(x) > 0$

$\Rightarrow$ increasing.

32. (d) □□□□□□□□□ — 2^{nd} place, 4^{th} place, 6^{th} place, 8^{th} place (even places)

Number of such numbers $= {}^4C_3 \times \frac{3!}{2!} \times \frac{6!}{2!4!} = 180$

33. (d) The number of four -digit numbers Starting with 5 is equal to $6^3 = 216$

Starting with 44 and 55 is equal to $36 \times 2 = 72$

Starting with 433, 434 and 435 is equal to $6 \times 3 = 18$

Remaining numbers are 4322, 4323, 4324, 4325 is equal to 4

So total numbers are 216 + 72 + 18 + 4 = 310

34. (d) $\frac{2^{403}}{15} = \frac{2^3(2^4)^{100}}{15} = \frac{8}{15}(15+1)^{100}$

$= \frac{8}{15}(15\lambda + 1) = 8\lambda + \frac{8}{15}$

$\because$ 8λ is integer

fractional part of $\frac{2^{403}}{15}$ is $\frac{8}{15} \Rightarrow k = 8$

35. (b)

a_1	a_2	a_3

Number of numbers $= 5^3 - 1$

a_4	a_1	a_2	a_3

2 ways for a_4

Number of numbers $= 2 \times 5^3$

Required number $= 5^3 + 2 \times 5^3 - 1 = 374$

36. (d) $\sum_{r=2}^{13}(7r+2) = 7 \cdot \frac{2+13}{2} \times 6 \times 2 \times 12$

$= 7 \times 90 + 24 = 654$

$\sum_{r=2}^{13}(7r+5) = 7\left(\frac{1+13}{2}\right) \times 13 + 5 \times 13 = 702$

Total 654 + 702 = 1356

37. (c) $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$

$\Rightarrow \sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i}\right)^3 = \frac{k}{21}$

$\Rightarrow \sum_{i=1}^{20}\left(\frac{i}{21}\right)^3 = \frac{k}{21}$

$\Rightarrow \frac{1}{(21)^3}\left[\frac{20(21)}{2}\right]^2 = \frac{k}{21}$

$\Rightarrow 100 = k$

38. **(a)** $f(k) = 3m$ (3,6,9,12,15,18)
for $k = 4,8,12,16,20$
6.5.4.3.2 ways
For rest numbers 15! ways
Total ways = (15)! × 6!

39. **(d)** No. of ways $= 10C_3 = 120$

40. **(c)** A→ 1; B→ 2; C→ 3; D→ 4

(A) z_k is 10^{th} root of unity $\Rightarrow \bar{z}_k$ will also be 10^{th} root of unity. Take z_j as $\bar{z}_k$.

(B) $z_1 \neq 0$ take $z = \frac{z_k}{z_1}$, we can always find z.

(C) $z^{10} - 1 = (z -_1)(z - z_1)...(z - z_9)$

$\Rightarrow$ $(z - z_1)(z - z_2)...(z - z_9) = 1 + z + z^2 + ... + z^9 \forall z \in$ complex number.

Put $z = 1$

$(1 - z_1)(1 - z_2)...(1 - z_9) = 10.$

(D) $1 + z_1 + z_2 + ... + z_9 = 0$

$\Rightarrow$ $\text{Re}(1) + \text{Re}(z_1) + ... + \text{Re}(z_9) = 0$

$\Rightarrow$ $\text{Re}(z_1) + \text{Re}(z_2) + ... + \text{Re}(z_9) = -1.$

$\Rightarrow$ $1 - \sum_{k=1}^{9} \cos \frac{2k\pi}{10} = 2.$

41. **(a)** A→ 1; B→ 2; C→ 1; D→ 4

(A) If $a = 1, b = 0$ then $\sin^{-1} x + \cos^{-1} y = 0$

$\Rightarrow$ $\sin^{-1} x = -\cos^{-1} y \Rightarrow x^2 + y^2 = 1.$

(B) If $a = 1, b = 1$ then $\sin^{-1} x + \cos^{-1} y + \cos^{-1} xy = \frac{\pi}{2}$

$\Rightarrow$ $\cos^{-1} x - \cos^{-1} y = \cos^{-1} xy$

$\Rightarrow$ $xy + \sqrt{1 - x^2}\sqrt{1 - y^2} = xy$ (taking sine on both the sides)

(C) If $a = 1, b = 2$ then $\sin^{-1} x + \cos^{-1} y + \cos^{-1}(2xy) = \frac{\pi}{2}$

$\Rightarrow$ $\sin^{-1} x + \cos^{-1} y = \sin^{-1}(2xy)$

$\Rightarrow$ $xy + \sqrt{1 - x^2}\sqrt{1 - y^2} = 2xy \Rightarrow x^2 + y^2 = 1$ (on squaring)

(D) If $a = 2, b = 2$ then $\sin^{-1}(2x) + \cos^{-1}(y) + \cos^{-1}(2xy) = \frac{\pi}{2}$

$\Rightarrow$ $2xy + \sqrt{1 - 4x^2}\sqrt{1 - y^2} = 2xy \Rightarrow (4x^2 - 1)(y^2 - 1) = 0$

42. **(a)** A→3; B→2,4; C→3,4; D→1,3

(A) $y = \frac{x^2 + 2x + 4}{x + 2}$

$\Rightarrow$ $x^2 + (2 - y)x + 4 - 2y = 0$

$\Rightarrow$ $y^2 + 4y - 12 \geq 0 \quad y \leq -6$ or $y \geq 2$

Minimum value is 2.

(B) $(A + B)(A - B) = (A - B)(A + B)$

$\Rightarrow$ $AB = BA$ as A is symmetric and B is skew symmetric

$\Rightarrow$ $(AB)' = -AB \quad \Rightarrow k = 1$ and $k = 3$

(C) $a = \log_3 \log_3 2 \Rightarrow 3^{-a} = \log_2 3$

Now $1 < 2^{-k + \log_2^3} < 2$

$\Rightarrow$ $1 < 3.2^{-k} < 2$

$\Rightarrow$ $\log_2\left(\frac{3}{2}\right) < k < \log_2(3)$

$\Rightarrow$ $k = 1$ or $k < 2$ and $k < 3$.

(D) sin = cos $\Rightarrow$ $\cos\left(\frac{}{2} - \right) = \cos$

$\frac{}{2}$ $2n$

$-\left(\pm \frac{}{2}\right) = -2n \quad \Rightarrow$ 0 and 2 are possible.

43. **(b)** A→1,2,3; B→1,4; C→3,4; D→1,2

(A) $x|x|$ is continuous, differentiable and strictly increasing in (−1, 1).

(B) $\sqrt{|x|}$ is continuous in $(-1, 1)$ and not differentiable at $x = 0$.

(C) $x + [x]$ is strictly increasing in $(-1, 1)$ and discontinuous at $x = 0$

$\Rightarrow$ not differentiable at $x = 0$.

(D) $|x - 1| + |x + 1| = 2$ in $(-1, 1)$.

$\Rightarrow$ the function is continuous and differentiable in $(-1, 1)$.

44. **(d)** X sin sin 3 ... sin 29

2(sin)X 1 cos 2 cos 2 cos 4 ... cos 28 cos 30

$\Rightarrow$ $X \quad \frac{1 \quad \cos 30}{2 \sin} \quad \frac{1}{4 \sin 2}$

45. **(d)** $AA^T = 9I$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = 9I$$

$$\Rightarrow \begin{bmatrix} 9 & 0 & a + 4 + 2b \\ 0 & 9 & 2a + 2 - 2b \\ a + 4 + 2b & 2a + 2 - 2b & a^2 + 4 + b^2 \end{bmatrix} \Rightarrow \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

Equation $a + 4 + 2b = 0$

$\Rightarrow$ $a + 2b = -4$ . . .(i)

and $2a + 2 - 2b = 0$

$\Rightarrow\ 2a - 2b = -2 \qquad \ldots(ii)$

$a^2 + 4 + b^2 = 0$

$\Rightarrow\ a^2 + b^2 = 5 \qquad \ldots(iii)$

Solving $a = -2, b = -1$

46. (c) $0.7 + 0.77 + 0.777 + \ldots.. + 0.777\ldots7$

$= \frac{7}{9}[0.9 + 0.99 + 0.999 + \ldots + 0.999\ldots9]$

$= \frac{7}{9}[(1-0.1) + (1-0.01) + (1-0.001\ldots1) + \ldots + (1-0.000\ldots1)]$

$= \frac{7}{9}\left[20 - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \ldots + \frac{1}{10^{20}}\right)\right]$

$= \frac{7}{9}\left[20 - \frac{1}{10}.\frac{1-\frac{1}{10^{20}}}{1-\frac{1}{10}}\right] = \frac{7}{9}\left[20 - \frac{1}{9}.\left(\frac{10^{20}-1}{10^{20}}\right)\right]$

$\frac{7}{81}\left[180 - \left(1 - \frac{1}{10^{20}}\right)\right] = \frac{7}{81}[179 + 10^{-20}]$

47. (c) $2x_1 + 3x_2 + 4x_3 = 11$

Possibilities are (0, 1, 2); (1, 3, 0); (2, 1, 1); (4, 1, 0).

$\therefore$ Required coefficients

$= ({}^4C_0 \times {}^7C_1 \times {}^{12}C_2) + ({}^4C_1 \times {}^7C_3 \times {}^{12}C_0) + ({}^4C_2 \times {}^7C_1 \times {}^{12}C_1) + ({}^4C_4 \times {}^7C_1 \times 1)$

$= (1 \times 7 \times 66) + (4 \times 35 \times 1) + (6 \times 7 \times 12) + (1 \times 7)$

$= 462 + 140 + 504 + 7 = 1113.$

48. (c) Coefficient of x^{10} in $(x + x^2 + x^3)^7$

Coefficient of x^3 in $(1 + x + x^2)^7$

Coefficient of x^3 in $(1-x^3)^7(1-x)^{-7} = {}^{7+3+7}C_3 - 7$

$= {}^9C_3 - 7 = \frac{9 \times 8 \times 7}{6} - 7 = 77$

49. (d) $\frac{\Sigma x_i}{16} = 16 \Rightarrow \Sigma x_i = 256$

$\frac{(\Sigma x_i) - 16 + 3 + 4 + 5}{18} = \frac{252}{18} = 14$

50. (c) Let m-men, 2-women

${}^mC_2 \times C = {}^mC_1\, {}^2C_1 \cdot 2 + 84$

$m^2 - 5m - 84 = 0$

$\Rightarrow\ (m - 12)(m + 7) = 0$

$\Rightarrow\ m = 12$

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