CHAPTER:- ELECTRIC CHARGES & FIELDS

COMPONENT/PARAMETER THEOREM/PRINCIPLE/CONDITION	FORMULA(S)	SI UNIT(S)
1) Electric Charge Q or q	$q = ne = it = \frac{V}{r}.t = \frac{w}{v}\ OR\ \frac{\vec{F}}{\vec{E}}$	C
2) Electric Field E	$E = q.V = \frac{d\emptyset}{dA} = \frac{Kq}{r^2} = \frac{\sigma}{\varepsilon_0}$	N/C
3) Linear Charge Density	$\lambda = \frac{Q}{L} = \frac{dQ}{dl}$	C/m
4) Surface Charge Density	$\sigma = \frac{Q}{A} = \frac{dQ}{dA}$	C/m^2
5) Volume Charge Density	$\rho = \frac{Q}{V} = \frac{dQ}{dV}$	C/m^3
6) Electric Flux	$\emptyset = \int E.da = \vec{E}.\vec{A} = E.A\,Cos\theta$	Nm^2/C
7) Gauss Theorem	$\emptyset = \int E.da = q/\varepsilon_0$	Nm^2/C
8) Electric Force	$F = \frac{q1.q2}{4\pi\varepsilon_0 r^2}$	N
9) Torque	$\tau = p\ X\ E = p.Esin\theta$	Nm
10) Electric Dipole Moment	$p = q.2l = q.d$	C-m
11) Acceleration of charged particles	$a = \frac{F}{m} = \frac{qE}{m}$	m/s^2
12) Electric Field due to Axial Line	$E_{axial} = \frac{2\vec{P}}{4\pi\varepsilon_0 x^3}$	N/C
13)) Electric Field due to Equitorial Line	$E_{equitorial} = \frac{\vec{P}}{4\pi\varepsilon_0 (x^2 + a^2)^{\frac{3}{2}}}$	N/C

Electrostatics
(Electric Field & Potenial)

ELECTROSTATICS / FIELD

Electrostatics is a branch of physics which deals with charge at rest charge is the origin of electromagnetic force. When charge is at rest then region around it called electric field while the region around the moving charge called magnetic field.

PROPERTIES OF CHARGE :

1. For a given closed system the total charge remains conserved.
2. Charge is always quantised i.e. $Q = \pm nc$; $n \in I$
   Charge on any body is always in the integral multiple of one electronic charge ($e = 1.6 \times 10^{-19}$ coul).
3. Charge is relativistically invariant.

TYPES MATERIAL

There are three kinds of material on the basis of conductivity.

(a) Conductor : Having large number of mobile electrons. It is approximately 10^{21} electrons/c.c.

(b) Bad conductor : Having very small number of free electrons, it is approximately 10^{7} electrons c.c

(c) Semi conductor : Conductivity lies between conductor and insulator. Number of free electrons is approximately 10^{4} electron/c.c.

COULOMB'S LAW :

When two point charges q_1 and q_2 are separated by a distance r then force of mutual intraction F is given by

$$F \propto q_1 q_2 \text{ when } r = \text{constant.}$$

and $$F \propto \frac{1}{r^2} \text{ when } q_1 q_2 = \text{constant}$$

Hence $F \propto \frac{q_1 q_2}{r^2}$ or $F = k.\frac{q_1 q_2}{r^2}$; K is proportionality constant

In SI units, $F = \frac{1}{4\pi \in_0} . \frac{q_1 q_2}{r^2}$; $\in_0 = 8.85 \times 10^{-12} F_m$ and $\frac{1}{4\pi \in_0} = 9 \times 10^9$ nt m^2 coul $^{-2}$

In cgs units, $F = \frac{q_1 q_2}{r^2}$; $\in_0$ is per mittivity of the vacuum.

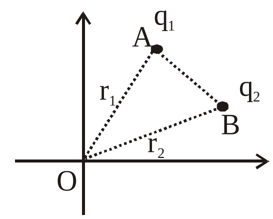
When there is a medium in the intervining region of two charges then

$$F = \frac{1}{4\pi \in_0 \in_r} \cdot \frac{q_1 q_2}{r^2}$$ ($\hat{I}_r$ is the relative permitivity of medium)

$\hat{I}_r$ is the relative permitivity which is the dimensionless qunatity that gives the factor by which force is reduced compared to vaccum.

$\in_r = \frac{F_0}{F_m}$, F_0 is the force in vacuum and F_m is force in medium.

$\in = \in_0 \in_r =$ Absolute permittivity of the medium.



$$\vec{F}_{AB} = \frac{1}{4\pi \in_0} \cdot \frac{q_1 q_2}{|\vec{BA}|^2} \cdot \frac{\vec{BA}}{|\vec{BA}|}$$

$$\vec{F}_{AB} = \frac{q_1 q_2}{4\pi \in_0 |\vec{BA}|^3} \cdot \vec{BA}$$

where $\vec{BA} = \vec{OA} - \vec{OB} - = \vec{r}_1 - \vec{r}_2$

$$\vec{F}_{AB} = \frac{q_1 q_2}{4\pi \in_0 |\vec{r}_1 - \vec{r}_2|^3} \cdot (\vec{r}_1 - \vec{r}_2)$$

when $q_1 q_2 > 0$, Force is attractive

and $q_1 q_2 < 0$, Force is repulsive

Process of Charging :

(1) By rubbing or friction - when two bodies are rubbed together there is transfer of electrons from body, which is surplus in electron to another body which is surplus in electrons and get positive charge of equal amount to negative charge.

(2) By conduction - when a charged body is in contact with another uncharged one threre is redistribution of charges on entire are is of both bodies followed by mechanical separation. The amount of charge redistribution on body depends on surface area.

(3) By induction - when an uncharged body is brought near charged on the charge opposite nature induced over the uncharged one.

The induced charge is always less than or equal to inducing charge. Induction is always followed by attraction, but attraction is not the surest test of induction.

If q be inducing charge, then charge induced on a body having dielectric constant K is given by

$q' = -q\left(1 - \frac{1}{K}\right)$, if charge is induced on the surface of a conductor then induced charge is

$q' = -q$ (As k is infinity for a conductor)

Distribution of Charges :

(a) Linear charge distribution : - If charge gets appeared on a body of linear dimension.

Linear charge density $(\lambda) =$ charge per unit length.

(b) Surface charge distrbution :- If charge gets appeared on a body having two dimensions.

Surface charge density $(\sigma) =$ charge per unit area

(c) Volume charge density : If charge is enclosed in a volume;

Volume charge density=charge per unit volume

Electric field : The site around the charge at rest called electric field.

Electric field strength or Electric intensity is defined as the force experienced by a unit positive charge.

$\vec{E} = \lim_{q_0 \to 0} \frac{F}{q_0}$ where q_0 is the positive test charge.

Unit of $\vec{E}$ is newton coulmob and dimension is $\left[MLT^{-3}A^{-1}\right]$. The resultant electric field at any point is equal to vector sum of electric field at that point due to various charges i.e. $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$

Resultant of two electric fields which are at an angle θ is given by

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\theta} \text{ and } \tan\alpha = \frac{E_2\sin\theta}{E_1 + E_2\cos\theta}$$

α is the angle of $\vec{E}$ with $\vec{E}_1$

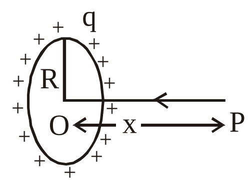
Electric field intensity $(\vec{E})$ for some body having uniformly continuous charge distribution.

1. Electric field strength due to a point charge at a distance r is given by

$$\vec{E}_1 = \frac{1}{4\pi\in_0}\frac{q}{r^2}.\hat{r}$$

2. Electric field strength due to a uniformly charged ring at a distance x from centre of the axis of ring..

$$\vec{E} = \frac{1}{4\pi\in_0}\frac{qx}{(x^2 + R^2)^{3/2}}$$



E becomes max at $x = \frac{R}{\sqrt{2}}$: Direction of $\vec{E}$ is away from centre along axis.

3. Electric field strength due to uniformally charged rod of length l at a distance r along perpendicular line from centre and linear charge density is λ.

$$E \neq \frac{q}{4\pi\in_0 r\sqrt{\frac{l^2}{4} + r^2}}$$

where λ = charge per unit length when $l \to \infty$

$$E = \frac{\lambda}{2\pi\in_0 r}$$

4. Electric field strength due to uniformally charged spherical shell of radius R at a distance r from, centre of shell

$E = 0$, if $r < R$

$E = \frac{Q}{4\pi\in_0 R^2} = \frac{\sigma}{\in_0}$; if $r \geq R$ where Q is charged on shell or σ is surface charge density.

5. Electric field strength due to uniformally charged solid sphere of radius R at a distance r from centre of sphere.

$E = \frac{Q}{4\pi\in_0 R^3}r = \frac{\rho}{3\in_0}r$; $r < R$; where Q is charged in solid sphere enclosed ;

ρ is volume charge density and $E = \frac{Q}{4\pi\in r^2}$ where $r \geq R$

ELECTRIC DIPOLE :

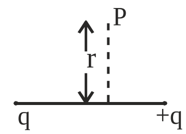
A system of two equal and opposite charges separated by a small distance called dipole. The dipole moment of dipole is defined as the product of charge and distance of separation and direction of dipole moment is from –ve charge to positive charge.

$p = q\Delta l$; Direction of dipole moment $(\vec{p})$ is from the charge to positive charge.

$\vec{E}$ due to dipole at a distance r from centre of dipole along axis of dipole, $\vec{E} = \dfrac{2\vec{P}}{4\pi \in_0 r^3}$.

$\vec{E}$ (Electric field strength) due to dipole at a distance r from centre of dipole along perpendicular bisector of line (Along equational line)

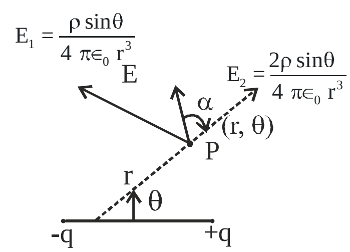
$$\vec{E} = \frac{-\vec{P}}{4\pi \in_0 r^3}$$



$\vec{E}$ (Electric field strength) at a point (r, θ) from centre of dipole.

$$E = \frac{P}{4\pi \in_0 r^3}\sqrt{1 + 3\cos^2\theta}$$

and α = angle made by $\vec{E}$ with $\vec{r} = \tan^{-1}\left(\dfrac{1}{2}\tan\theta\right)$



DIPOLE IN AN EXTERNAL ELECTRIC FIELD :

When dipole of dipole moment $\vec{P}$ is placed in an external electric field of field strength $\vec{E}$ then torque and potential energy at orientation θ are given by $\vec{\tau} = \vec{P} \times \vec{E}$ and $U = -\vec{P}.\vec{E}$ respectively. when dipole is placed in non uniform electric field then there no translational equilibrium and no rational equilibrium while for uniform field there is always translational equilibrium but no rational.

ELECTRIC FLUX :

In the region around the charge at rest there exists hypothetical electric lines, which measures the electric field strength at a point around charge at rest.

ELECTRIC FLUX DENSITY :

It is the number of electric lines cross through unit area held normally to the electric lines.

$$\phi_E = \int \vec{E}.\vec{ds} = \int E\cos\theta\, ds = \text{Electric flux through the area ds}$$

θ is the angle between $\vec{E}$ and area vector.

Gauss' law : The total electric flux through a closed loop is always equal to $\dfrac{1}{\in_0}$ lines the total charge enclosed.

$$\phi_{\in} = \oint \vec{E}\, \vec{ds} = \frac{1}{\in_0}(\text{charge enclosed})$$

APPLICATION OF GAUS'S LAW :

(a) The charge given to the conductor always get apeared on outside of the conductor.

(b) The electric field strength at a perpendicular bisector on uniformally charged rod of infinite length at a distance r from centre of rod

$$E = \frac{\lambda}{2\pi \in_0 r}.$$

(c) The electrostatic potential energy per unit volume (Energy density) = $\frac{1}{2}\in_0 E^2$ in air or vacuum.

$= \frac{1}{2}\in_0 \in_r E^2$ in medium of relative

permitivity or dielectric constant is $\in_r$ or k.

(d) The electric field strength $\vec{E}$ near the surface of uniformly conducting sheet of surface charge density

$$E = \frac{\sigma}{\in_0}$$

(e) The electric field strength E near the surface of infinite long plane thin non-conducting sheet of charge of surface charge density σ is $E = \frac{\sigma}{2\in_0}$

PROPERTIES OF ELECTRIC LINES :

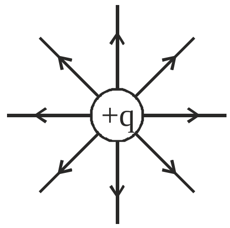
1. Electric lines are continuous curves that emanate from positive charge and terminate to negative charge.
2. Number of electric lines per unit area at a point gives the magnitudeof electric field strength while direction of electric field strength is given by the tangent drawn at the point along the direction of electric lines.
3. Direction and magnitude $\vec{E}$ at a point are unique.
4. No two electriclines intersect at a point.

DIPOLE INTERACTION

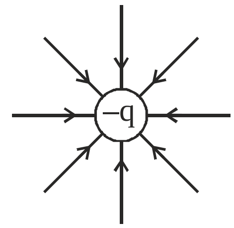
When two dipoles are placed along axial line having same direction of dipole moments then force of mutual attraction is $\frac{1}{4\pi}\frac{6P_1P_2}{r^4}$ where P_1 and P_2 are respective dipole moments while mutual potential energy is $\frac{1}{4\pi}\frac{2P_1P_2}{r^3}$; r = distance between centre S of dipole S when two dipoles are held parallel at a distance r where direction of dipole moments are same then mutual force (repulsion) and potential energy are given by $\frac{1}{4\pi}\frac{3P_1P_2}{r^4}$ and $\frac{1}{4\pi}\frac{P_1P_2}{r^3}$ respectively.

ELECTRIC LINES

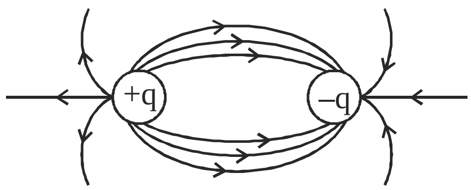
The region around charge at rest in which there hypothetically exists continuous lines called electric lines Electric line is an imaginary line along which a positive charge will move if left free.

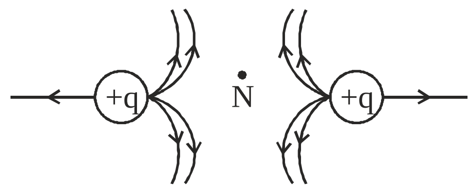


Radially outward



Radially inward





Electric lines never formed closed loop and meet the equipotential surface or conducting surface perpendicularly.

Electrostatic Potential Energy :

When there is a system of charges (assembing by two or more charges) there exists potential energy in the system. Electrostatic potential energy is the work done against electrostatic force to create a system of charges. When two charges q_1, q_2 are separated by a distance r then mutual potential energy

$U = \frac{1}{4\pi \in_o} \frac{q_1 q_2}{r}$ Electric potential at a point.

Electric Potential :

Electric potential at a point defined as the work required by an external agent to displace a unit positive charge from infinity to the point or also defined as work done by electrostatic force to displace from point to infinity.

$$V(r) = -\int_{\infty}^{r} \vec{E} \cdot \vec{dr}$$

Electric Potential and Electric Field Strength :

Negative rate of change of electric potential is the electric field strength along the line there is change in potential

$$-\frac{dv}{dr} = E \text{ or } \vec{E} = E_x\hat{i} + Ey\,\hat{j} + E_z\hat{k}$$

where $$E_x = -\frac{\partial v}{\partial x};\ Ey = -\frac{\partial v}{\partial y};\ E_z = -\frac{\partial v}{\partial z}$$

Negative sign gives the direction of $\vec{E}$ along the line in which increase in distance causes the decrease in potential.

Equipotential surface :

Every point on a surface in at same potential called equipotential surface. Electric lines always join the equipotential surface perpendicularly.

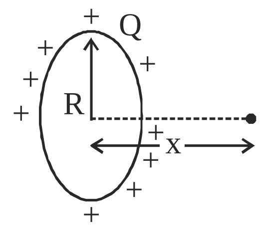
ELECTRIC POTENTIAL DUE TO SOME CHARGE DISTRIBUTIONS :

1. Electric Potential at a distance r due to a point charge q

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

2. Electric potential due to uniformally charged circular ring at a distance x from centre of ring along axis of ring

$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{\sqrt{x^2 + R^2}}$$

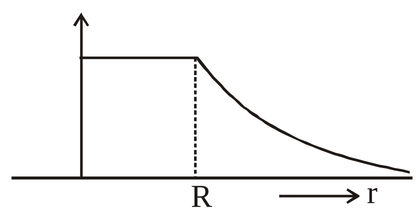


3. Electric potential due to uniformally charged hollow sphere or shell at a distance r

$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R};\ r \le R$$

$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2};\ r > R$$

R = Radius of the hollow sphere



4. Electric potential due to uniformally charged sphere of radius R and volume charge density (ρ)

$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r} = \frac{\rho R^3}{3\varepsilon_o r};\left(\rho = \frac{Q}{\frac{4}{3}\pi R^3}\right) \qquad ;\ r \ge R$$

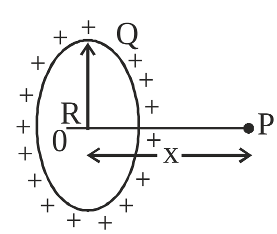
$$V = \frac{Q}{4\pi\varepsilon_o}\left[\frac{3R^2 - r^2}{2R^3}\right] = \frac{\rho\left(3R^2 - r^2\right)}{6\varepsilon_o} \qquad ;\ r < R$$

at centre of informally charged solid sphere,

$$V = \frac{3}{2}\times\frac{1}{4\pi\varepsilon_o}\frac{Q}{R} = \frac{3Q}{8\pi\varepsilon_o R}$$

5. Electrical potential at a distance x from uniformally charged disc

$$V = \frac{\sigma}{2\varepsilon_o}\left[\sqrt{x^2 + R^2} - x\right]$$



σ = surface charge density.

6. Electric Potential due to dipole

At axial point

$$V = \frac{1}{4\pi\varepsilon_o}\frac{P}{r^2};\ P = \text{Dipole moment,}$$

At equatorial point

$$V = 0$$

At general point (r, θ)

$$V = \frac{1}{4\pi\varepsilon_o}\frac{P\cos\theta}{r^2}$$

Current Electricity
(Electric Current, Thermal and Chemical Effect of Current)

ELECTRIC CURRENT

Consider the ends of the conductor be connected to a battery, i.e., an electric field is maintained within the conductor. Now the field acts on the electrons and gives them a resultant motion in the direction of $-\vec{E}$ because a free charge in electric field experiences a force. The flow of electrons constitutes an electric current.

The time rate of flow of charge through any cross section is called current. If a charge Δq passes through an area in time Δt, then the average electric current through the area in this time is defined as

$$i_{av} = \frac{\Delta q}{\Delta t} \qquad ...(1)$$

Now, the instantaneous current is given by

$$i = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt} \qquad ...(2)$$

The SI unit of current is ampere. If one coulomb of charge crosses an area in one second, the current is one ampere. For transient currnet $i = \frac{dq}{dt}$ while for steady current $i = \frac{q}{t}$

The conventional current is in opposite direction to the direction of movment of electrons.

CURRENT DENSITY

The current density j at a point is defined as a vector having magnitude equal to current-per unit area surrounding that point and normal to the direction of charge flow, i.e., direction in which current passes through that point.

If $\overrightarrow{\Delta S}$ be the area vector corresponding to area ΔS, then $\Delta i = \vec{j}.\overrightarrow{\Delta S}$

The total current through finite surface area S is

$i = \int_s \vec{j}.\overrightarrow{\Delta S}$, If current i is uniformally distributed over an areas and perpendicular to it then $i = \int \vec{j}.\overrightarrow{\Delta S}$

DRIFT VELOCITY

we know that a conductor contains a large number of free electrons or conduction electrons. When electrons leave their atoms and become free, the atoms of the conductor become positively charged and are called positive ions. So, the remaining material is a collection of relatively positive ions known as lattice.

In the absence of any external electric field, the electric current through this area is zero, otherwise the conductor will not remain equipotential.

When an electric field is established between the two ends of the conductor, the free electrons experience an electric force opposite to the field. Due to this force, the motion of electrons is accelerated.

The field does not give an accelerated motion to the electrons but it simply gives them a small constant velocity along the conductor which is superimposed on the random motion of the electrons. So, the electrons drift slowly opposite to the applied field. The net transfer of electrons across a cross section results in current. If the electron drifts a distance l in a long time t. we define drift velocity as

$$v_d = \frac{l}{d} \qquad \text{...(1)}$$

The drift velocity is the average uniform velocity by free electrons inside a conductor by the application of an electric field.

where e is charge of electron with mass m.

[Q force on electron due to electric field, F = e E and acceleration , a = F/m = (e E/m)]

$$\therefore \qquad v_d = \frac{eE}{m}.\tau \qquad \tau \text{ in time between two successive collision.} \qquad \text{...(3)}$$

RELATIONSHIP BETWEEN CURRENT DENSITY AND DRIFT

An electric field is maintained between the two ends of a conductor towards the left. The electrons move towards the right. Let the drift velocity of the electrons be v_d. Suppose there are n charge carriers per unit volume and each charge carrier has a charge e. In time dt, the electron advance a distance l which is given by

$$l = v_d\, dt$$

Now calculate the number of electrons crossing the length l of the conductor in time dt. This will be equal to the number of electrons contained in a volume Al, i.e. Av_d dt.

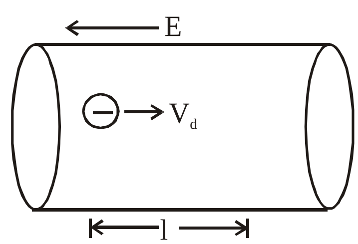
$\therefore$ number of electrons = volume × number of electrons per unit volume

$$= A\, v_d\, dt \times n$$

Hence charge crossing in time dt

= number of electrons × charge on the electron

$= Av_d$ dt n e



Further, current $i = \dfrac{\text{charge crossing in time dt}}{\text{time dt}}$

$$i = \frac{Av_d dt\, n\, e}{dt} = Av_d ne$$

So $\boxed{i = neAv_d}$ ...(1)

The current density is given by

$$j = \frac{i}{A} = \frac{neAv_d}{A}$$

or $\boxed{j = n\, e\, v_d}$ ...(2)

Mobility of free electron is defined in a conductor is drift velocity acquired per unit electric field strength.

applied across the conductor, $\mu = \dfrac{Vd}{E} \Rightarrow Vd = \mu E$

or $i = \mu(neA)E$

OHM'S LAW

The current flowing through a conductor is always directly proportional to the potential difference across its two ends.

$$V \propto i \quad \text{or} \quad V = Ri$$

where R is a constant of proportionality and is called as resistance of the conductor. So, the reistance of a conductor is defined as the ratio of the potential difference applied across the conductor to the current flowing through it, i.e. R = V/i. The value of resistance depends upon the nature of conductor, its dimensions and physical conditions.

We know that drift velocity v_d is given by

$$v_d = \left(\frac{eE}{m}\right)\tau$$

$$= \left(\frac{eV}{ml}\right)\tau \qquad \left(QE = \frac{V}{l}\right) \qquad ...(1)$$

We also know that relation betwen current i and drift velocity v_d is given by

$$i = neAv_d \qquad ...(2)$$

Substituting the value of v_d in eq. (2) from eq. (1), we have

$$i = neA\left(\frac{eV}{ml}\right)\tau = \left(\frac{ne^2A\tau}{ml}\right)V$$

or $$\frac{V}{i} = \frac{ml}{ne^2A\tau} = R \qquad \text{a constant}$$

R is constant for a given conductor, known as resistance of the conductor. Therefore,

$$V = R\,i$$

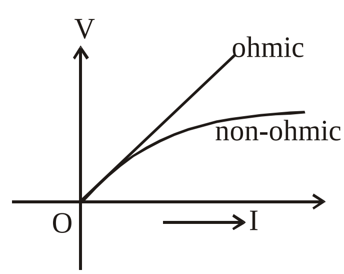
We know that

$$j = n\,e\,v_d$$

Further, $$v_d = \left(\frac{eE}{m}\right)\tau$$

$\therefore$ $$j = ne\left[\left(\frac{eE}{m}\right)\tau\right] = \frac{ne^2\tau}{m}E$$

or $$j = \sigma E \qquad \text{where } \sigma = \frac{ne^2\tau}{m}$$



The constant σ is called as electrical conductivity and is temperature dependent. So we have

. $$\boxed{j = \sigma E}$$

This equation is known as Ohm's law.

V-I line is not a straight line.

RESISTIVITY AND CONDUCTIVITY

The reistance of a conductor is directly proportional to its length l and inversely proportional to the area of cross section A, i.e.

$$R \propto l \qquad \text{and} \qquad R \propto \frac{1}{A}$$

$$R \propto \frac{l}{A} \qquad \text{or} \qquad R = \rho\left(\frac{l}{A}\right)$$

If l = 1 and A = 1 , then $R = \rho$

Therefore, specific resistance of the material of a conductor is equal to the resistance offered by the wire of unit length and unit area of corss section of the material of wire. Its unit is ohm-metre. This is constant for a material.

The reciprocal of resistivity of the material of a conductor is called as conductivity

$$\sigma = \frac{1}{\rho} = \frac{j}{E}$$

The unit of conductivity is ohm^{-1} metre $^{-1}$ $(\Omega m)^{-1}$. Good conductors of electricity have large conductivity than insulators.

FACTORS AFFECTING ELECTRICAL RESISTIVITY

The drift velocity v_d in magnitude of electrons is given by

$$v_d = \left(\frac{eE}{m}\right)\tau \qquad ...(1)$$

The current flowing through the conductor due to drift of electrons is given by

$$i = n A e v_d$$

$$= nAe\left(\frac{eE}{m}\right)\tau = \frac{nAe^2E}{m}\tau \qquad ...(2)$$

If V be the potential difference applied across the two ends of the conductor, then

$$E = \frac{V}{l} \qquad ...(3)$$

From eqs. (2) and (3) we get,

$$i = \frac{nAe^2V}{ml}\tau$$

or $$R = \frac{V}{i} = \frac{m}{ne^2\tau}\left(\frac{l}{A}\right) \text{ or } R = \rho\left(\frac{l}{A}\right) \qquad ...(4)$$

where $\rho = \text{resistivity} = \dfrac{m}{ne^2\tau}$

The resistivity ρ of the material of a conductor depends upon the following factors:

(i) It is inversely proportional to the number of free electrons per unit volume n of the conductor, i.e., depends on the nature of material.

(ii) It is inversely proportional to the average relaxation time τ of free electrons in the conductor. As τ is a function of temperature and hence the resistivity of a conductor depends on its temperature. The resistivity increases with the increase in temperature of conductor.

TEMPERATURE DEPENDENCE OF RESISTIVITY

Small temperature variations, the variation of resistivity can be expressed as

$$\rho(T) = \rho(T_0)\left[1 + \alpha(T - T_0)\right]$$

where $\rho(T)$ and $\rho(T_0)$ are the resistivities at temperature T and T_0 respectively and α is temperature coefficient of resistivity.

The resistance of a conductor is given by

$$R = \rho\left(\frac{l}{A}\right)$$ $\rho(T)$ = Resistivity at temperature T

$\rho(T_0)$ = Resistivity at temperature of T_0.

The resistance depends on the length and area of cross section besides resistivity. When the temperature increases, the length and area of cross section also increases are quite small and the factor (l/A) may be treated as constant. Therefore,

$$R \propto \rho$$ R(T) = Restance at temperature T.

Now, $R(T) = T(T_0)\left[1+\alpha(T-T_0)\right]$ R(T) = Resistance at temperature T_0.

where α is known as temperature coefficient of resistance.

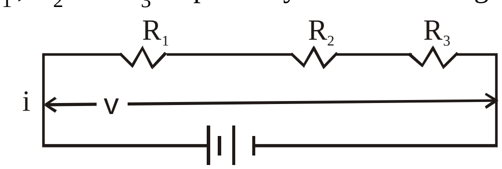
Grouping of Resistance

(A) Resistances in Series

In the shown figure. shows the series combination of three resistors having resistanceR_1 and R_3 and. A battery of e.m.f. E is connected across this combination. In this combination, the same current i is flowing through each resistor.

Let V_1, V_2 and V_3 be the potential differences across R_1, R_2 and R_3 respectively. Now according to Ohn's

V_1 = i R_1, V_2 = i R_2 and V_3 = i R_3



Further $V = V_1+V_2+V_3$

$= iR_1 + iR_2 + iR_3$

$= i(R_1 + R_2 + R_3)$ ...(1)

If R_2 be the equivalent resistance of series combination, then the potential difference V across the combination will be

$V = i R_S$ ...(2)

Compairing eqs. (1) and 2), we get

$R_S = R_1 + R_2 + R_3$ ...(3)

In series combination, the following points should be remembered

(i) The current is same in every part of the circuit

(ii) The total reistance of the circuit is equqal to the sum of indivd resistances connected in the circuit.

(iii) The total resistance of series combination is more than the greatest resistance of the circuit.

(iv) The potential difference across any resistor is proportional to its resistance, i.e. $v_1 : v_2 : v_3 = R_1 : R_2 : R_3$

(B) Resistance in Parallel

Fig. shows a parallel combination of three resistors having resistance R_1, R_2 and R_3 battery of e.m.f. E is connected points A and B. Let i be the current from the battery and i_1, i_2 and i_3 be the currents through resistance R_1, R_2 and R_3 respectively. Then

$i = i_1 + i_2 + i_3$ ...(5)

As shown in the figure, the potential difference across each resistance is V. Applying Ohm's law, we have

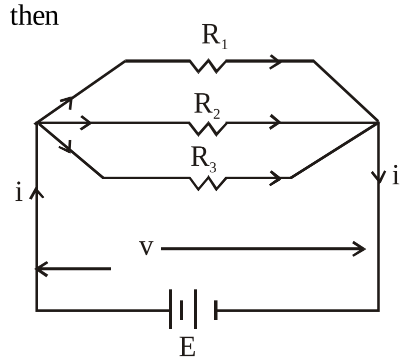
$V = i_1R_1 = i_2 R_2 = i_3 R_3$

or $i_1 = \frac{V}{R_1}$, $i_2 = \frac{V}{R_2}$ and $i_3 = \frac{V}{R_3}$

Substituting these values in eq. (5), we get

$$i = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \quad ...(6)$$

Let R_p be the equivalent resistance of the parallel combination, then



$$V = i\,R_p \quad \text{or} \quad i = \frac{V}{R_p} \quad ...(7)$$

Comparing eqs. (6) and (7), we get

$$\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

or

$$\boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \quad ...(8)$$

The reciprocal of equivalent resistance of parallel combination is equal to the sum of the reciprocals of the individual resistances.

The following points should be remembered in case of parallel combination:

(i) The potential difference across each resistance is the same

(ii) The current is different in different resistances. The sum of the currents in different resistances is equal to the main currents in the circuit, i.e.,

$$i = i_1 + i_2 + i_3 \quad ...(9)$$

(iii) The current through any registor is inversely proportional to its resistance.

(iv) The total resistance in parallel combination is less than the least resistance used in the circuit.

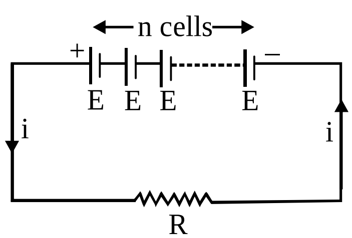
BATTERY AND ELECTROMOTIVE FORCE (E.M.F.)

A battery is a device which maintains a constant potential difference between its two terminals.

The potential difference between the two terminals. The potential difference between two terminals provides an electrostatic field Ee between tw terminals the emf is defined as the work done while by cell a unit positive charge flows from –ve plate to +ve plate.

GROUPING OF CELLS

(1) Sereis grouping : Fig shows a series combination of n cells each of e.m.f. E and internal resistance r.



$\therefore$ current through the circuit $= \dfrac{\text{total emf}}{\text{total resistance}}$

or

$$i = \frac{nE}{(R + nr)} \quad ...(1)$$

(i) If R>>r, r i.e., the effective internal resistance is as far far less than external resistance r can be neglected in comparison of R, then

$$i = \frac{nE}{R} = \text{n times the current drawn from single cell.} \quad ...(2)$$

(ii) If r >> R, i.e., the effective internal resistance is far far greater than external resistance, then R can be neglected in comparison to hr, then

$$i = \frac{nE}{nR} = \frac{E}{r} \qquad ...(3)$$

The current in the circuit is the same as due to a single cell, so n of useful

(iii) If in series grouping of n cells, s cells are reversed, then

$$E_{eq} = (n-s)E - sE = (n-2s)E$$

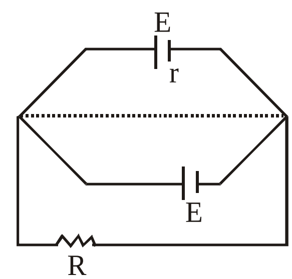
Total resistance of the circuit = (R+n r)

$$\therefore \qquad i = \frac{(n-2s)E}{(R+nr)} \qquad ...(4)$$

(2) Parallel grouping

$\therefore$ Total Resistance of the circuit $= [R + (r/n)]$ [$\because$ R and (r/n) are in series]

Now, current in the cirut, $i = \dfrac{E}{R + \left(\dfrac{r}{n}\right)}$ ...(5)



(i) If R>> (r/n), i.e. (r/n) can be neglected in comparison to R, then

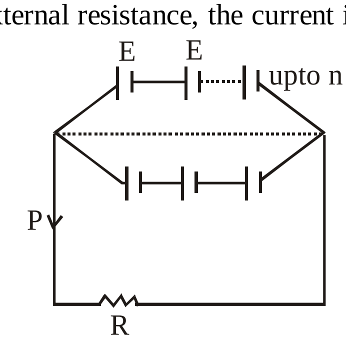
$$i = \frac{E}{R} \qquad ...(6)$$

Therefore, the current in the circuit is equal to the circuit current due to a single cell.

(ii) If (r/n) >> R, i.e., R can be neglected in comarison to (r/n), then

$$i = \frac{nE}{r} \qquad ...(7)$$

Therefore, if the effective internal resistance is greater than the external resistance, the current in the circuit is equal to n time the circuit current due to a single cell.



(3) Mixed Grouping

Total resistance of circuit $= R + \left(\dfrac{nr}{m}\right)$

The current i in the circuit is given by

$$i = \frac{nE}{R + (nr/m)} = \frac{nmE}{mR + nr} = \frac{NE}{mR + nr}$$

The current i in the circuit will be maximum when the factor (m R+ n r) in the denominator is minimum. The denominator is minimum when mR = n r

$$\therefore \qquad R = \left(\frac{nr}{m}\right)$$

Hence current will be maximum when external resistance is equal to the total internal resistance of all the cells.

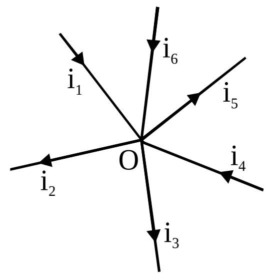
KIRCHHOFF'S LAW

Ohmm;s law is unable to give current in complicated cirucit. Kirchhoff's in1842, gave two general laws which are extremely useful in electrical circuits. There are.

(i) The algebraic sum of the currents at any junction in a circuit is zero, i.e.

$$\sum i = 0$$

This means that there is no accumulation of electric charge at any point in the circuit.



(ii) In any closed circuit,the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total emf in the circuit i.e.,

$$\sum iR = \sum E$$

The product of current and resistance is taken as positive when we traverse in the direction of current. The emf is taken positive when we traverese from negative to positive electrode through electrolyte.

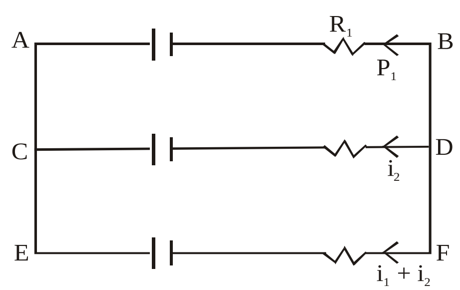
Let us apply Kirchhoff's second law to figure shown

For the mesh ACDBA,

$$i_1R_1 - i_2R_2 = E_1 - E_2 \quad ...(i)$$

For the mesh EFDCE

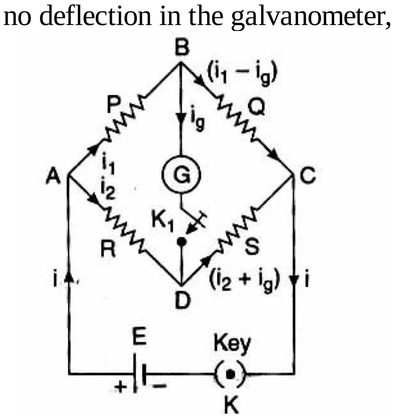
$$i_2R_2 + (i_1 + i_2)R_3 = E_2 \quad ...(ii)$$



From FFBAE, $i_1 R_1 + (i_1 + i_2) R_3 = E_1$ ...(iii)

CONDITION OF BALANCE IN WHEATSTONE'S BRIDGE

When there is no deflection in the galvanometer, the bridge is known as balanced. The condition of balance is given by

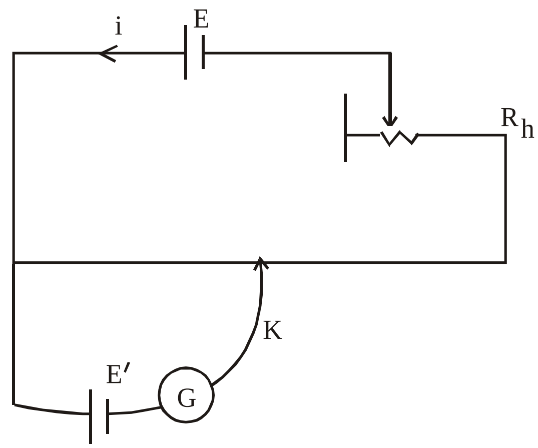


$$\frac{P}{Q}=\frac{R}{S} \text{ or } \frac{P}{R}=\frac{Q}{S}$$

When galvanometer and battery are exchanged then still the galvanometer shows no reflection.

POTENTIOMETER

Potentiometer is a device which is used to meosure the potential difference more accurately then an ideal voltmeter. The potentiometer does not any current from source. Hence it is equivalent to an draw ideal voltmeter.



Let v be potential difference across certain portion of wire. Let i be current through portion of stretched wire

$v = iR$ --- (i) $R = \rho.\frac{l}{A}$

$v = i\rho\left(\frac{l}{A}\right)$ For i = constant through wire of uniform cross-section

If L = total length of potentiometer wire

$\varepsilon =$ emf of driving cell or standard cell.

$$K = \frac{\varepsilon}{L}$$

$$V = K.l \Rightarrow v = \frac{\varepsilon}{L}.l$$

Comparision of emfs of two cells can found by potentiometer $\frac{\varepsilon_1}{\varepsilon_2}=\frac{l_1}{l_2}$ where l_1 and l_2 are balancing lengths while cells of ε_1 and ε_2 are attached respectively.

The internal resistance r of a cell is given by $r=\left(\frac{i_1}{i_2}-1\right)R;$ where l_1 and l_2 are balancing lengths and R is external resistance

HEATING EFFECT OF CURRENT

The phenomenon in which heat energy is produced in a conductor due to flow of electric current (flow of electrons) is known as heating effect of current.

Consider a resistor of resistance R. Let a potential difference V is maintained at its ends and a current i is flowing through it for a time t. If the charge q flows through it in time t, then

$q = i\,t$ [charge = current × time]

Now, the workdone by electric field on free electrons in time t is given by

$= V\,(i\,t)$ joule

$= (i\,R)\,(i\,t) = i^2\,R\,t$ joule

The workdone by electric field is converted in thermal energy of resistor through the collisions with ions or atoms. The thermal energy is generally referred to as heat produced in resistor. So, the amount of heat produced (H) is given by

$$\boxed{H = W = i^2 Rt \text{ joule}}$$

In calorie, the heat produced is given by

$$H = \frac{i^2 Rt}{4.18} \text{ calorie}$$ This is expression for joule's law of heating...(4)

JOULE'S LAWS OF HEATING

Joule's laws :

(a) The heat produced in a given resistor in a given time is proportional to the square of current flowing in it, i.e.,

$$H \propto i^2 \qquad ...(1)$$

(b) The heat produced in a given resistor in a given time by a given current is directly proportional to the resistance, i.e.,

$$H \propto R \qquad ...(2)$$

(c) The heat produced in a given resistor by a given current is proportional to time t for which the current is passed, i.e.,

$$H \propto t \qquad ...(3)$$

ELECTRIC POWER

The electric power is defined as the rate at which work is done by the source of e.m.f. in maintaining the current in an electric circuit.

If an amount of work W is done in maintaining electric current in a circuit for a time t, then electric power is given by

$$P = \frac{W}{t} \qquad ...(1)$$

Let a current i ampere flows through a conductor for a time t second under a potential difference V volt. The workdone for maintaining the current is given by

$$W = V\, i\, t \text{ joule} \quad ...(2)$$

So, the power of an electric circuit is one watt when one ampere current flows through it under a potential difference of one watt.

1 watt = 1 joule/sec.

The bigger units of electric power are

1 kW = 10^3 W and 1 MW = 10^6 W

Commercial unit of power is horse power (HP).

1 HP = 746 watt.

Other expression for power are :

$$P = i^2 R \text{ and } P = \frac{V^2}{R}$$

$$\boxed{P = Vi = i^2 R = \frac{V^2}{R}} \quad ...(3)$$

(i) When resistances are connected in series.

In this case, the current in each resistance will be the same. Hence from eq. (3), we have

$$P \propto V \text{ and } p \propto R$$

This shows that in series connections, the potential difference and power consumed will be more in larger resistance.

(ii) When resistances are connected in parallel.

In this case, the potential difference V across each resistance is same. Hence from eq. (3), we have

$$P \propto \left(\frac{1}{R}\right) \text{ and } i \propto \left(\frac{1}{R}\right)$$

This shows that in parallel connections, the current and power consumed will be more in smaller resistance.

APPLICATIONS OF HEATING EFFECT OF CURRENT

(1) Series combination of bulbs :

Consider a series combination of three bulbs of powers P_1, P_2 and P_3 which are manufactured for working on a supply of V volt. The resistances of these bulbs are respectively.

$$R_1 = \frac{V^2}{P_1}, R_2 = \frac{V^2}{P_2} \text{ and } R_3 = \frac{V^2}{P_3} \quad ...(1)$$

$$\therefore \text{ total resistance, } R = R_1 + R_2 + R_3 \quad ...(2)$$

Effective power $\frac{V^2}{P} = \frac{V^2}{P_1} + \frac{V^2}{P_2} + \frac{V^2}{P_3}$

or
$$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} \quad ...(3)$$

Current through each bulb

$$i = \frac{V}{R_1 + R_2 + R_3} \quad ...(4)$$

The brightness of these bulbs are

$$H_1 = i^2 R_1,\ H_2 = i^2 R_2 \text{ and } H_3 = i^2 R_3 \quad ...(5)$$

This shows that the bulb with highest resistance will glow with maximum brightness. Further $R \propto \frac{1}{P}$, therefore, the bulb of lowest power or wattage will have highest resistance and will glow with maximum brightness.

(2) Parallel combination of bulbs :

Consider a parallel combination of three bulbs of powers P_1, P_2 and P_3 respectively which are manufactured for working on a supply voltage V volt. In this case, we have

$$R_1 = \frac{V^2}{P_1},\ R_2 = \frac{V^2}{P^2} \text{ and } R_3 = \frac{V^2}{P_3} \quad ...(6)$$

Now $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ ...(7) (where R is effective resistance of the circuit)

From eqs. (6) and (7), we get

$$P = P_1 + P_2 + P_3 \quad ...(8)$$

The brightness of three bulbs will be respectively

$$H_1 = \frac{V^2}{R_1},\ H_2 = \frac{V^2}{R_2} \text{ and } H_3 = \frac{V^2}{R_3}$$

The resistance of highest wattage (power) bulb is minimum and hence the bulb of maximum wattage will glow with maximum brightness.

SEEBECK EFFECT

Seeback discovered that if two dissimilar metals (say bars or wires of copper and iron) are joined in series to form a closed circuit, and their two junctions are maintained at different temperatures, an e.m.f. is developed.

The current produced in this way without the use of a cell or a battery is known as thermoelectric current and the e.m.f. responsible for thermoelectric current is known as thermo e.m.f. This effect is known as Seebeck effect. The arrangement of wires is known as thermocouple.

Seebeck observed that the magnitude and direction of thermo e.m.f. depends on

(i) the nature of metals forming the thermocouple.

(ii) differnece in temperatures of two junctions.

Seebeck also observed that if the hot and cold junctions are interchanged then the direction of thermoelectric current is also reversed. This shows that seebeck effect is reversible effect.

Thermoelectric Series

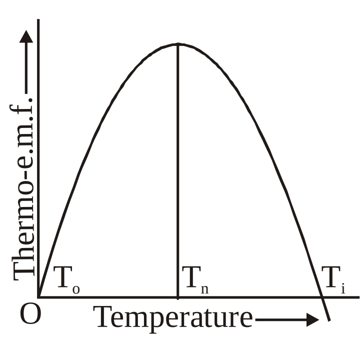
Seebeck arranged a large number of metals in a series such that when any two of these metals form a thermocouple, the current at the cold junction is from the metal occurring earlier in the series to the metal occurring later in the series. The series is known as themoelectric series. The series is as follows :

Antimony, nichrome, iron, zinc, copper, gold, silver, lead, aluminium, mercury, platinum, nickel constantan, bismuth.

Variation of thermo e.m.f. with temperature

(Neutral temperature and temperature of inversion) :

If a graph is plotted between the temperature of the hot junction and the thermo e.m.f. e, the cold junction being kept at 0°C, a parabolic curve is obtained as shown in fig. The thermo e.m.f.



increases with the temperature of hot junction and becomes maximum at a particular temperature. The temiperature of the hot junction at which thermo e.m.f., in a thermocouple is maximum is known as neutral temperature T_n for that couple.

Thus the temperature at which the thermo e.m.f. is zero is known as inversion temperature or temperature of inversion.

Beyond this temperature the e.m.f. again increases but in the reverse direction.

The temperature of inversion depends upon

(i) the nature of materials forming the thermocouple

(ii) the temperature of cold junction.

The thermo e.m.f., e varies with temperature according to the following equation.

$$e = aT + bT^2 \qquad ...(1)$$

$$\frac{de}{dT} = a + 2bT$$

at $T = T_n$, e is maximum, i.e., $\frac{de}{dT} = 0$. Thus

$$0 = a + 2bT_n$$

or $$T_n = -\frac{a}{2b} \qquad ...(2)$$

Further at $T = T_i, e = 0.$ Thus from equation (1)

$$0 = aT_i + bT_i^2$$

$$T_i = -\frac{a}{b} \qquad ...(3)$$

From equations (2) and (3)

$$T_i = 2T_n$$

Thus the inversion temperature T_i is as much above the neutral temperature as the temperature of the cold junction (0°C) is below it. T_i is therefore not a constant for the given thermocouple but depends upon the temperature of the cold junction.

If T_0 be the temperature of cold junction, then

$$T_i - T_n = T_n - T_0 \quad \text{or} \quad T_i = 2T_n - T_0$$

$$\therefore \quad \boxed{T_n = \frac{T_i + T_0}{2}}$$

Peltier's Effect :

Peltier discovered an effect which is the converse of Seebeck effect. When a current is passed across the junction of two dissimilar metals, heat is evolved at one junction and absorbed at the other, i.e., one junction is heated and the other is cooled. This effect is known as Peltier effect.

Peltier Coefficient :

The amount of heat (in joules) absorbed or evolved at a junction of two different metals when one coulomb of charge flows at the junction is called the Peltier coefficient. It is denoted by

$$\pi = \frac{\Delta H}{\Delta Q} = \frac{\text{Peltier heat}}{\text{charge flowing}}$$

This coefficient is not constant but varies as the absolute temperature of the junction. It also depends on the metal used.

If a charge q coulomb passes across a junction having a peltier coefficient π volt, then the energy absorbed or evolved at the junction = πq joule.

If V be the junctional P.D. in volt, then

energy absorbed or evolved = Vq joule

$$\therefore \quad \pi q = Vq$$

$$\pi = V$$

Hence the Peltier coefficient expressed in joule per coulomb is numerically equal to the junctional P.D. in volt.

Thomson effect :

Thomson observed that when two parts of a single conductor are maintained at different temperatures and a current is passed through it, heat may be absorbed or evolved in different sections of may be absorbed or evolved in different sections of the conductor. This effect is called Thomson effect.

According to Thomson effect, heat is absorbed or evolved in excess of Joule heat when a current is passed through an unequally heated conductor.

Thomson coefficient :

Thomson coefficient is defined as the amount of heat evolved or absorbed when a unit positive charge is passed through a part of the wire whose ends are maintained at a unit temperature difference. This is denoted by σ.

Let a charge ΔQ is passed through a small part of the wire having a temperature difference ΔT between the ends. Thomson heat is

$$\Delta H = \sigma(\Delta Q)(\Delta T)$$

or

$$\sigma = \frac{\Delta H}{(\Delta Q)(\Delta T)}$$

CHEMICAL EFFECT OF ELECTRIC CURRENT

It has been observed that some liquids allow the passage of current through them while some do not show such behaviour. On the basis of their electrical behaviour liquids can be divided into the following three categories :

(i) The liquids which do not allow the current to pass through them. For example distilled water, vegetable oil etc.

(ii) The liquids which allow the current to pass through them but do not dissociate into ions. For example, mercury.

(iii) The liquids which allow current to pass through them and also dissociate into ions. For example salt solutions, acid and bases. Such liquids are called electrolytes.

Thus when a current is passed through an electrolyte, it dissociates into ions. This is known as chemical effect of current.

FARADAY'S LAWS OF ELECTROLYSIS

The relation between quantity of electric charge passed and the amount of ion deposited at the electrode is given by Faraday's laws of electrolysis. There are two laws :

Faraday's first law :

According to Faraday's first law, the mass of the substance deposited or liberated in electrolysis is directly proportional to the charge passed through the electrolyte.

Let m be the mass of a substance deposited or liberated at an electrode when a charge q is passed through the electrolyte. Thus

$$m \propto q \quad \text{or} \quad m = Zq \qquad ...(1)$$

where Z is constant of proportionality and is known as electrochemical equivalent (E.C.E.) of the substance.

If i be the current passed through the electrolyte for a time t, then

$$q = i\,t \qquad ...(2)$$

From eqs. (1) and (2)

$$m = Z\,i\,t \qquad ...(3)$$

If q = 1 coulomb, then Z = m

Thus the electrochemical equivalent (E.C.E.) of a substance may be defined as the mass of the substance liberated or deposited on an electrode during electrolysis when one coulomb of charge is passed through the electrolyte.

The S.I. unit of E.C.E. is kg/coulomb. But generally this is expressed in gram/coulomb (gC^{-1}). The value of E.C.E. of copper and silver are 3294×10^{-7} gC^{-1} and 11180×10^{-7} gC^{-1} respectively.

Faraday's second law :

According to Faraday's second law, when the same amount of charge is passed through different electrodes, the masses of different substances deposited or libarated at the electrodes are proportional to their chemical equivalents.

If m_1 and m_2 be the masses of the substances deposited or liberated and E_1 and E_2 be their respective chemical equivalent, then

$$\frac{m_1}{m_2} = \frac{E_1}{E_2} \quad \text{or} \quad \frac{Z_1 i\,t}{Z_2 i\,t} = \frac{E_1}{E_2} \quad \text{or} \quad \frac{Z_1}{Z_2} = \frac{E_1}{E_2}$$

The chemical equivalent of the substance is defined as the ratio of atomic weight to the valency. Thus

$$E = \frac{\text{atomic weight}}{\text{valency}}$$

The atomic weight of silver is 108 and its valency is 1. Therefore, its chemical equivalent is 108. Similarly, the chemical equivalent of copper is 31.75.

FARADAY CONSTANT

From Faraday's second law

$$\frac{Z_1}{Z_2} = \frac{E_1}{E_2} \quad \text{or} \quad \frac{E_1}{Z_1} = \frac{E_2}{Z_2}$$

$$\therefore \quad \frac{E}{Z} = \text{a constant} = F \text{ (Faraday constant)}$$

Thus the ratio of $\left(\frac{E}{Z}\right)$ is same for all substances and is called as Faraday constant.

Now, $$F = \frac{E}{Z} = \frac{E}{\left(\frac{m}{q}\right)} = \frac{Eq}{m}$$

So, the Faraday constant is equal to the charge required to liberate one gram equivalent of substance at an electrode during electrolysis. Its value is 96500 C/gram equivalent.

In case of copper, E.C.E. = 0.0003294 gC^{-1} and E = 31.75 g

$\therefore$ Faraday constant $= \dfrac{31.75}{0.0003294}$

$= 96500$ C/gram equivalent.

The charge of 1 mole of electrons is called one faraday. So

one faraday $= N_A \times e$

$= (6.023 \times 10^{23}) \times (1.602 \times 10^{-19}\ \text{C})$

$= 96500$ C.

Therefore, faraday is unit of charge (1 faraday = 96500 C) while the quantity charge per mole of electrons is called Faraday constant (F = 96500 C/mole or 1 faraday).

Moving Charges and Magnetism

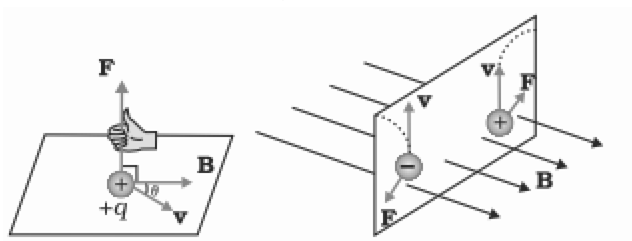
When charge moves it also produces magnetic field. According to Oersted, he concluded that moving charges or currents produced a magnetic field in the surrounding space.

Formulae:

Lorentz Force: Combination of two forces –electrostatic force and magnetic force. Hence,

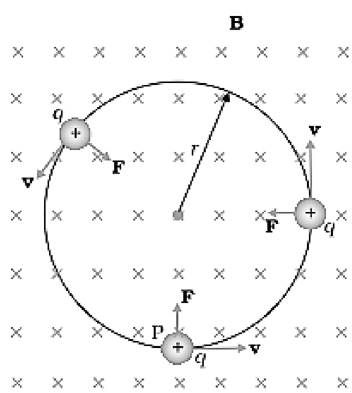
$$\vec{F} = \vec{F}_{Electric} + \vec{F}_{Magnetic} = q[\vec{E} + (\vec{v} \times \vec{B})] = qE + qvB\sin\theta$$

- Magnetic force on a straight current-carrying conductor: $\vec{F} = I(\vec{l} \times \vec{B})$
- For an any shaped wire: $\vec{F} = I\sum_j \vec{dl}_j \times \vec{B}$



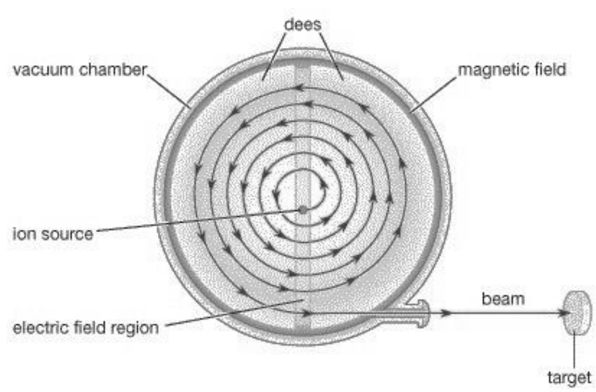
Motion in Magnetic Field:

- When moving charge is kept in motion in magnetic field then it follows helical path.



Hence, $\frac{mv^2}{r} = qvB$, $r = \frac{mv}{qB}$

- Angular Frequency, $\omega = 2\pi v = \frac{qB}{m}$



Cyclotron:

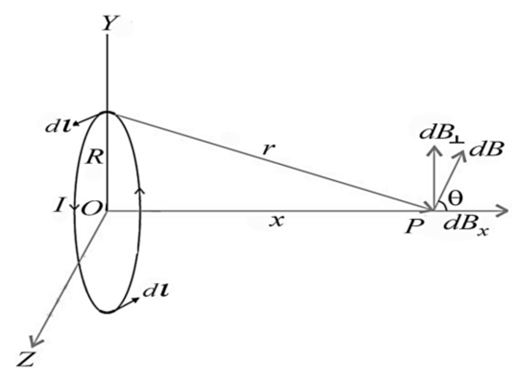
- Cyclotron Frequency, $v_c = \dfrac{qB}{2\pi m}$
- Time Period, $T = \dfrac{2\pi m}{qB}$
- Condition for resonance: Applied voltage frequency(V_a)=Cyclotron Frequency(V_c)
- Velocity of accelerated ion, $v = \dfrac{qBR}{m}$
- Kinetic Energy of ions: $\dfrac{1}{2}mv^2 = \dfrac{q^2B^2R^2}{2m}$

Bio Savart's Law:

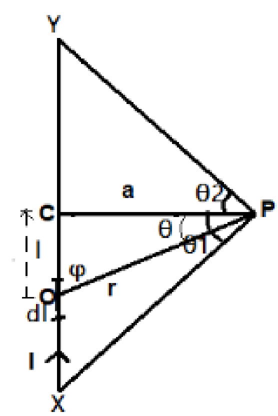
- Magnetic field due to current element dl at a distance r, $dB = \dfrac{\mu_o}{4\pi}\dfrac{|\vec{dl}|\times\vec{r}}{r^3}$,

  $|dB| = \dfrac{\mu_o}{4\pi}\dfrac{|dl|\sin\theta}{r^2}$

  where



$\dfrac{\mu_o}{4\pi} = 10^{-7}\,Tm/A$ and μ_o=permeability of free space



- Magnetic field at the axis of circular current carrying wire:

  $B = B_x\hat{i} = \dfrac{\mu_o IR^2}{2(x^2+R^2)^{3/2}}\hat{i}$

  And, at x=0, center of the loop $B_o = \dfrac{\mu_o I}{2R}\hat{i}$

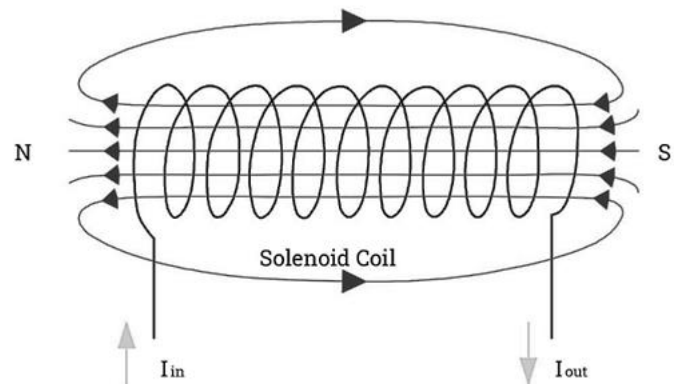
- Magnetic field due to a current in straight conductor:

  $B_0 = \dfrac{\mu_0 i}{4\pi r}\left(\sin(\theta_1) + \sin(\theta_2)\right)$

- Magnetic field at perpendicular bisector of a straight current carrying conductor: $B_o = \frac{\mu_o}{4\pi}.\frac{i}{r}(2\sin\theta)$
- Magnetic field due to a semi-infinite wire: $B_o = \frac{\mu_o}{4\pi}.\frac{i}{r}$
- Magnetic field due to a wire at the axial position: B=0

Ampere's Circuital Law: $\oint B.dl = \mu_o I$

- Magnetic field due to an infinite long straight wire of radius ***a,*** current ***I*** and at a point ***r***:
    a. At r<a, $B = \frac{\mu_o Ir}{2\pi a^3}$
    b. r=a, $B = \frac{\mu_o I}{2\pi a}$
    c. r>a, $B = \frac{\mu_o I}{2\pi r}$
- Magnetic field due to solenoid:



$B = \mu_o nI = \frac{\mu_o NI}{L}, N =$ number of turns and L= length of rod

- Magnetic field due to Toroid: $B = \mu_o nI = \frac{\mu_o NI}{2\pi R}$
- Force between two parallel currents carrying wire I_1 and I_2 at distance d:

$$F_{21} = I_2 L B_2 = \frac{\mu_o I_1 I_2}{2\pi d} L \text{ and } F_{21} = -F_{12}$$

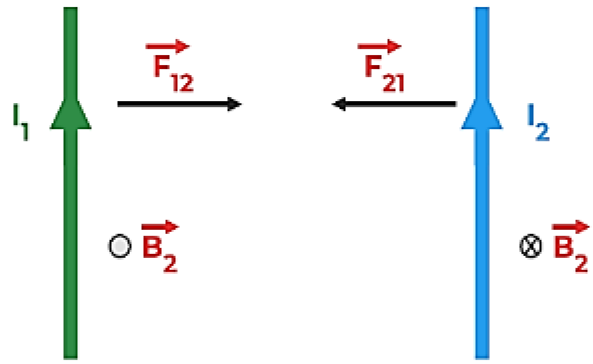
Torque on current loop, magnetic dipole:

- Torque on a rectangular current loop in uniform magnetic field:
    a. When plane of the loop is along with magnetic field: $\tau = IAB$, where A= area of rectangle and B= magnetic field.
    b. When plane of the loop is not along with magnetic field: $\tau = IAB\sin\theta$
- Magnetic moment of current loop is: $m = IA$
- Therefore, torque would be: $\tau = m \times B$
- In case of electrostatic then it has electric dipole of dipole moment p_e in electric field E: $\tau = p_e \times E$

Circular Current loop as a magnetic dipole: Magnetic field in terms of magnetic moment:

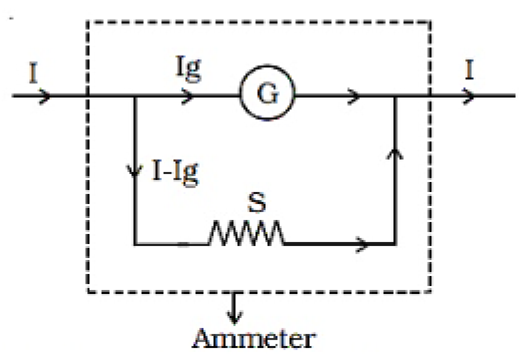
$$B = \frac{\mu_o m}{2\pi x^3}$$, where *m=IA* and *x*= distance from the dipole.

The magnetic dipole moment of a revolving electron: $\mu_l = \frac{e}{2m_e}(m_e vr) = \frac{e}{2m_e} l$ where $l = m_e vr$ and *l*= magnitude of the angular momentum, from Bohr's hypothesis $l = \frac{nh}{2\pi}$.



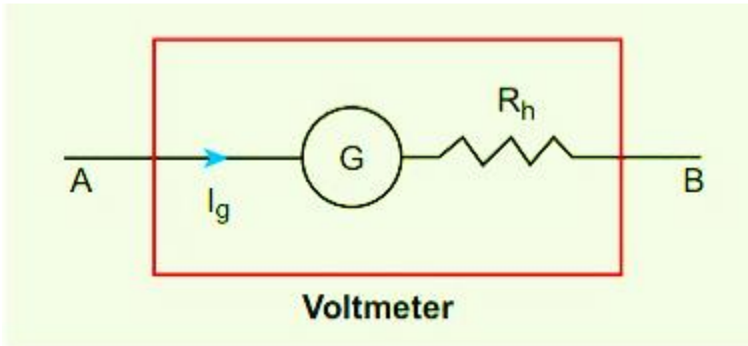
The moving coil galvanometer:

- Torque produced when current flows through the coil: $\tau = NIAB$
- Counter torque produced by spring that balances the magnetic torque: $k\phi = NIAB$ where k= torsional constant of the spring i.e., the restoring torque per unit turns and ϕ is deflection on the scale by a pointer attached to spring. Hence, $\phi = \frac{NAB}{k} I$
- **For measuring currents**, the galvanometer has to be connected in series. In this case we connect a low resistance called shunt in parallel with the galvanometer coil.



- **For measuring voltage**, the galvanometer also can be used as a voltmeter to measure the voltage throughout a given phase of the circuit. For this the voltmeter should be connected in parallel with that phase of the circuit.

$$\frac{\phi}{V}=\left(\frac{NAB}{k}\right)\frac{I}{V}=\left(\frac{NAB}{k}\right)\frac{1}{R}$$



CHAPTER: MAGNETISM & MATTER

102) Magnetic Field strength at a point due to Bar Magnet	$B = \frac{\mu_0}{4\pi} . \frac{M}{r^3} . \sqrt{1 + 3Cos^2\theta}$	*T*
103) Magnetic Field along axial line of short Bar Magnet	$B = \frac{\mu_0}{4\pi} . \frac{2Mr}{4\pi(r^2 - l^2)^3}$	*T*
104) Magnetic Moment	$M = m. 2l$	
105) Magnetic Field along 7quatorial line of short Bar Magnet	$B = \frac{\mu_0}{4\pi} . \frac{M}{4\pi(r^2 + l^2)^{\frac{3}{2}}}$	*T*
106) Torque on Magnetic Dipole in Uniform Magnetic Field	$\tau = MB\ Sin\theta$	*Nm*
107) Time period of oscillation of Bar magnet in Uniform Magnetic Field	$T = 2\pi \sqrt{\frac{I}{MB}}$	*s*
108) Potential Energy of Dipole due to current loop in Magnetic Field	$U = -\vec{M}.\vec{B} = -MB\ Cos\theta$	*J*
109) Workdone in rotating Magnetic Field	$W = \Delta U = -MB(Cos\theta 2 - Cos\theta 1)$	*J*
110) Angle of Dip Vs Horizontal & Vertical components of Earth's Magnetic Field	$B_V = B\ Sin\delta,\quad \frac{B_V}{B_H} = Tan\delta,$ $B = \sqrt{{B_V}^2 + {B_H}^2}$	*°, T*
111) Atom as Magnetic Dipole	$M = n.\frac{eh}{4\pi m} = n.\mu_B$ $\mu_B \rightarrow Bohr\ magneton = 9.27\ X\ 10^{-24} J/T$	*Am^{-2}*
112) Relative Permeability	$\mu_r = \frac{B}{B_0} = \frac{\mu}{\mu_0}$	*WbA^{-1}*
113) Magnetizing Force or Magnetic Intensity	$H = ni = \frac{B}{\mu}$	*Am^{-1}*
114) Intensity of Magnetization	$\vec{I} = \frac{\vec{M}}{V}, I = \frac{m}{A}$	*Am^{-1}*

115) Magnetic Susceptibility	$$\chi = \frac{\boldsymbol{I}}{\boldsymbol{H}}$$	***NA***
116) Curie's Law	$$\chi_m = \frac{\boldsymbol{I}}{\boldsymbol{H}} = \frac{\boldsymbol{C}}{\boldsymbol{T}}$$	***NA***

EMI

Magnetic Flux:

(i) The magnetic flux through a small area $d\vec{A}$ placed in a magnetic field $\vec{B}$ is defined as:

$$d\phi = \vec{B}.d\vec{A} = B(dA)\cos\theta$$

(ii) The magnetic flux can be positive, negative or zero depending on the angle θ. For $\theta = 90^o$ and $\phi = 0$. Thus, whenever the angle between area vector, and magnetic field is 90°, the flux is zero, i.e., whenever the plane of the surface is parallel to $\vec{B}$, the flux is zero. The flux is positive for $0^o \le \theta \le 90^o$ and negative for $180^o \ge \theta \ge 90^o$.

(iii) The magnetic flux through a closed surface is always zero, i.e.,

$$\phi = \oint \vec{B}.d\vec{A} = 0$$; This equation suggests, there is no existance of monopoles.

Laws of electromagnetic induciton:

(i) First law: Whenever there occurs a change in the magnetic flux linked with a coil, there is produced an induced e.m.f. in the coil. The induced e.m.f. lasts so long as the change in flux is taking place. There is an induced current only when coil circuit is complete.

(ii) Second Law : The magnitude of induced e.m.f. is equal to the rate of change in the magnetic flux, i.e. $e \propto (d\phi/dt)$. For N turns, $e \propto N(d\phi/dt)$

Lenz's Law :

The direction of the induced current is such that it tends to oppose the cause of change in magnetic flux.

(a) Combining with Faradays law of EMI, we have $e = -N.\frac{d\phi}{dt}$ for N number of turns.

(b) Lenz's law is based on law of conservation of energy.

Some other important points:

(i) The induced e.m.f. in a circuit does not depend on the resistance of the circuit as $e = -\frac{d\phi}{dt}$. However, the induced current in the circuit does depend on the resistance.

$$I = \frac{e}{R} = -\frac{1}{R}\left(\frac{d\phi}{dt}\right)$$

(ii) The induced charge that flows in the circuit depends on the change of flux only and not on how fast or slow the flux changes.

$$\frac{dq}{dt} = -\frac{1}{R}\left(\frac{d\phi}{dt}\right) \quad \text{or} \quad dq = \frac{d\phi}{R}$$

On integrating, the total charge that flows in the circuit is found to be:

$$q = \frac{(\phi_1 - \phi_2)}{R}$$

Induced E.M.F. across a conducting rod:

(i) Conducting rod moving in a uniform magnetic field: When a conducting rod of length l moves with a velocity υ in a uniform magnetic field of induction B such that the plane containing $\vec{\upsilon}$ and l makes an angle θ with $\vec{B}$ thent he magnitude of the average induced e.m.f. |e| is given by : $|e| = \upsilon Bl\sin\theta$

(ii) Conducting rod rotating with angular velocity ω in a uniform magnetic field : When a rod of length l rotates with angular velocity ω in a uniform magnetic field B, then induced e.m.f. across the ends of the rotating rod is : $e = (1/2)B\omega l^2 = B\pi f l^2 = BAf$

where $A = \pi l^2$ = area swept by the rod in one rotation and f is the frequency of rotation.

Self-inductance :

(i) When a current I flows through a coil, it produces a magnetid flux ϕ through it. Then $\phi \propto I$ or $\phi = LI$, where L is constant, called the coefficient of self-in-duction or self-inductance of the coil.

(ii) Further, $e = -(d\phi/dt) = -\frac{d}{dt}(LI) = -L(dI/dt)$

(iii) Self-inductance L of a solenoid ofN turns, length l, area of cross-section A, with a core material of relative permeability μ_r is given by : $L = \mu_r\left(\frac{\mu_0}{4\pi}\right)\frac{4\pi N^2 A}{l}$

Mutual inductance :

(i) When a current I flowing in the primary coil produces a magnetic flux ϕ in the secondary coil, then $\phi \propto I$ or $\phi = MI$, where M is a constant, called the coefficient of mutual induction or mutual inductance.

(ii) $e = -\frac{d\phi}{dt} = -\frac{d}{dt}(MI) = -M\left(\frac{dI}{dt}\right)$

(iii) Mutual inductance M of two coaxial solenoid is given by : $M = \mu_r\left(\frac{\mu_0}{4\pi}\right)\frac{4\pi N_1 N_2 A}{l}$

where N_1 and N_2 represent the total number of turns in the primary coil and the secondary coil.

Series and parallel combination of inductances :

(i) Two inductors of self-inductances L_1 and L_2 are kept so far apart that their mutual inductance is zero. These are connected in series. Then the equivalent inductance is : $L = L_1 + L_2$

(ii) Two inductors of self-inductances L_1 and L_2 are connected in series and they have mutual inductance M. Then the equivalent inductance of the combination is : $L = L_1 + L_2 \pm 2M$

The plus sign occurs if windings in the two coils are in the same sense, while minus sign occurs if windings are in opposite sense.

(iii) Two inductors of self-inductances L_1 and L_2 are connected in parallel. The inductors are so far apart that their mutual inductance is negligible. Then their equivalent inductance is :

$$L = \frac{L_1 L_2}{L_1 + L_2} \quad \text{or} \quad \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$

(iv) If two coils of self-inductances L_1 and L_2 are wound over each other, the mutual inductance is given by: $M = K\sqrt{L_1 L_2}$ (where K is called coupling constant). It is equal to zero if there is no coupling. It is equal to zero if there is no coupling. It is equal to 1 for maximum coupling. The maximum coupling occurs when the two coils are wound over each other, over a ferromagnetic core.

Growth and decay of current in LR circuit :

(i) When a switch in an LR circuit is closed, the current does not become maximum immediately but it takes some time, i.e. there is a time lag.

(ii) If R be the resistance present in the circuit, then current I at any instant is given by : $E - L (dI/dt) = IR$

(a) At start, I = 0, so (dI/dt) is maximum and $(dI/dt)_{max.} = E/L$.

(b) Finally, (dI/dt) = 0, therefore I is maximum and $I_{max.} = E/R$ i.e. final current in the circuit is independent of inductance L.

(iv) The instantaneous current in the circuit during its growth is given by : $I = \frac{E}{R}(1 - e^{-\frac{R}{L}t})$

Here, (L/R) = time constant of LR circuit. The time constant is the time in which current rises to 0.6321 times the maximum current whichis equal to (E/R).

(v) When the switch in an LR circuit is opened, the instantaneous current I is given by $I = \left(\frac{E}{R}\right)e^{-\frac{R}{L}t}$

Hence, the time consant of an LR circuit may also be defined as the time in which the current falls to 0.3679 times of its initial current.

(vi) Decay or growth of current in LR circuit is fast when L/R is small and slow when (L/R) is large.

Transformer :

(i) The transformer was inverted by Henry. It works on the principle of mutual induction and is used in AC only. It suitably changes AC voltage.

(ii) A transformer consists of a (a) primary coil of turns N_p, (b) seconcdary coil of turns N_s and (c) a laminated soft iron core.

(iii) If V_p and V_s denote the voltage across the primary coil and the secondary coilrespectively. then $(V_s/V_p) = (N_s/N_p)$.

(iv) In an actual transformer,

Output power $\leq$ input power but in an ideal transformer

Output power = input power i.e. $V_sI_s = V_pI_p$

(I_p and I_s are the current in primary and secondary coils respectively).

$$\frac{V_s}{V_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$$

(v) There are two types of transformers :

(a) Step-up trnasformer : Here, $N_s > N_p$, so $V_s > V_p$ and $I_s < I_p$.

(b) Step-down transfomer : Here, $N_s < N_p$, so $V_s < V_p$ and $I_s > I_p$.

ALTERNATING CURRENT

The instantaneous value of an AC is given by : $I = I_0 \sin \omega t$

and the alternaing voltage is given by : $E = E_0 \sin \omega t$

Here, ω is the angular frequency of Ac and $(\omega / 2\pi)$ is the frequency of Ac. $(2\pi / \omega)$ represents the time period of AC.The frequency of AC represents the number of cycles of AC completed in one second. AC supplied in India has a frequency of 50Hz.

Mean Value :

The mean value of AC represented by the equation, $I = I_0 \sin \omega t$, is zero over one complete cycle and is meaningless. In practice, mean value of alternating current refers to its average value over half cycle.

$$I_{mean} = \frac{2I_0}{\pi}$$

- A moving coil galvanometer, connected to an AC source of 50 Hz AC, shows a steady zero reading of the pointer. If the frequency is 2 Hz, the pointer oscillates with equal amplitude on either side of zero position.

  RMS or Virtual value : The RMS value of defined as the square root of the mean of square of the instantaneous value of current over the complete cycle. It may also be defined as the direct current which produces the same heating effect in a resistor as the actual AC in the same time.

$$I_V = \frac{I_0}{\sqrt{2}} = I_{rms}$$

- AC ammeter or voltmeter measures virtual current or virtual voltage (these are hot wire instrument)
- Form Factor $= \frac{\text{Virtual value}}{\text{Mean value}} = \frac{e_0 / \sqrt{2}}{2e_0 / \pi} = \frac{\pi}{2\sqrt{2}} = 1.1$

(A) AC through pure resistor R :

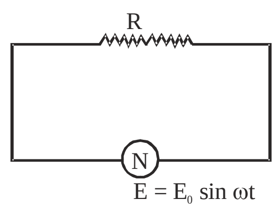
(i) Alternating e.m.f. of the source : $E = E_0 \sin \omega t$

(ii) A resistance oppose current but does not oppose a change in current. Hence, current is in phase with e.m.f.

(iii) The instantaneous value of the current is given by :

$$I = \frac{E_0 \sin \omega t}{R}$$

The virtual value of current I_V is given by $I_V = \frac{E_V}{R}$



(B) AC through pure inductor of inductance L :

(i) Alternating e.m.f. of the source : $E = E_0 \sin \omega t$

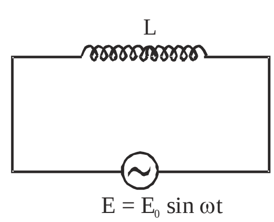
(ii) An inductor does not opposte current but opposes a change in current.

(iii) Since the voltage changes continuously, Hence, the current reacts to the change and inductive reactance

$$X_L = \omega L$$

(iv) The current lags behind the voltage by $\pi / 2$

(v) The instantaneous current is given by : $I = \left(\frac{E_0}{\omega L}\right) \sin\left(\omega t - \frac{\pi}{2}\right)$



The virtual value of the current is given by $I_V = E_V / \omega L$

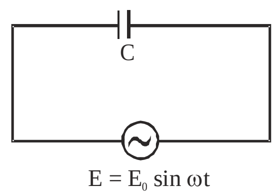
(C) AC through a pure capacitor of capacitance C :

(i) Alternating e.m.f. of the source. $E = E_0 \sin \omega t$.

(ii) A capacitor has infinite resistance for a DC source. With an AC source, voltage changes, hence charge ont he plates of the capacitor changes with time i.e. there is a current. The current leads the voltage by $\pi / 2$.

(iii) The capacitor has a capacitive reactang X_C in AC circuit, given by : $X_C = (1/\omega C)$

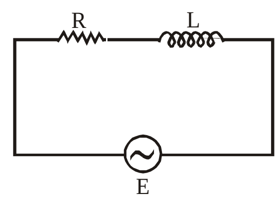
(iv) The instantaneous valeu of current is



$$I = \frac{E_0}{\left(\frac{1}{\omega C}\right)} \sin\left(\omega t + \frac{\pi}{2}\right)$$

The virtual value of current I_V is given by : $I_V = \frac{E_V}{(1/\omega C)}$

(D) AC through RL circuit :

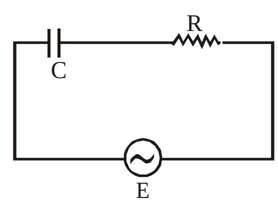


(i) In this case, we have impedance.

$$Z = \sqrt{R^2 + (X_L)^2} \quad \text{and} \quad I = \frac{E_0}{z} \sin(\omega t - \phi)$$

(ii) Phase angle, $\tan \phi = (X_L / R) = (\omega L / R)$

(E) AC through CR circuit :

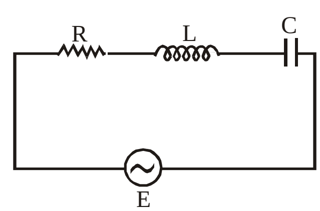


(i) In this case, we have impedence

$$Z = \sqrt{R^2 + (X_C)^2} \quad \text{and} \quad I = \frac{E_0}{z} \sin(\omega t + \phi)$$

(ii) Phase angle, $\tan \phi = (X_C / R) = (I / \omega CR)$

(F) AC source connected to resistor R, an inductor L and a capacitor C in series :



(i) The virtual current I_V is given by :

$$I_V = \frac{E_V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

(ii) The current and voltage have a phase difference ϕ given by :

$$\tan \phi = \frac{\omega L - \frac{1}{\omega C}}{R} \quad \text{and So, current} \quad I = \frac{E_0}{z} \sin(\omega t + \phi)$$

(iii) The impedance which represents the effective resistance of the circuit to AC source is represented by Z. The impedance Z is the vector sum of resistance and reactances in AC series circuit.

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

(iv) In AC series combination, virtual voltage are added vectorially i.e.

$$Ev = \sqrt{V_R^2 + (V_L - V_C)^2}$$

where $V_R = I_V R$, $V_C = I_V \, (1/C\omega)$ and $V_L = I_V (\omega L)$

Power in an AC circuit :

(i) The power in an electric circuit is the rate at which electric energy is consumed in the circuit.

$$\text{Average power, } \langle P \rangle = E_{rms} \times I_{rms} \times \cos\phi$$

So, the product of rms value of voltage and current when multiplied by $\cos\phi$ gives the power dessipated.

(ii) $\cos\phi$ is known as power factor. For a LCR series circuit.

$$\cos\phi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{R}{z}$$

(iii) Wattless current : we know that the average power in a circuit is given by :

$\langle P \rangle = E_{rms} \times I_{rms} \times \cos\phi$; Here $(I_{rms} \cos\phi)$ is called watt less current which does not contribute in power dessipation.

Resonance in Series R-L-C circuit :

(i) A particular frequency of AC at which impedance of a series LCR circuit becomes minimum or the current becomes maximum is called the resonant frequency and the circuit is called as series resonance circuit.

(ii) At resonance frequency

$$\omega_0 L = \frac{1}{\omega_0 C} \quad \text{and} \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad \text{or} \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

(iii) At resonance $\tan\phi = \dfrac{\omega_0 L - \dfrac{1}{\omega_0 C}}{R} = \text{zero}$ i.e. $\phi = 0$ i.e. phase difference is zero. So, $\cos\phi = +1$, i.e. power factor is maximum.

(iv) At resonance, $I_{max} = \dfrac{E_V}{Z_{min}} = \dfrac{E_V}{R}$

In this case, $V_L = I_V \omega_0 L$, $\quad V_C = I_V \, (1/\omega_0 C)$

$$Q \; V_L = V_C \quad \text{and} \quad E_V = \sqrt{V_R^2 + (V_L - V_C)^2} \quad \text{hence,} \quad E_V = V_R$$

i.e. at resonance with L and C in series, the current is maximum through the L and C combination but potential difference across the combination is zero.

(v) $Q = \dfrac{\omega L}{R} \text{ or } \dfrac{1}{\omega CR}$ i.e. the quality factor may be defined as the ratio of reactance of either inductance or capacitance at resonance frequency to the resistance of the circuit.

Choke coil :

(i) We know that in purely resistive circuit, the power loss is maximum because power factor $\cos\phi = (R/Z) = 1 \; (Q \; Z = R)$. Hence, the use of resistance is avoided in AC circuits to control current.

(ii) A choke coil is a coil which has high inductance and negligible resistance. Thus, the power factor is almost zero. So a choke coil controls the alternating current without an apreciable energy loss. This is used with fluorescent tubes to control the current.

ELECTROMAGNETIC WAVES

Maxwell's Equations

(a) $\oint \vec{E}.d\vec{s} = \frac{q}{\epsilon_0}$ (Gauss's theorem in electrostatics)

(b) $\oint \vec{B}.d\vec{s} = 0$ (Gauss's law in magnetism)

(c) $\oint \vec{E}..d\vec{s} = -\frac{d}{dt}\int \vec{B}.d\vec{s}$ (Faraday's law of electromagnetic induction)

Important Features of Electromagnetic waves

- E.M. waves are transverse waves in which there are sinusoidal variations of electric and magnetic fields. These two fields exist at right angles to each other as well as at right angles to the direction of wave propagation.
- Both these fields vary with time and space and have the same frequency of variation.
- These waves can travel through vacuum also, hence these waves are non-mechanical.
- Velocity of electromagnetic wave in free space (vaccum) is constant and given by

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.854 \times 10^{-12}}} = 3 \times 10^8 \, ms^{-1}$$

- Direction of wave propagative is given by the direction of $\vec{E} \times \vec{B}$.
- Examples of electromagnetic waves are radio waves, microwaves, infrared rays, light waves, ultraviolet rays, X-rays and $\gamma -$rays.
- The amplitudes of electric and magnetic fields in free space, in electromagnetic waves are related by $E_0 = cB_0$

Energy Density of Electromagnetic Eave

- The average energy density of electric field is, $U_E = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0 \left(E_0/\sqrt{2}\right)^2 = \frac{1}{4}\epsilon_0 E_0^2$
- The average density of magnetic field is, $U_B = \frac{B^2}{2\mu_0} = \frac{\left(B_0/\sqrt{2}\right)^2}{2\mu_0} = \frac{B_0^2}{4\mu_0}$
- In EM waves the average energy density due to either field are equal i.e. $U_E = U_B$

Momentum of Electromagnetic wave

- The electromagnetic wave has linear momentum associated with it. The linear momentum p carried by the portion of wave having energy U is given by $p = \frac{U}{c}$. As per plank, $p = \frac{hv}{c} = \frac{h}{\lambda}$

Production of Electromagnetic Waves

- An electromagnetic wave is emitted when an electron orbiting in higher stationary orbit of an atom jumps to one of the lower stationary orbits of that atom.
- Accelerated charge (e.g. LC oscillator) produces EM waves.
- Some electromagnetic waves (i.e. X-rays) are also produced when fast moving electrons are suddenly stopped by a metal surface having high atomic number.

Electromagnetic Spectrum

The major components spectrum with their wavelength λ ranges in incrasing order are

1. Gamma rays $[\lambda = 6 \times 10^{-19} \, m \, to 10^{-11} m]$

2. X-rays $[\lambda = 6\times10^{-19}\,\text{m to}\,3\times10^{-8}\,\text{m}]$
3. Ultraviolet $[\lambda = 6\times10^{-10}\,\text{m to}\,4\times10^{-7}\,\text{m}]$
4. Visible light $[\lambda = 4\times10^{-7}\,\text{m to}\,8\times10^{-7}\,\text{m}]$
5. Infra red $[\lambda = 8\times10^{-7}\,\text{m to}\,3\times10^{-5}\,\text{m}]$
6. Heat radiations $[\lambda = 8\times10^{-5}\,\text{m to}\,10^{-1}\,\text{m}]$
7. Micro waves $[\lambda = 10^{-3}\,\text{m to}\,0.03\,\text{m}]$
8. Ultra high frequency $[\lambda = 10^{-1}\,\text{m to}\,1\,\text{m}]$
9. Very high ratio frequency $[\lambda = 1\,\text{m to}\,10\,\text{m}]$
10. Radio frequencies $[\lambda = 10\,\text{m to}\,10^{4}\,\text{m}]$

Optics
(Ray Optics)

REFLECTION :

Light is a form of energy which is propagated as electromagnetic waves. It does not require a medium for its propagation. Its speed in free space (i.e., vacuum) is 3×10^8 m/s.

As in visible region the possible number of different wavelengths (or frequencies) between 4000 Å and 7000 Å are infinite and different frequencies produce the sensation of different colours, the number of colours in visible region is infinite. However, our eye can distinguish only following six colours.

Eye is most sensitive to yellow-green light, light of wavelength 5550 Å.

Persistence of eye is 1/10 s, i.e., if time interval between two successive light pulses is lesser than 0.1 s, eye cannot distinguish them separately. Also the resolving limit of the eye is one minute, two objects separated by distance d will not be distinctly visible to the eye if the angle, θ subtended by them at the eye,

$$0 < 0°1' \quad \text{i.e.,} \quad \frac{d}{D} < \frac{\pi}{(180 \times 60)}$$

When light passes from one medium to the other, velocity and wavelength m change, amplitude may decrease or remain constant, but frequency and colour of light do not change, i.e., colour of light is determined by its frequency (a not wavelength), e.g., if red light passes from air to water (or glass) its velocity and wavelength in water (or glass) will be different from that in air frequency and colour remain the same.

If the velocity of light in a medium (such as water or glass) is same in all directions, the medium is called isotropic. However, if the velocity of light is different in different directions (e.g., in calcite or quartz, etc.), the medium is said to be anisotropic for that light.

Principle of Reversibility of Light:

If a light ray is reversed, it always retraces its path. Object and image positions are interchangeable. The points corresponding to object and image are called conjugate points.

Optical Path :

It is defined as distance travelled by light in vacuum in the same time in which it travels a given path length in a medium. If light travels a path length d in a medium at speed v, the time taken by it will be (d/v). So optical path length.

$$L = c \times \left[\frac{d}{v}\right] = \mu d \; ; \; \left[\text{as} \frac{c}{v} = \mu\right]$$

As for all media $\mu > 1,$ optical path length is always greater than actual path length.

REFLECTION:

Laws of Reflection :

The incident-ray, reflected-ray and normal to the reflecting surface at the point of incidence all lie in the same plane. The angle of reflection is equal to the angle of incidence, i.e., $\angle i = \angle r$.

Reflection from Plane-surfaces

The image is always erect, virtual and of same size as the object. It is at the same distance behind the mirror as the object is in front of it.

- Some points regarding plane reflecting surface.
- If the rays after refraction or reflection actually converge at a point, the image's said to be real and it is not only to be seen but also obtained on a suitably placed screen. However, if the rays do not actually converge but appear to do so and actually diverge, the image is said to be virtual. A virtual image can only be seen and cannot be obtained on a screen at the position of the image.
- If an object moves towards a plane mirror at speed v, the image will also approach (or recede) at same speed v, the speed of image relative to object will be v – (–v) = 2v. Similarly if the mirror is moved towards or (away from) the object with speed v the image will move towards (or away from) the object with speed 2v.
- Deviation δ is defined as the angle between directions of incident ray and emergent ray. So if light is incident at an angle of incidence i, $\delta = 180 - (\angle i + \angle r) = (180 - 2i)$

  If keeping the incident ray fixed, the mirror is rotated by an angle θ, about an axis in the plane of mirror, the reflected ray is rotated through an angle 2θ.
- The image formed by a plane mirror suffers lateral-inversion in the image formed by a plane mirror left is turned into right and vice-versa with respect to object as.
- To see his full image in a plane mirror a person requires a mirror of at least half of his height.
- To see a complete wall behind himself a person requires a mirror of at least (1/3) the height of wall and he must be in the middle of wall and mirror.
- If there are two plane mirrors inclined to each other at an angle θ, the number of images of a point object formed are determined as follows:
- If $(360/\theta)$ is even integer (say m) number of images formed

  n = (m – 1), for all positions of object
- If $(360/\theta)$ is odd integer (say m) number of images formed

  n = m, if the object is not on the bisector of mirrors
  n = (m – 1), if the object is on the bisector of mirrors
- If $(360/\theta)$ is a fraction, the number of images formed will be equal to its integral part.

Reflection at Spherical Surface

- The focal length of a spherical mirror of radius R is given by f = (R/2)
- The power of a mirror is defined as $P = -\frac{1}{f\,(\text{in m})} = -\frac{100}{f\,(\text{in cm})}$
- If a thin object of linear size O is situated vertically on the axis of a mirror at a distance u from the pole and its image of size I is formed at a distance v (from the pole), magnification (transverse) is defined as :

$$m = \left[\frac{I}{O}\right] = -\left[\frac{v}{u}\right]$$

–ve magnification implies that image is inverted with respect to object* +ve magnification means that image is erect with respect to object.

- If an object is placed at a distance u from the pole of a mirror and its image is formed at a distance v

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

- If the 1-D object is placed with its length along the principle axis, the so called longitudinal magnification becomes

$$m_L=\frac{I}{O}=-\frac{(v_2-v_1)}{(u_2-u_1)}=-\frac{dv}{du}=\left[\frac{v}{u}\right]^2=m^2$$

NEWTON'S FORMULA:

- In case of spherical mirrors if object distance (x_1) and image distance (x_2) are measured from focus instead of pole u (f + x_1) and v = (f + x_2), the mirror formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \text{ reduces to } \frac{1}{(f+x_2)}+\frac{1}{(f+x_1)}=\frac{1}{f}$$

which on simplification gives $x_1x_2 = f^2$

In case of spherical mirrors if we plot a graph between.

(1/u) and (1/v), it will be a straight line with intercept (1/f) with each axis as $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ becomes y + x = c with $c=\frac{1}{f}$.

Graph between u and v will be a hyperbola, as for $u = f, v=\infty$ and for $u=\infty, v = f$. A line u = v will cut this hyperbola at (2f, 2f).

Concave mirror behaves as convex lens (both convergent) while convex mirror behaves as concave lens (both divergent).

USASGE OF CONVEX / CONCAVE MIRRORS:

As convex mirror gives erect, virtual and diminished image. In convex mirror, the field of view is increased as compared to plane mirror. This is why it is used as rear-view mirror in vehicles. Concave mirrors give enlarged, erect and virtual image (if object is between F and P), so these are used by dentists for examining teeth. Further due to their converging property concave mirrors are also used as reflectors in automobile head lights and search lights and by ENT surgeons in ophthalmoscope.

REFRACTION :

Law of Refraction:

1. Incident ray, Refracted ray and normal at a point lie in same plane.
2. Product of refractive index and the sine of angle made by light ray with the normal remains constant when light travels from one medium to another.

i.e. $\mu\times\sin i=\text{constant}$

$$\mu_1\times\sin i=\mu_2\sin r$$

If light passes from rarer to denser medium $\mu_1=\mu_R$ and $\mu_2=\mu_D$ so that in passing from rarer to denser medium, the ray bends towards the normal.

If light passes from denser to rarer medium $\frac{\sin i}{\sin r} = \frac{\mu_R}{\mu_D} < 1$

in passing from denser to rarer medium, the ray bends away from the normal.

APPARENT DEPTH AND NORMAL SHIFT

$$\frac{d_{Ac}}{d_{Ap}} = \frac{\mu_1}{\mu_2}$$

Object in a denser medium is seen from a rerer medium.

The distance between object and its image, called normal shift and with $d_{Ac} = t$, will be

$$x = d_{Ac} - d_{Ap} = t - (t/\mu) = t[1 - (1/\mu)]$$

Object in a rarer medium is seen from a denser medium

$$x = d_{Ap} - d_{Ac} = [(\mu - 1)]t$$

Further in passing through a medium of thickness t and refractive index μ, a ray incident at a small angle θ is displaced parallel to itself by 'y' called lateral displacement.

$$y = \left[\frac{\mu - 1}{\mu}\right]t$$

If there are number of liquids of different depths, one over the other

$$d_{Ac} = d_1 + d_2 + d_3 \ldots \text{ and } \qquad d_{Ap} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3} + \ldots$$

$$\mu = \frac{d_{Ac}}{d_{Ap}} = \frac{d_1 + d_2 + \ldots}{(d_1/\mu_1) + (d_2/\mu_2) + \ldots} = \frac{\Sigma d_i}{\Sigma(d_i/\mu_i)}$$

$$\mu = \left[\frac{2\mu_1\mu_2}{\mu_1 + \mu_2}\right] = \text{Harmonic mean.}$$

TOTAL INTERNAL REFLECTION :

If light is passing from denser to rarer medium through a plane boundary, then $\mu_1 = \mu_D$ and $\mu_2 = \mu_R$; so with $\mu = (\mu_D/\mu_R)$,

$$\sin i = \frac{\mu_R}{\mu_D}\sin r \quad \text{i.e.,} \quad \sin i = \frac{\sin r}{\mu}$$

$$\sin i \propto \sin r \quad \text{with} \quad (\angle i) < (\angle r)$$

So as angle of incidence i increases angle of refraction r will also increase and for certain value of i (<90°) r will become 90°. The value of angle of incidence for which r = 90° is called critical angle and is denoted by θ_C and in the light will be given by

$$\sin\theta_C = \frac{\sin 90}{\mu} \quad \text{i.e.,} \quad \sin\theta_C = \frac{1}{\mu}$$

the total light incident on the boundary will be reflected back into the same medium from the boundary. This phenomenon is called total internal reflection. Here it is worthy to note that :

For total internal reflection to take place light must be propagating from denser to rarer medium.

In case of total internal reflection, as all (i.e., 100%) incident light is reflected back into the same medium there is no loss of intensity while in case of reflection from mirrors or refraction from lenses there is some loss of intensity as all light can never be reflected or refracted.

From Snell's law, $\theta_C = \sin^{-1}(1/\mu)$

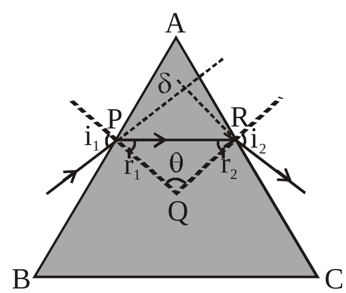
PRISM-THEORY:

Prism is a transparent medium bounded by any number of surfaces in such a way that the surface on which light is incident and the surface from which light emerges are plane and non-parallel.

Angle of prism or refracting angle of prism means the angle between the faces on which light is incident and from which it emerges.

Angle of deviation means the angle between emergent and incident rays.

The angle of deviation δ will be



$$\delta = (i_1 - r_1) + (i_2 - r_2) \qquad \left[r_1 + r_2 + \theta = 180^o \Rightarrow A + 90^o + \theta + 90^o = 360 \Rightarrow A + \theta = 180^o \right.$$

$$\delta = (i_1 + i_2) - (r_1 + r_2) \qquad \left. r_1 + r_2 + \theta = A + \theta \Rightarrow r_1 + r_2 = A \right]$$

$$\delta = [i_1 + i_2 - A] \qquad (r_1 + r_2) = A$$

This is the required result and holds good if emergent ray exists.

If angle of prism A is small, r_1 and r_2 (as $r_1 + r_2 = A$) and hence i_1 and i_2 will also be small. Since for small angles sin $\theta = \theta$. Snell's law at first and second surfaces of prism gives respectively:

$$i_1 = \mu r_1 \quad \text{and} \quad \mu r_2 = i_2$$

$$(i_1 + i_2) = \mu(r_1 + r_2) = \mu A$$

So for small angle of prism.

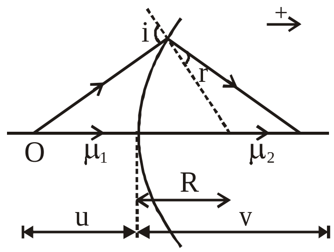
Deviation will be maximum when angle of incidence i_1 is maximum

$$\delta_{max} = 90 + i_2 - A$$

MINIMUM DEVIATION :

Minimum deviation of a ray through the prism is given by: $\mu = \dfrac{\sin[(A + \delta_m)/2]}{\sin(A/2)}$

REFRACTION AT SPHERICAL SURFACES



Refraction at single spherical surface is shown in the figure above.

Here light rays starting form an object O, placed in a medium of R.I. μ_1, move into another medium of R.I. μ_2 after being refracted at a curved surface of radius R. Object distances (u) and image distance (v) as shown in the figure, are measured from pole. Radius of curvature (R) is also measured from the pole, as shown. Then

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

For refraction at single surface and for real objects,

μ is always –ve

v is +ve if image forms on otherside, ie in next medium.

v is –ve if image forms on same side, ie in first medium

R is +ve if surface is convex as seen from the object

The image is real if formed inside the second medium, because such image is due to actual meeting of rays after refraction. If formed on the same side, there is no actual meeting of rays, hence the image is virtual.

Lateral magnification, $m = \frac{\text{Image size}}{\text{Object size}} = \frac{\mu_1 v}{\mu_2 v}$

REFRACTION THROUGH LENS

Lens maker's formula for thin lens

Let R_1 and R_2 be radii of curvature of the first and second spherical surface. Let f be the focal length of the lens and μ be the refractive index of lens material w.r.t. the medium in which the lens is placed. Then

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{or} \quad P = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$ where P is power of the lens

Linear Magnification for a lens

Magnification $m = \frac{v}{u}$

We know that $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ or $\frac{v}{v} - \frac{v}{u} = \frac{v}{f} \Rightarrow 1 - \frac{v}{u} = \frac{v}{f} \Rightarrow \frac{v}{u} = \frac{f - v}{f} \Rightarrow m = \frac{f - v}{f}$

We can also express m in terms of u and f.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \text{or} \quad \frac{u}{v}-\frac{u}{u}=\frac{u}{f} \quad \Rightarrow \quad \frac{u}{v}=\frac{u+f}{f} \quad \Rightarrow \quad m=\frac{f}{f+u}$$

Power of a Lens

$$P=\frac{1}{f}$$

Since focal length of a convex lens or a converging lens is positive, therefore its power is positive. Similarly, the power of a concave lens or a diverging lens is negative.

Opticians express the power of a lens in terms of a unit called the DIPOTRE. It is regarded as the SI unit of optical power. The power of a lens is said to be one dioptre if the focal length of the lens is 1 metre.

When focal length is in cm, $P=\frac{100}{f}$ dioptre.

Lens Formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

This equation holds good for both convex and concave lenses, whether the image formed is real or virtual.

For real image, v is +ve because real image always forms or other side of the lens. For virtual image v is –ve.

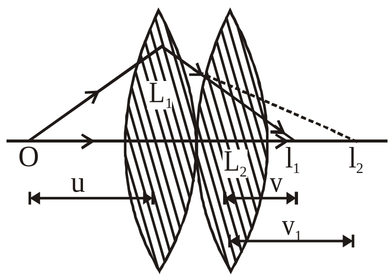
Focal length f is +ve for convex lens and –ve for concave lens. Object distance u is always -ve for a real object and u is always +ve for a virtual object.

Convex Lens

Position of object	Position of image	Real/Virtual	Inverted/erect	Magnification and size of image
at infinity	at focus	real	inverted	m < 1 greatly diminished
beyond 2f	between f and 2f	real	inverted	m = 1 same size
at 2f	at 2f	real	inverted	m = 1 same size
between f and 2f	beyond 2f	real	inverted	m > 1 magnified
at f	at inifinity	real	inverted	$m \Rightarrow \infty$ magnified
between optical centre and focus	at a distance greater than the object distance and on the same side object	virtual	erect	m> 1 magnified
		Concave Lens		
at inifinity	at focus (v = f)	virtual	erect	m < 1 diminished
between infinity and optical centre	between optical centre and focus	virtual	erect	m < 1 diminished

Focal Length of Combination of two thin lenses in contact

Let two lenses be in contact, as shown in the figure. Focal length of the combination is given by



$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

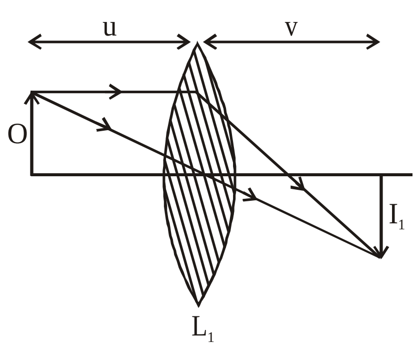
If P is the power of the combination, then $P = P_1 + P_2$ where P_1 and P_2 are the powers of the individual lenses. If two lenses are separated by distance d then focal length of system of two lenses is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Displacement Method for finding the focal length of a convex lens

In this method, the distance between the object and the screen must be greater than 4f, where f is the focal length of the convex lens. The image on the screen can be formed corresponding to two different positions of the lens. Figure (i) shows the magnified image of size I_1 for the position L_1 of the lens.

$$m_1 = \frac{I_1}{O} = \frac{v}{u}$$



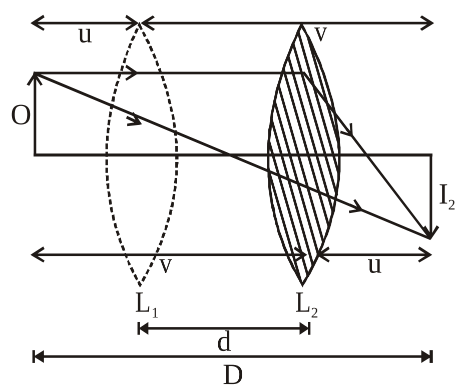


Figure (ii) shows the diminished image of size I_2 for the position L_2 of the lens.

$$m_2 = \frac{I_2}{O} = \frac{v}{u}$$

From (1) and (2),

$$\frac{I_1 I_2}{O^2} = \frac{v}{u} \times \frac{u}{v} = 1$$

or $\quad O = \sqrt{I_1 I_2} \Rightarrow$ object size = $\sqrt{(\text{1st image size}) \times (\text{2nd image size})}$

WAVE OPTICS

NATURE OF LIGHT :

Light is an electromagnetic wave which is sinusoidally varying electric and magnetic fields propagated from one part to another part. The electric and the magnetic field are given by

$$E_y = E_0 \sin(kx \pm wt)$$

$$B_z = B_0 \sin(kx \pm wt)$$

It propagates as transverse non mechanical wave in a medium at a speed given by $V = \frac{1}{\sqrt{\mu \in}}$;

The electric and magnetic fields are related as $E_0 = VB_0$

REFRACTIVE INDEX OF A MEDIUM :

Refractive index of a medium is defined as

$$\mu = \frac{\text{speed of light in vaccum}}{\text{speed of light in med}} \Rightarrow \mu = \frac{c}{v}$$

INTERFERENCE :

The modification in the distribution of intensity of light in the region of superposition of two coherent light waves is called interference. At some points the waves superpose in such a way that the resultant intensity is greater than the sum of the intensities due to separate waves (constructive interference) while at some other points intensity is less than the sum of the separate intensities (destructive interference).

YOUNG' DOUBLE SLIT EXPERIEMENT :

Let the two waves each of angular frequency ω from sources S_1 and S_2 reach the point P. Equations are given by $y_1 = A_1 \sin[\omega t - kx]$ and, $y_2 = A_2 \sin[\omega t - k(x + \Delta x)]$

So the resultant wave at P by principle of superposition will be

$$y = y_1 + y_2 = A_1 \sin(\omega t - kx) + A_2 \sin(\omega t - kx - \phi);$$ where ϕ is initail phase difference

$$y = A \sin(\omega t - kx - \alpha)$$ where, $A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos\phi$

and, $\alpha = \tan^{1}[A_2 \sin\phi / (A_1 + A_2 \cos\phi)]$

Superposition of two waves of equal frequencies propagating almost in the same direction, results in harmonic wave of same frequency ω and wavelength $(\lambda = 2\pi / k)$ but amplitude A. The intensity of resultant wave

$$I = K\left[A_1^2 + A_2^2 + 2A_1A_2 \cos\phi\right]$$

$$I = I_1 + I_2 + 2\left(\sqrt{I_1 I_2}\right)\cos\phi$$

The resultant intensity at P is not just the sum of intensities due to separate waves $(I_1 + I_2)$ but different and depends on phase difference ϕ and the position of the point P.

CONDITION FOR INTERFERENCE

(a) Intensity will be maximum when :

$$\cos\phi = \max. = 1$$

or $\phi = \pm 2\pi n$ with n = 0, 1, 2

$$\frac{2\pi}{\lambda}(\Delta x) = \pm 2\pi n$$

$$\Delta x = \pm n\lambda$$

$$I_{max} = \left(I_1 + I_2 + 2\sqrt{I_1 I_2}\right)$$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \propto (A_1 + A_2)^2$$

Intensity will be maximum at those points where path difference is an integral multiple of wavelength λ and maximum intensity is greater than the sum of two intensities ($I_1 + I_2$). These points are called points of constructive interference or interference maxima.

(b) Intensity will be minimum when :

$$\cos\phi = \min. = -1;\ \phi = \pm\pi, \pm 3\pi, \pm 5\pi;\ \phi = \pm(2n-1)\pi;\ \frac{2\pi}{\lambda}(\Delta x) = \pm(2n-1)\pi;\ \Delta x = \pm(2n-1)\lambda/2$$

$$I_{min} = \left(I_1 + I_2 - 2\sqrt{I_1 I_2}\right) \Rightarrow I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 \propto (A_1 - A_2)^2$$

Intensity will be minimum at those points where path difference is an odd integral multiple of $(\lambda/2)$ and minimum intensity is less than the sum of two intensities ($I_1 + I_2$). These points are called points of destructive interference or interference minima.

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \qquad I_{min} = \left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2$$

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2};\ \frac{I_1}{I_2} = \frac{A_1^2}{A_2^2}$$

- All maxima are equally spaced (as path difference between two consecutive maxima is λ) and equally bright the two waves from S_1 and S_2 have same frequency and start in the same phase at P they have a constant phase difference $\phi = (2\pi/\lambda)\Delta x$, developed due to different paths traversed by them. Such waves are said to be 'Coherent' and produce sustained interference effects.

If d is the separation between the slits and D (>>d) is the distance of screen from the plane of slits as

$$\Delta x = d\sin\theta \qquad \sin\theta = (\Delta x/d)$$

for small θ, $\sin\theta = \tan\theta = \theta = (y/D)$

$$\frac{y}{D} = \frac{\Delta x}{d} \qquad y = \frac{D}{d}(\Delta x)$$

- If the point P is nth bright fringe, $\Delta x = n\lambda$ and hence

$$\left(y_n\right)_{Bright} = \frac{D}{d}(n\lambda) \qquad n = 0, 1, 2, \text{etc.}$$

- If the point P is nth minima

$$\left(y_n\right)_{Dark} = \frac{D}{d}(2n-1)\frac{\lambda}{2} \qquad n = 1, 2, ... \text{etc.}$$

- Fringe-width β is defined as the distance between two consecutive maxima (or minima) on the screen

$$\beta = \Delta y \qquad \Delta x = \lambda$$

$$\beta = \frac{D}{d}(\lambda)$$

- As linear position y is related to the angular position θ by $\theta = (y/D)$, i.e., $\Delta\theta = (\Delta y/D)$, the so called angular fringe-width

$$\theta_0 = \frac{\beta}{D} = \frac{\lambda}{d}$$ Fringe-width is independent of n.

- If the transparent sheet of refractive index μ and thickness t is introduced in one of the paths of interfering waves, optical path will become μt instead of t for the portion in which glass is inserted so the optical path will increase by $(\mu - 1)t$. Due to this, a given fringe from its present position y will shift to a new position y', the lateral shift of the fringe is

$$y_0 = y' - y = \frac{D}{d}(\mu-1)t = \frac{\beta}{\lambda}(\mu-1)t$$

entire fringe-pattern is displaced by y_0 towards the side in which the thin sheet is introduced without any change in fringe width.

DIFFRACTION :

The flaring out or encroachment of light in the shadow zone as it passes around obstacles or through small apertures is called diffraction.

As a result of diffraction, the edges of the shadow do not remain well defined and sharp but become blurred and fringed. If the width of the aperture is comparable to the wavelength of light, most of the incident wavefront will be obstructed and in accordance with Huygens' wave theory a cylindrical (or spherical) wavefront depending on the aperture (slit or hole) will originate from it as the direction of wave motion is normal to the wavefront, after passing through the aperture light will flare out. This flaring out or spreading of light is the so called diffraction.

In case of diffraction at single slit theory shows that intensity at a point on the screen is given by:

$$I_{(\theta)} = I_m\left[\frac{\sin\alpha}{\alpha}\right]^2 ; \alpha = \frac{\phi}{2} = \frac{\pi}{\lambda}(d\sin\theta)$$

From this it is clear that I will be minimum when for $\alpha \neq 0$,

$$\sin\alpha = 0, \quad \alpha = n\pi \qquad n = 1, 2, ...$$

- Angular position of minima in case of diffraction at single slit is given by:

$$\frac{\pi}{\lambda}(d\sin\theta) = n\pi \qquad d\sin\theta = n\lambda$$

And as central maximum extends between first minima on either side, for small θ, the angular width of central maximum will be: $\theta_0 = 2\theta_1 = (2\lambda / d)$

- At centre as $\theta = 0$ and hence $(\sin\alpha / \alpha) \to 1$. This in turn means that intensity at centre is always maximum and equal to I_m. This maximum of intensity is called central maximum.
- At the position of a minima, wavelets from the two ends of the slit reach in phase differeing by an integral multiple of 2π as in this situation path difference $(d \sin\theta)$ condition of minima is $n\lambda$.
- Subsidiary maxima are approximately midway between two consecutive minima and of decreasing inteisty. The position of nth subsidiary maxima will therefore be given by:

$$[d \sin\theta]_{max} = \frac{n\lambda + (n+1)\lambda}{2} = \left(n + \frac{1}{2}\right)\lambda$$

- The angular width of subsidiary maximum $\theta_s = (\theta_n - \theta_{n-1}) = (\lambda / d)$ is half that of central maximum $[\theta_0 = 2(\lambda / d)]$.
- Due to diffraction at a circular aperture, a converging lens can never form a point image of an object but it produces a bright disc called Airy disc surrounded by dark and bright concentric rings. The minimum radius of the image disc is given by

$$r = 1.22 \frac{\lambda}{d}(f)$$

- Diffraction limits the ability of optical instruments to form distinct images of objects when they are close to each other. According to Rayleigh (called rayleigh's criterion), two objects of equal intensity will be just resolved (i.e., distinctly visible) by an optical instrument if the central diffraction maximum of one lies at the first minimum of the other. So the angular limit of resolution will be equal to the angular separation between the centre of central maximum and first minimum, which for a single slit will be

$$\theta_R = \frac{\lambda}{d} - 0, \qquad \theta_R = \frac{\lambda}{d}$$

for circular aperture such as lens, θ_R is found to be $1.22(\lambda / d)$; so two objects at a distance D with separation y will be distinctly visible only if

$$\theta \geq \theta_R, \qquad (y / D) \geq 1.22(y / d)$$

- A diffraction-grating consists of large number of equally spaced parallel slits. If light is incident normally on a transmission grating, the direction of principal maxima (PM) is given by

$$d \sin\theta = n\lambda$$

where d is the distance between two consecutive slits and is called grating element and n order of principle maxima.
- The dispersive and resolving power of a grating are given by

$$DP = \frac{d\theta}{d\lambda} = \frac{n}{d \cos\theta} \qquad RP = \frac{\lambda}{d\lambda} = nN$$

closely spaced lines on a grating give greater dispersion while greater number of lines increase its resolving power.

POLARISATION :

An ordinary beam of light consists of a large number of waves emitted by the atoms or molecules of the light source. Each atom produces a wave with its own orientation of electric vector $\vec{E}$ Because all directions of vibrations of $\vec{E}$ are equally probable the resultant electromagnetic wave is a superposition of waves produced by the individual atomic sources. This resultant wave is called unpolarised light and is symmetrical about the direction of wave propagation. If somehow (say using polaroids or Nicol-prism) we confine the vibrations of electric vector in one direction perpendicular to the direction of wave motion the light is said to be plane polarised or linearly-polarised and the phenomenon of confining the vibrations of a wave in a specific direction perpendicular to the direction of wave motion is called polarisation. The plane containing the direction of vibration and wave motion is called plane of polarisation.

All the vibrations of an unpolarised light at a given instant can be resolved in two mutually perpendicular directions and hence, an unpolarised light is equivalent to the superposition of two mutually perpendicular identical plane polarised lights.

- If in case of unpolarised light, electric vector in some plane is either more or less, than in its perpendicular plane, the light is said to be 'partially polarised'
- If an unpolarised light is converted into plane polarised light, its intensity reduces to half.
- A part from partially polarised and plane (i.e., linearly) polarised, light can also be circularly or elliptically polarised, that too left-handed or right handed. Elliptically and circularly polarised lights result due to superposition of two mutually perpendicular plane polarised lights differing in phase by $(\pi/2)$ with.

Methods of Obtaining Plane Polarised Light :

By Reflection :

Brewster discovered that when light is incident at a particular angle on a transparent substance, the reflected light is completely plane polarised with vibrations in a plane perpendicular to the plane of incidence. This specific angle of incidence is called polarising angle θ_p and is related to the refractive index μ of the material through the relation: $\tan\theta_p = \mu$. This is known as Brewster's law.

By Scattering :

When light is incident on atoms and molecules, the electrons absorb the incident light and re-radiate it in all directions. This process is called scattering. it is found that scattered light in directions perpendicular to the direction of incident light is completely plane polarised while transmitted light is unpolarised. Light in all other directions is partially polarised.

- If plane polarised light of intensity $I_0\left(= KA^2\right)$ is incident on a polaroid and its vibrations of amplitude A make an angle θ with the transmission axis, then the component of vibrations parallel to transmission axis will be A cos θ while perpendicular to it A sin θ. Now, as polaroid will pass only those vibrations which are parallel to its transmission axis, i.e., A cos θ, so the intensity of emergent light will be

  $$I = K\left(A\cos\theta\right)^2 = KA^2\cos^2\theta$$

  $$I = I_0\cos^2\theta$$

  This law is called Malus law.
- If an unpolarised light is converted into plane polarised light (say by passing it through a polaroid or a Nicol-prism), its intensity becomes half.
- If light of intensity I_1 emerging from one polaroid called polariser is incident on a second polaroid (usually called analyser) the intensity of the light emerging from the second polaroid in accordance

with Malus law will be given by $I_2 = I_1 \cos^2 \theta'$

where θ' is the angle between the transmission axis of the two polaroids.

- Optically activity of a substance is measured with the help of polarimeter in terms of 'specific rotation' which is defined as the rotation produced by a solution of length 10 cm (1 dm) and of unit concentration (i.e., 1 g/cc) for a given wavelength of light at a given temperature,

$$[\alpha]_{t^\circ C}^{\lambda} = \frac{\theta}{L \times C}$$

where θ is the rotation in length L at concentration C.

Formulae For DUAL NATURE AND RADIATION

CHAPTER FORMULAS & NOTES

Important Terms, Definitions & Formulae

1 **Electric Discharge:** The passage of an electric current through a gas is called electric discharge.

2 **Discharge Tube:** A hard glass tube along with the necessary arrangement, which is used to study the passage of electric discharge through gases at low pressure, is called a discharge tube.

3 **Cathode Rays.** Cathode rays are the stream of negatively charged particles, electrons which are shot out at a high speed from the cathode of a discharge tube at pressure below 0.01 mm of Hg.

4 **Work Function.** The minimum amount of energy required by an electron to just escape from the metal surface is known as work function of the metal.

5 **Electron Emission.** The minimum amount of energy required by an electron to just escape from the metal surface is known as work function of the metal.

(i) **Thermionic emission.** Here electrons are emitted from the metal surface with the help of thermal energy.

(ii) **Field or cold cathode emission.** Electrons are emitted from a metal surface by subjecting it to a very high electric field.

(iii) **Photoelectric emission.** Electrons emitted from a metal surface with the help of suitable electromagnetic radiations.

(iv) **Secondary emission.** Electrons are ejected from a metal surface by striking over its fast moving electrons.

6 **Forces Experienced by an Electron in Electric and Magnetic Fields.**

(a) **Electric field:** The force F_E experienced by a electron e in an electric field of strength (intensity) E is given by

$$F_E = eE$$

(b) **Magnetic field**: The force experienced by an electron e in a magnetic field of strength B weber/m^2 is given by

$$F_B = Bev$$

where v is the velocity with which the electron moves in the electric field and the magnetic field, perpendicular to the direction of motion.

If the magnetic field is parallel to the direction of motion of electron, then, $F_B = 0$.

7 **Photoelectric Effect:** The phenomenon of emission of electrons from the surface of substances (mainly metals), when exposed to electromagnetic radiations of suitable frequency, is called photoelectric effect and the emitted electrons are called photoelectrons.

8 **Cut Off or Stopping Potential:** The value of the retarding potential at which the photoelectric current becomes zero is called cut off or stopping potential for the given frequency of the incident radiation.

9 **Threshold Frequency:** The minimum value of the frequency of incident radiation below which the photoelectric emission stops altogether is called threshold frequency.

10 **Laws of Photoelectric Effect.**

(i) For a given metal and a radiation of fixed frequency, the number of photoelectrons emitted is proportional to the intensity of incident radiation.

(ii) For every metal, there is a certain minimum frequency below which no photoelectrons are emitted, howsoever high is the intensity of incident radiation. This frequency is called *threshold frequency.*

(iii) For the radiation of frequency higher than the threshold frequency, the maximum kinetic energy of the photoelectrons is directly proportional to the frequency of incident radiation and is independent of the intensity of incident radiation.

(iv) The photoelectric emission is an *instantaneous* process.

11 **Einstein's Theory of Photoelectric Effect.** Einstein explained photoelectric effect with the help of Planck's quantum theory. When a radiation of frequency ν is incident on a metal surface, it is absorbed in the form of discrete packets of energy called quanta or photons.

A part of energy $h\nu$ of the photon is used in removing the electrons from the metal surface and remaining energy is used in giving kinetic energy to the photoelectron.

Einstein's photoelectric equation is

$$KE = \frac{1}{2}mv^2 = h\nu - w_o$$

where w_o is the work function of the metal.

If ν_0 is the threshold frequency, then $w_o = h\nu_o$

$$KE = \frac{1}{2}mv^2 = h\,(\nu - \nu_0)$$

All the experimental observations can be explained on the basis Einstein's photoelectric equation.

12 **Compton Scattering.** It is the phenomenon of increase in the wavelength of X-ray photons which occurs when these radiations are scattered on striking an electron. The difference in the wavelength of scattered and incident photons is called Compton shift, which is given by

$$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\phi)$$

where ϕ is the angle of scattering of the X-ray photon and m_0 is the rest mass of the electron.

13 **$\frac{e}{m}$ of an Electron by Thompson's Method.**

J. J. Thomson devised an experiment to determine the velocity (v) and the ratio of the charge (e) to the mass (m) i.e., $\frac{e}{m}$ of cathode rays. In this method electric field $\vec{E}$ and magnetic field $\vec{B}$ are applied on the cathode rays.

In the region where they are applied perpendicular to each other and to the direction of motion of cathode rays,

Force due to electric field, F_E = Force due to magnetic field F_B,

Or $$eE = Bev \Rightarrow v = \frac{E}{B}$$

Also $$\frac{e}{m} = \frac{E}{B^2 R} = \frac{V/d}{B^2 R} = \frac{Vx}{B^2 \ell L d}$$

where V = Potential difference between the two electrodes (i.e., P and Q)
d = distance between the two electrodes
R = radius of circular arc in the presence of magnetic field B
x = shift of the electron beam on the screen
ℓ = length of the field
L = distance between the centre of the field and the screen.

14 **Milliken's Oil Drop Method.** This determines the charge on the electron. Let ρ be the density of oil, σ is the density of the medium in which oil drop moves and η the coefficient of viscosity of the medium, then the radius r of the drop is

$$r = \sqrt{\frac{9}{2}\frac{\eta v_0}{(\rho - \sigma)g}}$$

where v_0 is the terminal velocity of the drop under the effect of gravity alone. At the terminal velocity v_0, the force due to viscosity becomes equal to the electric weight of the body. The charge on oil drop is

$$q = \frac{18\pi\eta(v_1 + v_0)}{E}\sqrt{\frac{\eta v_0}{2(\rho - \sigma)g}}$$

where v_1 is the terminal velocity of the drop under the influence of electric field and gravity and E is the applied electric field.

15 **Photocell.** It is an arrangement which converts light energy into electric energy. It works on the principle of photoelectric effect. It is used in cinematography for the reproduction of sound.

16 **Dual Nature of Radiation:** Light has dual nature. It manifests itself as a wave in diffraction, interference, polarization, etc.,

while it shows particle nature in photoelectric effect, Compton scattering, etc.

17 **Dual Nature of Matter:** As there is complete equivalence between matter (mass) and radiation (energy) and the principle of symmetry is always obeyed, de Broglie suggested that moving particles like protons, neutrons, electrons, etc., should be associated with waves known as de Broglie waves and their wavelength is called de Broglie wavelength. The de Broglie wavelength of a particle of mass m moving with velocity v is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

where h is Planck's constant.

18 **Davison and Germer Experiment.** This experiment confirms the existence of de Broglie waves associated with electrons.

19 **de Broglie Wavelength of an Electron.** The wavelength associated with an electron bean accelerated through a potential

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{12.3}{\sqrt{V}} \mathring{A}$$

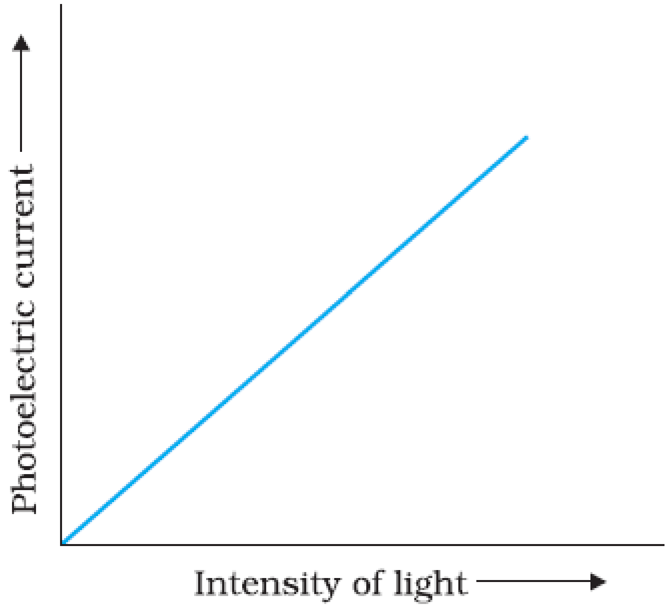
20 **Electron Microscope**: It is a device which makes use of accelerated electron beams to study very minute objects like viruses, microbes and the crystal structure of solids. It has a magnification of $\sim 10^5$.

<u>TOP Formulae</u>

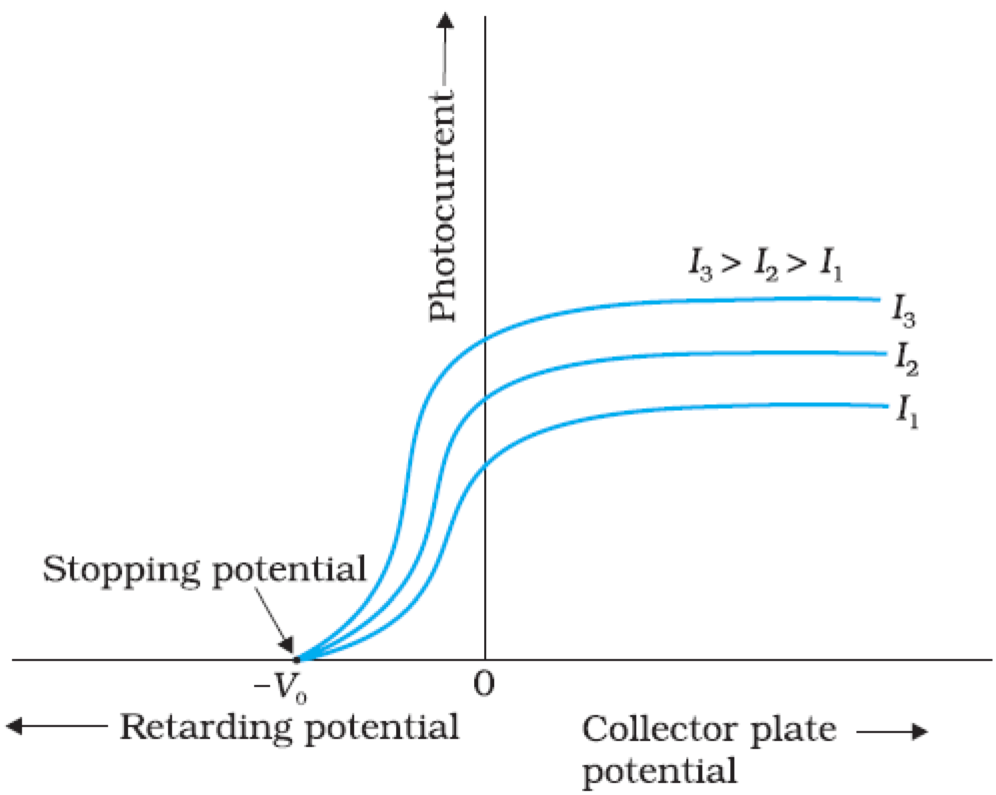
1. Maximum kinetic energy of the photoelectrons emitted from the metal surface: $K_{max} = eV_o = h\nu - \phi_o$ (Einstein's Photoelectric equation)
2. Work function of a metal surface:
   $w_o = \phi_o = h\nu_o$
3. de Broglie wavelength associated with the particle of momentum p is given as: $\lambda = \frac{h}{p} = \frac{h}{mv}$
   $\lambda = \frac{1.227}{\sqrt{V}} nm$, where V is the magnitude of accelerating potential
4. Heisenberg uncertainty principle:
   $\Delta x.\Delta p \approx h/2\pi$, where Δx is uncertainty in position & Δp is uncertainty in momentum

TOP Diagrams & Graphs:

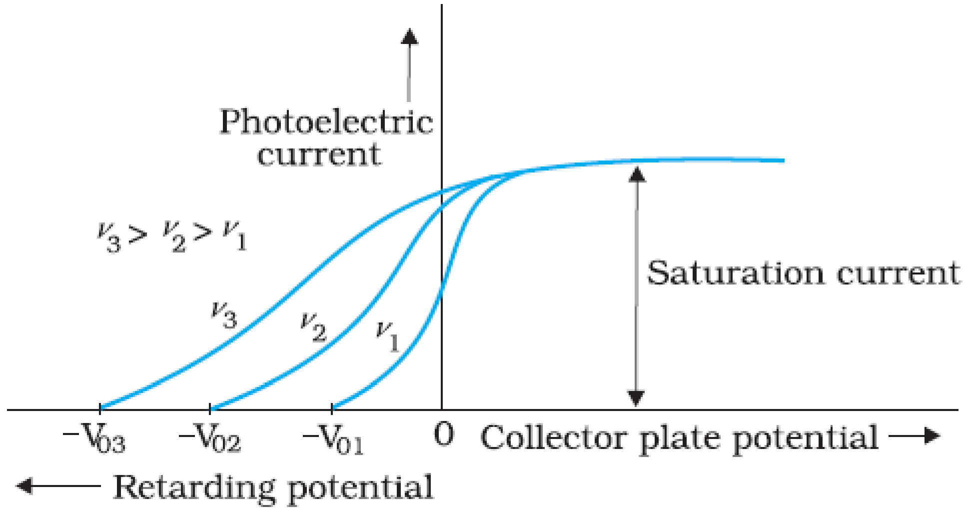
1. Variation of Photoelectron current with intensity of incident light:



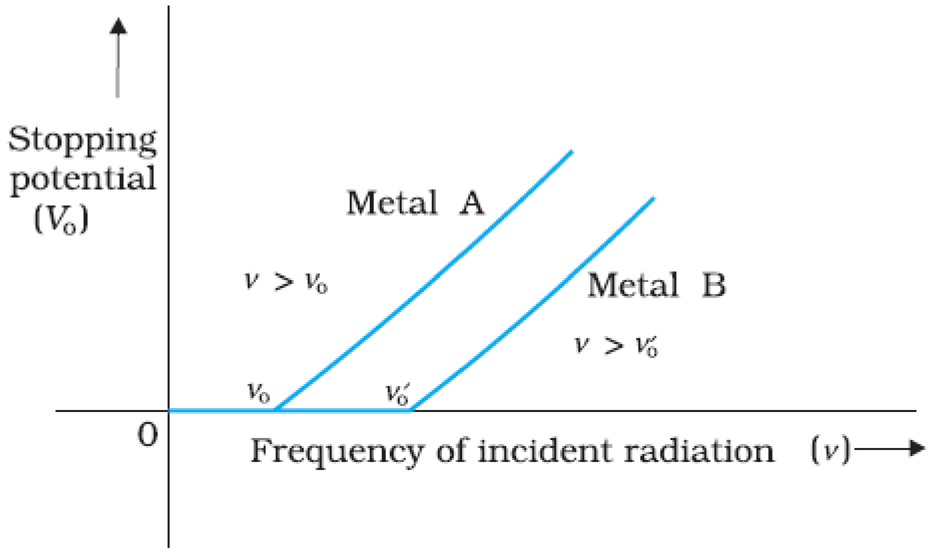
2. Variation of Photoelectron current with collector plate potential for different intensity of incident radiation



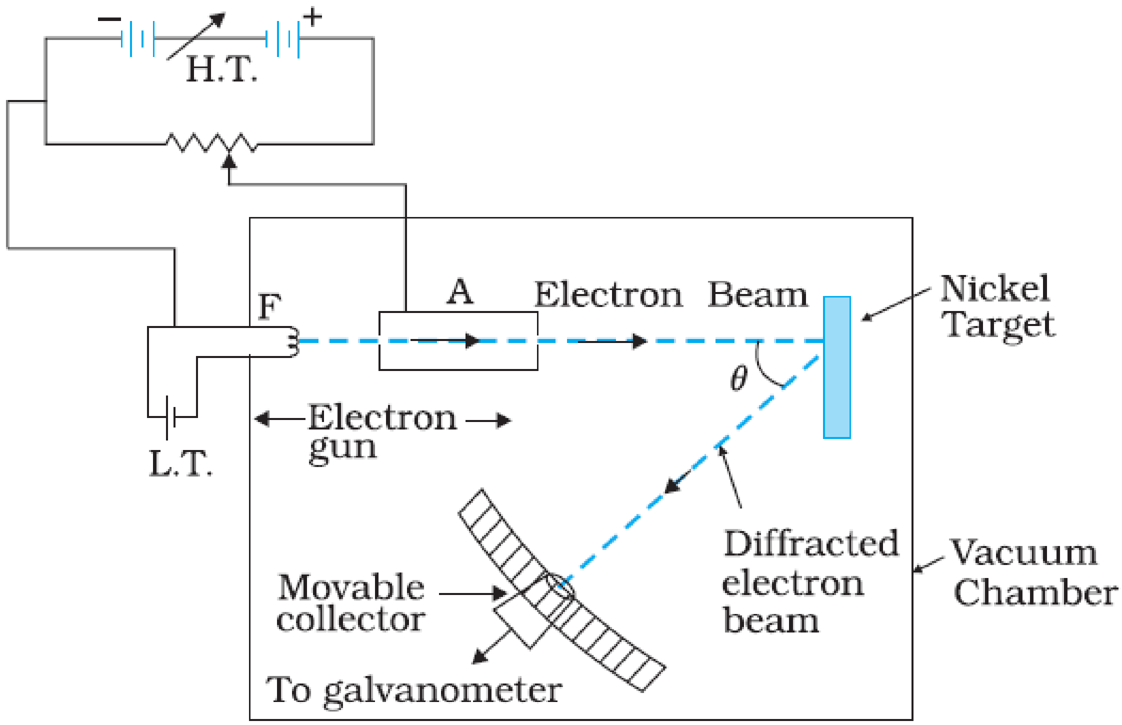
3. Variation of Photoelectron current with collector plate potential for different frequencies of incident radiation



4. Variation of stopping potential V_o with frequency ν of incident radiation for a given photosensitive material



5. David-Germar electron diffraction arrangement


H.T.
−
+
F
A
Electron Beam
Nickel Target
θ
Electron gun
L.T.
Diffracted electron beam
Vacuum Chamber
Movable collector
To galvanometer

Modern Physics

Rutherford Seattening :

Rutherford seattering experiments helped in understanding structure of atom. Rutherford bombarded a narrow beam of α – particles on a gold foil and observed visible light scintillations on a zinc surplide screen. Rutherford observed that :

(1) Most of the α – particles were either undeflected or deflected through small angles of the order of 1°.

(2) A few α – particles were deflected through angles as large as 90° or more.

Bohr Model of Hyrogen atom

Bohr proposed his theory of structure of atom on the basis of following assumptions :

(1) Electrons move in circular orbits about proton under the influence of coulomb force of attraction. These orbits are stationary states in which electrons do not continuously radiate electromagnetic energy.

(2) The emission or absorption of electron takes place only when there is a transition of electrons between two stationary states.

(3) The angular momentum of this system in a stationary state is an integral multiple of $\frac{h}{2\pi}(=h)$.

On the basis of these assumptions :

(a) Radius of nth Bohr's orbit

$$r_n = \frac{4\pi \in n^2 h^2}{me^2} = n^2 a_0$$

(b) Velocity of electron

$$V_n = \frac{1}{n}\frac{e^2}{4\pi \in_0 h} = \frac{V_0}{n} = \frac{1}{n}\frac{h}{ma_0}$$

(c) Energy of electron in nth orbit,

$$E = -\frac{e^2}{8\pi \in_0 r_n} = -\frac{e^2}{8\pi \in_0 a_0 n^2} = -\frac{E_0}{n^2} = \frac{13.6\,eV}{n^2}$$

A quanta of light is emitted when an atom in excited state decays to a lower energy state.

$$h\upsilon = E_f - E_i$$

(d) Frequency, wavelength, wave number of transitions

$$\frac{1}{\lambda} = \frac{\nu}{c} = \frac{E_f - E_i}{hc}$$

$$= -\frac{E_0}{hc}\left[\frac{1}{n_i^2} - \frac{1}{n_i^2}\right] \qquad = R\left[\frac{1}{n_i^2} - \frac{1}{n_f^2}\right]$$

Where $R = \frac{E_0}{hc}$ = Rydberg constant = $1.097373 \times 10^7\ m^{-1}$

X-RAYS

These are electromagnetic wave whose wavelengths typically range from 0.01 to 1 nm.

When an electron strikes a metallic target, before stopping it makes several collisions with atoms. Electrons may interact with the atom in either of the two ways :

(i) Due to strong nuclear electric field, electron is decelerated. In the process it radiates electromagnetic energy. Electron emits a series of photons with varying energy. These photons are x-rays. The x-rays produced in this process are called continuous x-rays.

(ii) When the high energy electron collides with one of the lower shell K electrons in a target atom, if enough energy can be transfered to this electron, the atom may be ionised. An electron from one of the higher shells will change its state and fill the inner shell vacancy at lower energy emitting radiation. The emitted radiation in heavy atoms is x-ray. Photons emitted in this way is called characteristic x-ray.

De Broglie or Matter Waves :

De Broglie proposed that material particles also have both wave and particle properties.

The wavelength to be associated with a particle is given by Planck's constant divided by the particle's momentum.

$$\lambda = \frac{h}{p}$$

This relation for photons was extended to all particles by De Broglie. Waves associated with particles are called matter waves, and the wavelength is called the de Broglie wavelength of a particle.

- The DeBroglie wavelength of the electron is large enough to be observed. Because of their small mass, electrons can have a small momentum and in turn a large wavelength.
- If m is the mass and v the velocity of the material particle, then

$$p = mv$$

$$\lambda = \frac{h}{mv}$$

- If E is the kinetic energy of the material particle, then

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$p = \sqrt{2mE}$$

Therefore, the de Broglie wavelength is given by $\lambda = \frac{h}{\sqrt{2mE}}$

- For electrons ($m_e = 9.1 \times 10^{-31}$ kg)

$$\lambda = \frac{h}{\sqrt{2mqV}} = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}V}}\,m = \frac{12.27}{\sqrt{V}}\,\mathring{A}$$

Bohr's Quantisation Condition :

The angular momentum of the electron in this orbit is L = rp. Using the above relation,

$$L = rp = \frac{nh}{2\pi} = n\hbar$$ which is Bohr's quantisation condition.

Atomic Nucleus

Nuclear size is of the order of femtometre (1 fm = 10^{-15} m).

The radius of a nucleus is given by $R = R_0 A^{1/3}$

The value of R_0 is, $R_0 \approx 1.2 \times 10^{-15}\ m \approx 1.2\ fm$

Radioactivity

1. Radioactive decay is a statistical process; we cannot precisely predict the timing of a particular radioactivity of a particular nucleus. The nucleus can disintegrate immediately or it may take infinite time. We can predict the probability of the number of nuclei disintegrating at an instant.
2. Radioactivity is independent of all the external conditions. We caannot induce radioactivity by applying strong electrical field, magnetic field, high temperature, high pressure, etc.
3. The energy liberated during radioactive decay comes from within individual nuclei.
4. When a nucleus undergoes alpha or beta decay, its atomic number changes and it transforms into a new element. Thus elements can be transformed from one to another.
5. The rate, at which a particular decay process occurs in a radioactive sample, is proportional to the number of radioactive nuclei present (i.e., those nuclei that have not decayed). If N is the number of radioactive nuclei present at some instant, the rate of change of N is,

$$\frac{dN}{dt} = -\lambda N$$

where λ is called decay constant. The minus sign indicates that $\frac{dN}{dt}$ is negative.

$$N = N_0 e^{-\lambda t}$$

where the constant N_0 represents the number of nuclei or radioactive nuclei at $t = 0$.

- Half life of a radioactive substance is the time it takes half of a given number of radioactive nuclei to decay. Setting $N = N_0/2$ and $t = T_{1/2}$ we get

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

Writing the above equation in the form $e^{\lambda T_{1/2}} = 2$ and taking natural logarithm of both sides, we get

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

- Mean life (average life) τ is defined as the average time the nucleus survives before it decays.

$$\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693}$$

PHOTOELECTRIC EFFECT

The phenomenon of emission of electrons from a metallic surface by the use of light (or radiant energy) of certain minimum frequency (or maximum wavelength) is called photoelectric effect.

The incident photon interacts with a single electron and loses its energy in two parts.

(a) Firstly, in getting the electron released from the bondage of the nucleus.

(b) Secondaly, to importing kinetic energy to emitted electron.

Accordingly, if hv is the energy of incident photon, then

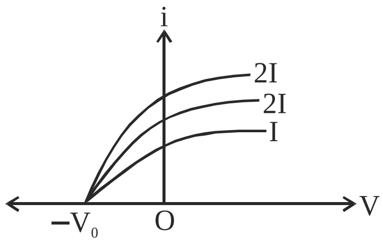
$$h\nu = \phi_0 + E_{max}$$

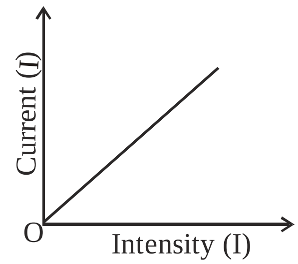
$$h\nu = h\nu_0 + K_{max.} = h\nu = e\nu_0$$

This is Einstein's photoelectric equation, where ϕ_0 is work function and $E_{max} = \frac{1}{2}mv_{max}^2 = eV_s$ is the maximum kinetic energy of photo-electrons emitted. ν_0 is the reverse potential differenic required to stop the electron from ejecting, called stopping potential.

Effect of Intensity

For a given frequency, if intensity of incident light is increased, the photoelectric current increases but the stopping potential remains the same. In photoelectric effect current (i) is directly proportional to intensity(I) of incident light.

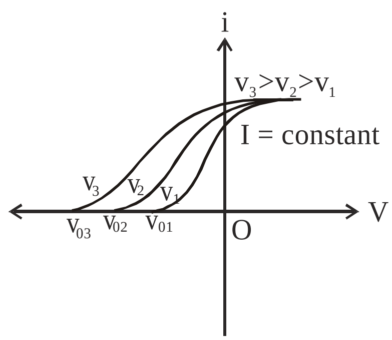




The intensity of incident light affects the photoelectric current but leaves the maximum kinetic energy of photoelectrons unchanged.

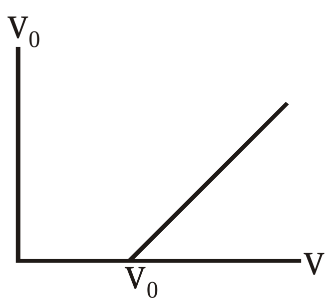
Effect of Frequency

When the intensity of incident light is kept fixed and frequency is increased, the photoelectric current remains the same but the stopping potential increases. If the frequency is decreased, the stopping potential decreases and at a particular frequency of incident light, the stopping potential becomes zero. This value of frequency of incident light for which the stopping potential is zero is called threshold frequency v_0, No photoelectric emission takes place below this frequency. Thus the increase of frequency increases maximum kinetic energyof photoelectrons but leaves the photoelectric current unchanged.



Effect of Photo-Metal

- When frequency and intensity of incident light are kept fixed and photo-metal is changed, we observe that stopping potential (V_0) versus frequency (ν) graphs are similar, cutting, frequency axis at different points. This shows that threshold frequency are different for different metals, the slope $\left(\frac{\nu_0}{\nu}\right)$ for all the metal is same and hence a universal constant.



Effect of Time

There is no time lag between incidence of light and the emission of photo-electrons.

CHAPTER 12

Atoms

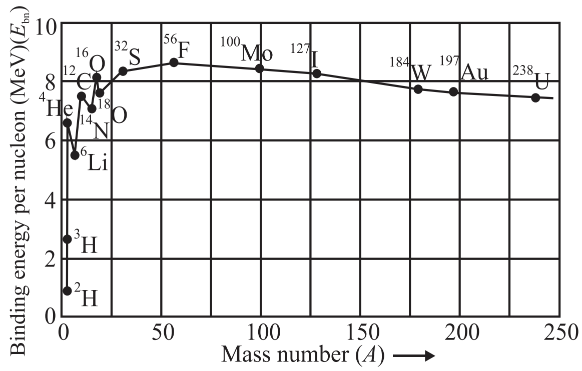
Sr. No.	Concept	Formulas	Other information
1.	**Distance of closest approach**	K.E. of α – particle, $K = \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}.\frac{2Ze^2}{r_0}$ Distance of closest approach, $r_0 = \frac{1}{4\pi\varepsilon_0}.\frac{2Ze^2}{K} = \frac{1}{4\pi\varepsilon_0}.\frac{4Ze^2}{mv^2}$	m = mass of α particle Z = atomic number e = charge of electron v = velocity of particle ε_0 = permittivity of free space
2.	**Bohr's atomic model**	Angular momentum of an electron; $L = mvr = \frac{nh}{2\pi},\ n = 1, 2, 3.......$ When electron jumps from higher energy level to lower energy level: $h\upsilon = E_2 - E_1$	n = principal quantum number v = frequency of emitted radiation. h = Planck's constant E_1, E_2 = energies associated with the orbits of principal quantum numbers n_1, n_2, respectively ($n_2 > n_1$).
3.	**Bohr's theory of hydrogen atom**	Radius of n^{th} orbit, $r_n = \frac{n^2h^2}{4\pi^2mke^2} = r_0n^2$ r_0 = 0.53Å Speed of electron in n^{th} orbit, $v = \frac{2\pi\, ke^2}{nh} = \alpha\frac{c}{n} = \frac{1}{137}.\frac{c}{n}.$ Total energy of an electron in n^{th} orbit is $E_n = K.E. + P.E. = \frac{ke^2}{2r} - \frac{ke^2}{r} = -\frac{ke^2}{2r}$ or $E_n = -\frac{2\pi^2mk^2e^4}{n^2h^2}$ $= \frac{-2.17\times10^{-18}Z^2}{n^2}\text{J} = -\frac{13.6}{n^2}eV$ Frequency of electron; $\upsilon_0 = \frac{1}{T} = \frac{v}{2\pi r} = \frac{4\pi^2k^2me^4}{n^3h^3}$ $\therefore\ \upsilon_0 \propto \frac{1}{n^3}$	R = Rydberg's constant $= 1.0973 \times 10^7\ m^{-1}$ h = Planck's constant m = mass of electron $k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{Nm}^2\text{C}^{-2}$ e = charge of electron (1.6×10^{-19} C)

Sr. No.	Concept	Formulas	Other information
4.	**For hydrogen like atom (He$^+$, Li^{++} etc)**	$r_n = \frac{r_0 n^2}{Z}$ $v_n = \frac{Z}{n}\left(\frac{c}{137}\right)$ $E_n = \frac{-13.6\, Z^2}{n^2} eV$	r_n = radius of n^{th} orbit v_n = speed of electron in n^{th} orbit E_n = energy of electron in n^{th} orbit c = 3 × 10^8 m/s Z = atomic number of element of hydrogen like atom.
5.	**Electron energy in hydrogen atom**	$E_n = \frac{-13.6}{n^2} eV$ Relation between $K.E$ and $P.E$; $K.E = -E_n$ $P.E = 2E_n$ or $P.E = -2K.E$	E_n = energy of the n^{th} orbit. n = orbit number K.E. = kinetic energy of electron in n^{th} orbit. P.E. = potential energy of electron in n^{th} orbit.
6.	**Spectral series of hydrogen atom**	Frequency, $\upsilon = \frac{me^4}{8\epsilon_0^2 h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$ Wave number, $\bar{\upsilon} = \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$ $R = \frac{me^4}{8\epsilon_0^2 h^3 c} = 1.097 \times 10^7\, m^{-1}$	υ = frequency of electron n_1 = principal quantum number of lower energy orbit. λ = wavelength of emitted spectral line. n_2 = principal quantum number of higher energy orbit.
7.	**Different spectral series for hydrogen atom**	(i) Lyman Series: Here $n_2 = 2,3,4,\ldots$ and $n_1 = 1$. $\bar{\upsilon} = \frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n_2^2}\right]$ (ii) Balmer Series. Here $n_2 = 3,4,5,\ldots$ and $n_1 = 2$. $\bar{\upsilon} = \frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right]$ (iii) Paschen Series. Here $n_2 = 4,5,6,\ldots$ and $n_1 = 3$. $\bar{\upsilon} = \frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right]$ (iv) Brackett Series. Here $n_2 = 5,6,7,\ldots$. and $n_1 = 4$. $\bar{\upsilon} = \frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n_2^2}\right]$ (v) Pfund Series. Here $n_2 = 6,7,8,\ldots$ and $n_1 = 5$ $\bar{\upsilon} = \frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n_2^2}\right]$	$\bar{\upsilon}$ = wave number R = Rydberg's constant.

CHAPTER 13

Nuclei

Sr. No.	Concept	Formulas	Other information
1.	**Atomic and mass number**	Symbol of nucleus: ${}_ZX^A$ or ${}_Z^AX$ Atomic number (Z) = Number of protons Mass number $(A) = Z + N$ $(N) = A - Z$	N = number of neutrons X = symbol of element
2.	**Size of nucleus**	Volume, $V = \frac{4}{3}\pi R^3 \propto A \Rightarrow R \propto A^{1/3}$ $\Rightarrow \boxed{R = R_0 A^{1/3}} \Rightarrow R_0 = 1.2 \times 10^{-15}$	R = radius of nucleus A = mass number of the nucleus.
3.	**Nuclear density**	Density of nuclear matter $= \frac{\text{mass of nucleus}}{\text{volume of nucleus}}$ $\rho = \frac{mA}{\frac{4}{3}\pi R_0^3 A}$ or $\rho = \frac{3m}{4\pi R_0^3} = 2.38 \times 10^{17}$ kg m^{-3}	m = average mass of a nucleon = 1.67×10^{-27} kg A = mass number $R_0 = 1.2 \times 10^{-15}$ m
4.	**Mass defect of nucleus**	Δm = (Mass of protons + Mass of neutrons) – Mass of the nucleus $\Delta m = [Zm_p + (A-Z)m_n - m_N]$ a.m.u. 1 a.m.u. $\boxed{(\text{or } 1\text{u}) = 1.66 \times 10^{-27} \text{ kg}}$ Packing fraction = $\boxed{\frac{\text{Actual mass of nucleus} - \text{Mass number}}{\text{Mass number}} = \frac{m-A}{A}}$	Z = atomic number m_p = mass of a proton = 1.0073 a.m.u. = 1.673×10^{-27} kg m_n = mass of a neutron = 1.0087 a.m.u. = 1.675×10^{-27} kg m_N = mass of the nucleus
5.	**Energy-mass relation**	According to Einstein, $E = mc^2$	E = Energy equivalent of mass m c = speed of light
6.	**Binding energy of nucleus**	Binding energy $= \Delta mc^2 = \left[\{Zm_p + (A-Z)m_n\} - m_N\right].c^2$ If Δm is measured in a.m.u. $E_b = \Delta m \times 931$ MeV Where, $\Delta m = \left[\{Zm_p + (A-Z)m_n\} - m_N\right]$a.m.u. Binding energy per nucleon $= \frac{\Delta m \times 931 \text{ MeV}}{A}$	E_b = binding energy Δm = mass defect A = mass number 1 a.m.u. $\simeq 931$ MeV

Sr. No.	Concept	Formulas	Other information
7.	**Binding energy curve**		E_{bn} = binding energy per nucleon
8.	**Nuclear reaction**	Nuclear fission; $^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n + Q.$ Nuclear fusion. $^{2}_{1}H + ^{2}_{1}H \longrightarrow ^{4}_{2}He + 24MeV.$	Q = Amount of heat released
9.	**Q-Value**	Energy of nuclear fission or fusion = (Mass of reactants – Mass of products) × 931 MeV	(Here all mass values are taken in a.m.u.)

SEMICONDUCTORS

Semiconductors : Electrical properties of which lie between conductor and insulators.

Semiconductors are materials that have a small energy gap of the order of 1eV. At 0K (absolute zero), the semiconductors behave like insulators.

Intrinsic Semiconductors (pure) : Semiconductor which are free from inpurity.

Intrinsic Semiconductors have an equal number of electrons in conduction band and holes in valence band

$n_e = n_h$

where n_e = number of electrons per unit volume

n_h = number of holes per unit volume

$n_e \times n_h = n_i^2$

n_i = intrinsic charge carrier density or intrinsic charge carrier concentration

Doped or Extrinsic Semiconductors : Semi conductors doped or added with certian impurity to increase its conductivity.

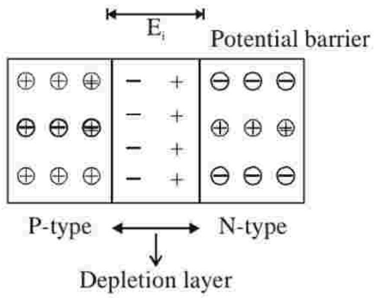
N-Type : In N type of semiconductor electrons are majority charge carriers and holes are minority charge carriers.

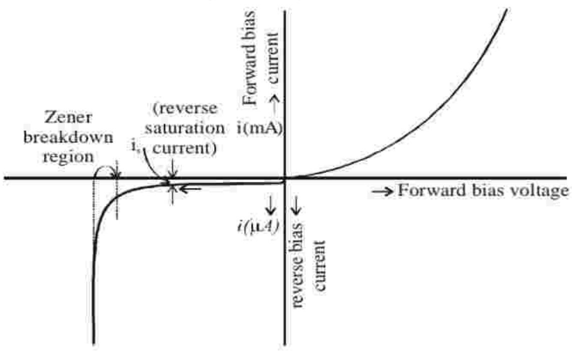
P Type : In P type semiconductors holes are majority charge carriers whereas electrons are minority charge carriers.

Semiconductor Devices

The P-N junction Diode :

P side of P-N junction has holes as a majority charge carriers and electrons as a minority charge carriers whereas N side has electrons as a majority charge carriers and holes as a minority charge carriers. holes diffuse from P side to N side whereas electrons diffuse from N side to P side.





P-N Junction Characteristics

Dynamic Resistance :

$$R = \frac{\Delta V}{\Delta i}$$

Where ΔV denotes a small change in the applied potential difference and Δi denotes corresponding small change in current.

Dynamic Resistance is equal to the reciprocal of the slope of the i-V characteristic.

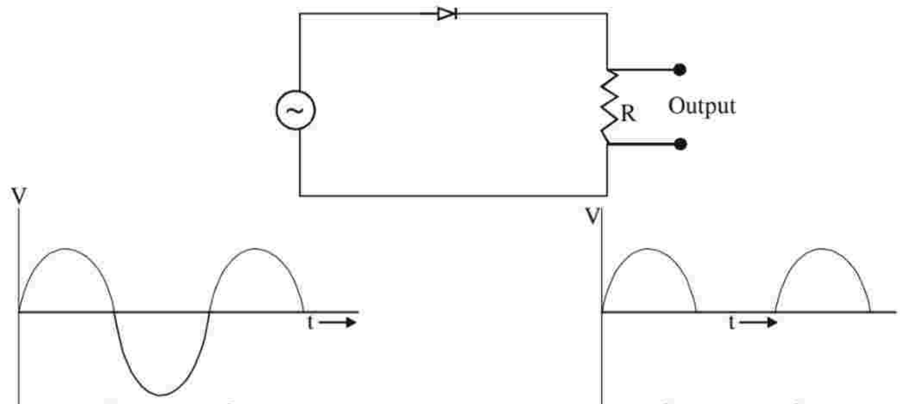
Photodiode : When a light of proper wavelength falls on the junction, new electron-hole pairs are created. The number of charge carriers increases and hence the conductivity of the junction increases. If the junction is connected in some circuit, the current in the circuit is controlled by the intensity of the incident light.

Light-emitting Diode (LED) : When a conduction electron makes a transition to the valence band to fill up a hole in P-N junction, the extra energy is emitted as a photon. If the wavelength of this photon is in the visible range one can see the emitted light. Such a P-N junction is known as light emitting diode (LED).

Zener diode : A diode operated in Zener break down mode is called Zener diode. In this type of mode of operation current increases rapidly but voltage remains almost constant. Thus it is used to obtain constant voltage output.

P-N Junction as a Rectifier : PN Junction can be used to convert A.C into unidirectional current. (DC)

(a) Half wave rectifier



Input wave form

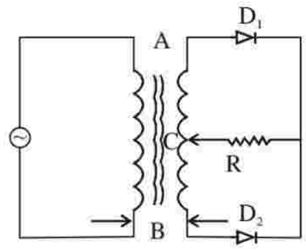
Output wave form

Average out put current $= \frac{I_0}{\pi}$

where I_o = amplitude of the input current

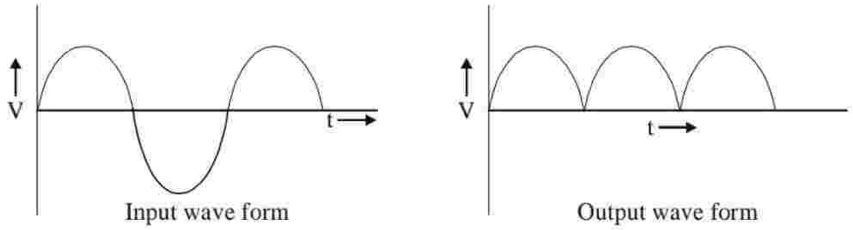
R.M.S. Value of output current $= \frac{I_0}{2}$

(b) Full Wave rectifier



If $V_A > V_C > V_B$ — D_1 Conducts

If $V_B > V_C > V_A$ — D_2 Conducts



Input wave form — Output wave form

Average output current $= \frac{2I_0}{\pi}$

R.M.S. Value of output current $= \frac{I_0}{\sqrt{2}}$

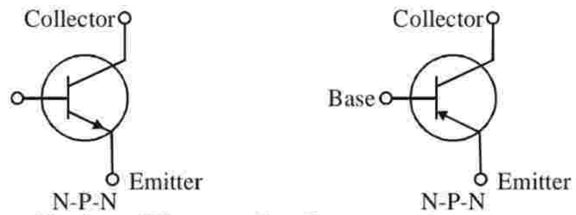
Junction transistor

It has three terminals

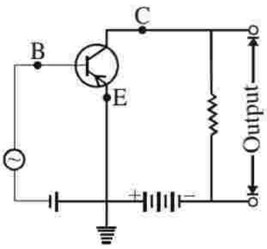
(a) Emitter (b) Base (c) Collector

Emitter is heavily doped, collector is moderately doped and base is thin and lightly doped.

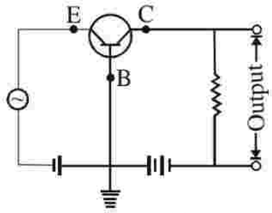
Symbol



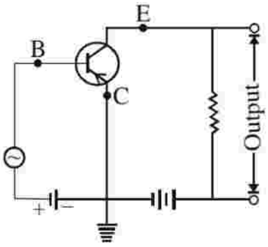
A transistor can be operated in three different modes. Common-emitter, common collector and common base.



Common Emitter



Common Base



Common Collector

Transistor as an Amplifier

1. Common base :

In this type amplifier, base to emitter junction is forward biased whereas base to collector is reverse biased

Transistor parameter

Current again $= \alpha = \dfrac{I_0}{I_E}$; AC current gain $= \dfrac{\Delta I_c}{\Delta I_e}$

Voltage gain $= A_v = \dfrac{\Delta v_0}{\Delta v_i} = \dfrac{I_0 R_{out}}{I_E R_{in}}$ = Current gain × Resistance gain $= \alpha \times \dfrac{R_o}{R_i}$

Where R_o = Resistance of the output circuit

R_i = Resistance of the input circuit

Power gain $= \alpha^2 \times$ Resistance gain

2. Common emitter amplifier

Current gain $\beta = \dfrac{\Delta I_c}{\Delta I_b}$

Voltage gain $(A_v) =$ Current gain × Resistance gain $= \beta \times A_v$

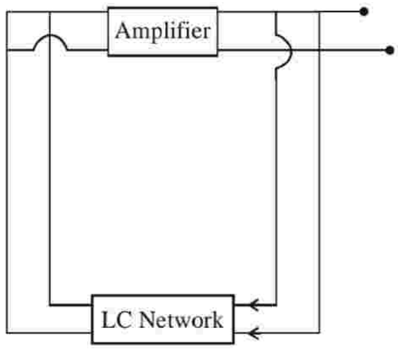
Power gain $= \beta^2 \times$ Resistance gain

Resistance gain $= \dfrac{R_o}{R_i}$

Transistor used as an oscillator converts D.C. into A.C.

Amplifier section is just a transistor used in common-emitter mode.

[5]



$$f_0 = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

Part of output energy is sent back in phase to input circuit. This is also called positive feed back.