CHAPTER 1

Electric Charges and Fields

Electric Charge

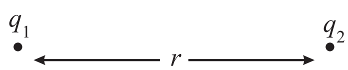
Charge is a property of matter due to which two bodies interact with each other electromagnetically. There are two kinds of charges- positive and negative. SI unit is coulomb. Charge is quantized and additive.

Coulomb's Law

Force between two charges

$$\vec{F} = \frac{1}{4\pi \in_0 \in_r} \frac{q_1 q_2}{r^2} \hat{r}$$

$\in_r$ = dielectric constant of the medium, $\in_0$ = permittivity of free space



- ❖ Coulomb's Law is applicable only for static point charges.

Principle of Superposition

Force on a point charge due to many charges is given by

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \text{........}$$

- ❖ The force due to one charge is not affected by the presence of other charges.

Electric Field or Electric Field Intensity

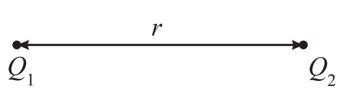
$$\vec{E} = \frac{\vec{F}}{q}$$

Unit is N/C or V/m.

Electric Field due to Point Charge Q

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \frac{1}{4\pi \in_0} \frac{Q}{r^2} \hat{r}$$

Null Point for Two Charges



If $|Q_1| > |Q_2|$

$\Rightarrow$ Null point is near Q_2

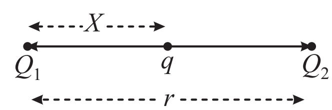
$$x = \frac{\sqrt{Q_1}\, r}{\sqrt{Q_1} \pm \sqrt{Q_2}}$$

$x \rightarrow$ distance of null point from Q_1 charge

(+) for like charges [null point will be in between Q_1 & Q_2]

(–) for unlike charges [null point will be outside Q_1 & Q_2 and near weaker charge]

Equilibrium of Three Point Charges



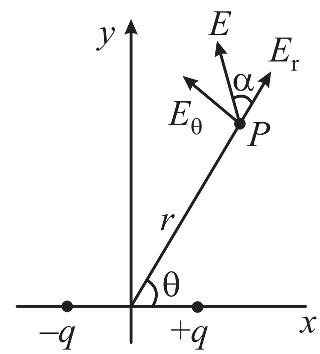
(i) Q_1 & Q_2 must be of like nature.

(ii) q should be of unlike nature.

$$x = \frac{\sqrt{Q_1}}{\sqrt{Q_1} + \sqrt{Q_2}} r \quad \text{and} \quad q = \frac{-Q_1 Q_2}{\left(\sqrt{Q_1} + \sqrt{Q_2}\right)^2}$$

Electric Dipole

- ❖ Electric dipole moment $\vec{p} = q\vec{d}$, where $\vec{d}$ is distance from negative to positive charge.
- ❖ Torque on dipole placed in a uniform electric field $\vec{\tau} = \vec{p} \times \vec{E}$
- ❖ Electric field at a general point due to a dipole.



Electric field: $E = \frac{1}{4\pi \in_0} \frac{p\sqrt{1 + 3\cos^2\theta}}{r^3}$

Direction: $\tan\alpha = \frac{E_\theta}{E_r} = \frac{1}{2}\tan\theta$

- Electric field at an axial point (or End-on) of dipole

$$\vec{E} = \frac{1}{4\pi \in_0}\frac{2\vec{p}}{r^3}$$

- Electric field at an equatorial position (Broad-on) of dipole

$$\vec{E} = \frac{1}{4\pi \in_0}\frac{(-\vec{p})}{r^3}$$

Electric Lines of Force

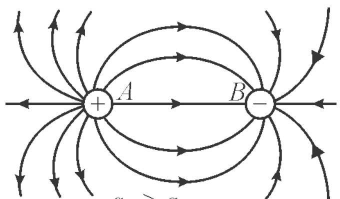
Electric lines of electrostatic field have following properties.

(i) Imaginary

(ii) Can never cross each other

(iii) Can never form closed loops

(iv) The number of lines originating or terminating on a charge is proportional to the magnitude of charge.



$q_A > q_B$

(v) Lines of force end or start normally at the surface of a con ductor.

(vi) If there is no electric field there will be no lines of force.

(vii) Lines of force per unit area normal to the area at a point represents magnitude of intensity. Crowded lines represent strong field while distant lines represent weak field.

(viii) Tangent to the line of force at a point in an electric field gives the direction of Electric Field.

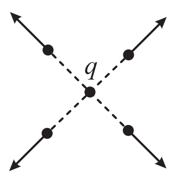
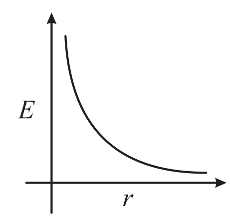
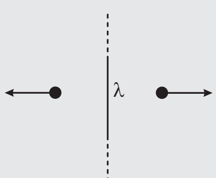
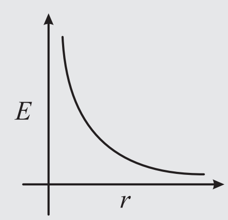
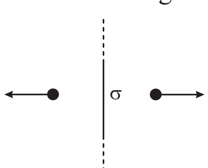
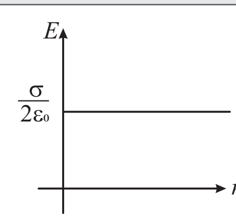
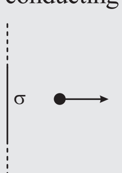
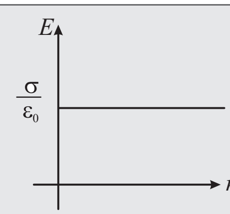
Gauss' Law

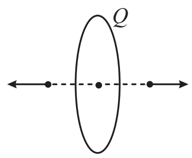
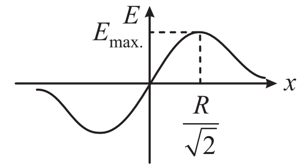
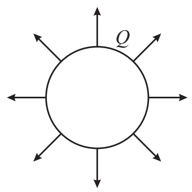
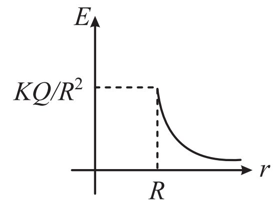
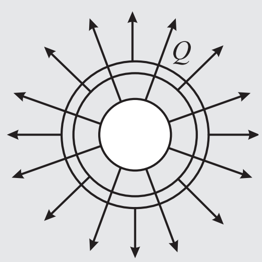
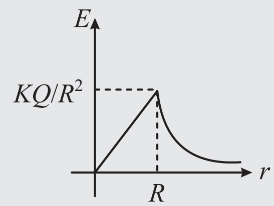
Electric flux: $\phi = \int \vec{E}.d\vec{s}$

Expression for Gauss' Law: $\oint \vec{E}.d\vec{s} = \frac{\sum q_{\text{enlosed}}}{\epsilon_0}$

(Applicable only on closed surface)

Net flux emerging out of a closed surface is $\frac{q_{en}}{\varepsilon_0}$

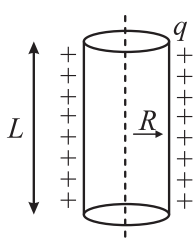
Name/Type	Formula	Note	Graph
Point charge 	$\vec{E} = \frac{kq}{\lvert\vec{r}\rvert^2}\hat{r}$	❖ q is source charge. ❖ $\hat{r}$ is unit vector drawn from source charge to the point. ❖ Outwards due to + ve charges and inwards due to – ve charges.	
Infinitely long line charge 	$\frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$ or $\frac{2k\lambda}{r}\hat{r}$	❖ λ is linear charge density (assumed uniform) ❖ r is perpendicular distance of point from line charge. ❖ $\hat{r}$ is radial unit vector drawn from the line charge to the point.	
Infinite non-conducting thin sheet 	$\frac{\sigma}{2\varepsilon_0}\hat{n}$	❖ σ is surface charge density. (assumed uniform) ❖ $\hat{n}$ is unit normal vector	
Infinite non-conducting thin sheet 	$\frac{\sigma}{\varepsilon_0}\hat{n}$	❖ σ is surface charge density. (assumed uniform) ❖ $\hat{n}$ is unit normal vector	

Uniformly charged ring 	$E = \dfrac{kQx}{\left(R^2 + x^2\right)^{3/2}}$ $E_{\text{centre}} = 0$	❖ Q is total charge of the ring ❖ x = distance of point on the axis from center of the ring. ❖ Electric field is always along the axis. (away from ring if Q is +ve, towards ring if Q is –ve.)	
Uniformly charged hollow conducting/non-conducting/ solid conducting sphere 	(i) for $r \geq R$ $\vec{E} = \dfrac{kQ}{\lvert\vec{r}\rvert^2}\hat{r}$ (ii) For r < R $E = 0$	❖ R is radius of the sphere. ❖ $\vec{r}$ is vector drawn from center of sphere to the point. ❖ Sphere acts like a point charge placed at center for points outside the sphere. ❖ $\vec{E}$ is always along radial direction. ❖ Q is total charge ($= \sigma 4\pi R^2$). (σ = surface charge density)	
Uniformly charged solid non-conducting sphere (insulating material) 	(i) for $r \geq R$ $\vec{E} = \dfrac{kQ}{\lvert\vec{r}\rvert^2}\hat{r}$ (ii) for $r \leq R$ $\vec{E} = \dfrac{kQ\lvert\vec{r}\rvert}{R^3}\hat{r}$	❖ $\vec{r}$ is vector drawn from center of sphere to the point ❖ Sphere acts like a point charge placed at the center of points outside the sphere ❖ $\vec{E}$ is always along radial direction ❖ Q is total charge $\left(\rho \cdot \frac{4}{3}\pi R^3\right)$. ($\rho$ = volume charge density) ❖ Inside the sphere $E \propto r$. ❖ Outside the sphere $E \propto 1/r^2$.	

For a charged Long Conducting Cylinder of length L

❖ For $r \geq R : E = \dfrac{q}{2\pi \epsilon_0 rL}$

❖ For $r < R : E = 0$



❖ Electric field intensity at a point near a charged conductor $E = \dfrac{\sigma}{\epsilon_0}$

❖ For conducting infinite sheet of surface charge density σ,

$E = \dfrac{\sigma}{\epsilon_0}$

❖ Energy density in electric field $U = \dfrac{\epsilon_0}{2}E^2$

CHAPTER

2 Electrostatic Potential and Capacitance

Relation between $\vec{E}$ & V

$$\vec{E} = -\text{grad}\,V = -\nabla V$$

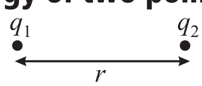
$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}, \Delta V = -\int_{\vec{r}_1}^{\vec{r}_2} -\vec{E}.\vec{d}r$$

For Spherical and cylindrical symmetry

$$E = \frac{-dv}{dr}$$

Electric Potential Energy of two point Charges

$$U = \frac{1}{4\pi \epsilon_0}\frac{q_1 q_2}{r}$$



Electric Potential due to an Electric Dipole

- At a point which is at a distance r from midpoint of dipole and making angle θ with dipole axis.

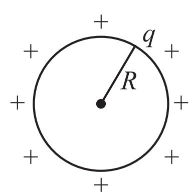
Potential $V = \frac{1}{4\pi \epsilon_0}\frac{p\cos\theta}{r^2}$

Equipotential Surface and Equipotential Region

In an electric field the locus of points of equal potential is called an equipotential surface. An equipotential surface and the electric field lines meet at right angles. In a region where $E = 0$, potential of the whole region must remain constant as no work is done in displacement of charge in it. It is called as equipotential region. Material of conductors is an equipotential region.

Potential due to Various Bodies

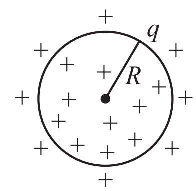
For a Conducting Sphere



For $r \geq R : E = \frac{1}{4\pi \epsilon_0}\frac{q}{r^2}, V = \frac{1}{4\pi \epsilon_0}\frac{q}{r}$

For $r < R : E = 0,\ V = \frac{1}{4\pi \epsilon_0}\frac{q}{R}$

For a non-conducting Sphere

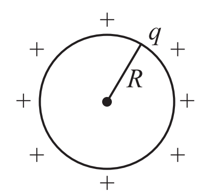


For $r \geq R : E = \frac{1}{4\pi \epsilon_0}\frac{q}{r^2}, V = \frac{1}{4\pi \epsilon_0}\frac{q}{r}$

For $r < R : E = \frac{1}{4\pi \epsilon_0}\frac{qr}{R^3}, V = \frac{1}{4\pi \epsilon_0}\frac{q(3R^2 - r^2)}{2R^3}$

$$V_{\text{center}} = V_{\max} = \frac{3}{2}\frac{kq}{R} = 1.5V_{\text{surface}}$$

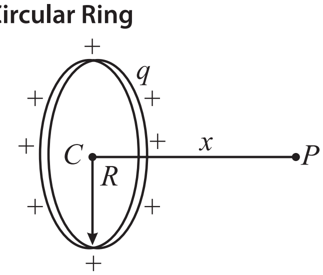
For a Conducting/non Conducting Spherical shell



For $r \geq R : E = \frac{1}{4\pi \epsilon_0}\frac{q}{r^2}, V = \frac{1}{4\pi \epsilon_0}\frac{q}{r}$

For $r < R : E = 0,\ V = \frac{1}{4\pi \epsilon_0}\frac{q}{R}$

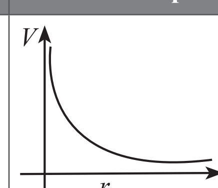
For a Charged Circular Ring

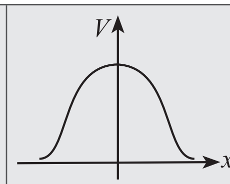
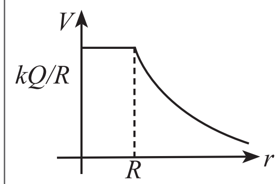
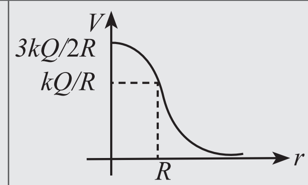


$$E_P = \frac{1}{4\pi \epsilon_0}\frac{qx}{(x^2 + R^2)^{3/2}}, V_p = \frac{q}{4\pi \epsilon_0 (x^2 + R^2)^{1/2}}$$

Electric field will be maximum at $x = \pm\frac{R}{\sqrt{2}}$

Electric potential will be maximum at $x = 0$

Name/Type	Formula for Potential	Note	Graph
Point charge	$\frac{kq}{r}$	❖ q is source charge. ❖ r is the distance of the point from the point charge.	

Ring (uniform/ non uniform charge distribution)	At center: $\frac{kQ}{R}$ At the axis $\frac{kQ}{\sqrt{R^2+x^2}}$	❖ Q is source charge. ❖ x is the distance of the point on the axis from center of ring	
Uniformly charged hollow conducting/ non conducting/solid conducting sphere	For $r \geq R$, $V = \frac{kQ}{r}$ For $r \leq R$, $V = \frac{kQ}{R}$	❖ R is radius of sphere ❖ r is the distance from center of sphere to the point ❖ Q is total charge = $\sigma 4\pi R^2$.	
Uniformly charged solid nonconducting sphere	For $r \geq R$, $V = \frac{kQ}{r}$ For $r \leq R$, $V = \frac{kQ(3R^2 - r^2)}{2R^3}$ $= \frac{\rho}{6\varepsilon_0}(3R^2 - r^2)$	❖ R is radius of sphere ❖ r is distance from center to the point ❖ $V_{\text{center}} = \frac{3}{2} V_{\text{surface}}$. ❖ Q is total charge $= \rho \frac{4}{3}\pi R^3$. ❖ Inside the sphere, potential varies parabolically ❖ Outside the sphere potential varies hyperbolically.	
Infinite line charge	Not defined	❖ Absolute potential is not defined. ❖ Potential difference between two points is given by formula: $V_B - V_A = -2k\lambda \ln(r_B/r_A)$	
Infinite nonconducting thin sheet	Not defined	❖ Absolute potential is not defined. ❖ Potential difference between two points is given by formula $V_B - V_A = -\frac{\sigma}{2\varepsilon_0}(r_B - r_A)$	
Infinite charged conducting thin sheet	Not defined	❖ Absolute potential is not defined. ❖ Potential difference between two points is given by formula $V_B - V_A = -\frac{\sigma}{\varepsilon_0}(r_B - r_A)$	

CHAPTER

3 Current Electricity

Electric Current

I_{av}(average current) $= \dfrac{\Delta q}{\Delta t}$

I (instantaneous current) $= \lim\limits_{\Delta t \to 0} \dfrac{\Delta q}{\Delta t} = \dfrac{dq}{dt}$

SI unit : Ampere

Electric Current in a Conductor

$I = nqAv_d$

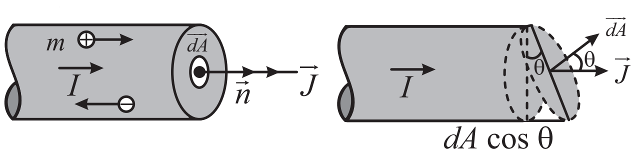
where I = current, n = number of charge carriers per unit volume, A = area of cross section, v_d = drift velocity.

$$\vec{v}_d = -\frac{e\vec{E}}{m}\tau$$

where e = charge of electron, m = mass of electron, $\vec{E}$ = electric field, τ = relaxation time.

Current Density(J) and Mobility (μ)

$$\vec{J} = \frac{ne^2}{m}\tau\vec{E}$$



$$I = \int \vec{J}.\vec{dA}$$

$$\mu = \frac{|v_d|}{E} = \frac{e\tau}{m}$$

(**SI unit:** m^2/Vs)

Electrical Resistance (R) and Ohm's Law

$$I = neAv_d = neA\left(\frac{eE}{m}\right)\tau$$

$$E = \frac{V}{l}$$

So, $I = \left(\dfrac{ne^2\tau}{m}\right)\left(\dfrac{A}{l}\right) \times V = \left(\dfrac{A}{\rho l}\right) \times V = \dfrac{V}{R}$

$\Rightarrow V = IR$

ρ is called resistivity (it is also called specific resistance) and

$\rho = \dfrac{m}{ne^2\tau} = \dfrac{1}{\sigma}$, σ is called conductivity

SI Units: $R \to$ ohm (Ω), $\rho \to$ ohm–meter (Ω-m), $\sigma \to \Omega^{-1}m^{-1}$.

Dependence of Resistance on Temperature:

$R = R_0\,(1 + \alpha\;(T - T_0))$.

α = thermal coefficient of resistivity (**positive for conductors and negative for semi conductors and insulators**)

Electrical Power

$P = VI$

Energy $= \int P dt$

$P = I^2 R = VI = \dfrac{V^2}{R}$, Heat: $H = VIt = I^2\;Rt = \dfrac{V^2}{R}t$

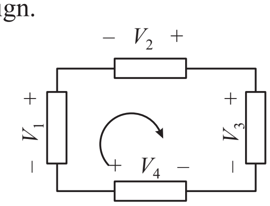
Kirchhoff's Laws

I. Law (Junction law or Nodal Analysis): This law is based on law of conservation of charge. It states that "The algebraic sum of the currents meeting at a junction is zero" or 'total currents entering a junction equals total current leaving the junction'.

$\Sigma I_{in} = \Sigma I_{out}$. It is also known as **KCL (Kirchhoff's current law)**.

II. Law (Loop analysis): The algebraic sum of all the voltages in closed circuit is zero.

$\Sigma\; IR + \;\Sigma\; EMF = 0$ in a closed loop . The closed loop can be traversed in any direction. While traversing a loop if higher potential point is entered, put a +ve sign in expression or if lower potential point is entered put a negative sign.



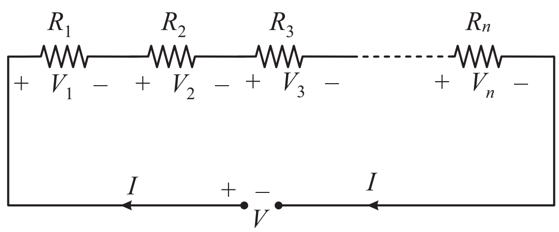
$-V_1 - V_2 + V_3 - V_4 = 0$. Boxes may contain resistor or battery or any other element (linear or non-linear).

It is also known as **KVL (Kirchhoff's voltage law).**

Combination of Resistances

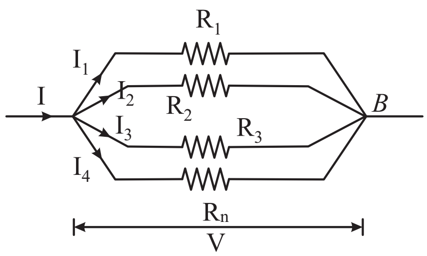
I. Resistances in Series:

$R = R_1 + R_2 + R_3 + ... + R_n$

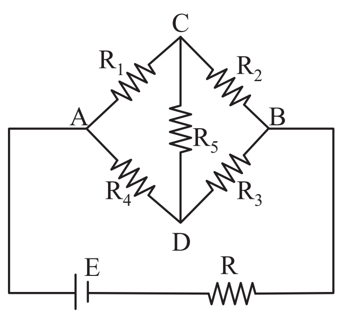


II. Resistances in Parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$$



Wheatstone Network

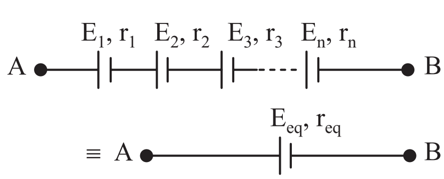


Current through the R_5 is zero (null point or balance point) if

$$\frac{R_1}{R_4} = \frac{R_2}{R_3}$$

Grouping of Cells

I. Cells in Series:



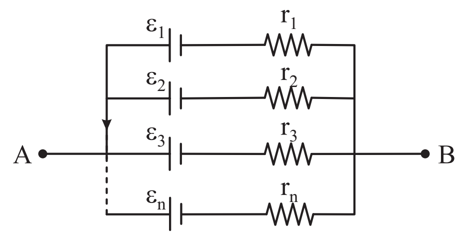
Equivalent EMF $E_{eq} = E_1 + E_2 + ... + E_n$ [write EMF's with polarity]

Equivalent internal resistance $r_{eq} = r_1 + r_2 + r_3 + r_4 + ... + r_n$

II. Cells in Parallel:

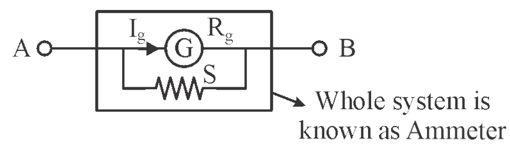
$$E_{eq} = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + ... + \frac{\varepsilon_n}{r_n}}{\frac{1}{r_1} + \frac{1}{r_2} + ... + \frac{1}{r_n}}$$ [Use emf with polarity]

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + ... + \frac{1}{r_n}$$

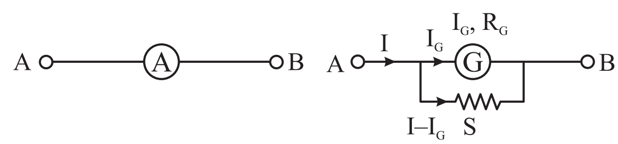


Ammeter

A shunt (small resistance) is connected in parallel with galvanometer to convert it into ammeter. An ideal ammeter has zero resistance



Ammeter is represented as follows :



If maximum value of current to be measured by ammeter is I then

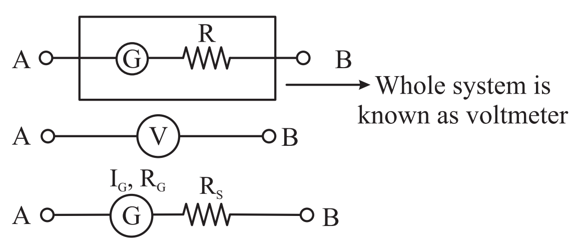
$I_G \cdot R_G = (I - I_G)\, S$

$$S = \frac{I_G \cdot R_G}{I - I_G} \Rightarrow S = \frac{I_G \times R_G}{I} \text{ (if } I >> I_G\text{)}.$$

where, I = Maximum current that can be measured using the given ammeter.

Voltmeter

A high resistance is put in series with galvanometer. It is used to measure potential difference across a resistor in a circuit.



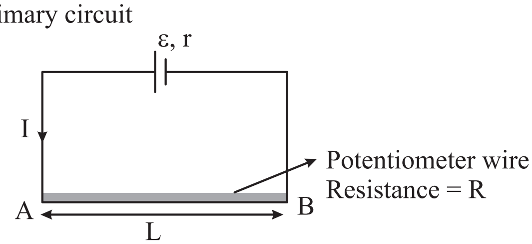
For maximum potential difference

$V = I_G \cdot R_S + I_G R_G$

If $R_G << R_S \Rightarrow R_S \approx \dfrac{V}{I_G}$

Potentiometer

Primary circuit



$$I=\frac{\varepsilon}{r+R};\quad V_A-V_B=\frac{\varepsilon}{R+r}\cdot R$$

Potential gradient (x) → Potential difference per unit length of wire

$$x=\frac{V_A-V_B}{L}=\frac{\varepsilon}{R+r}\cdot\frac{R}{L}$$

Applications of potentiometer

(*a*) To find emf of unknown cell and compare emf of two cells.

In case I,

In figure, (1) is joint to (2) then balance length = l_1

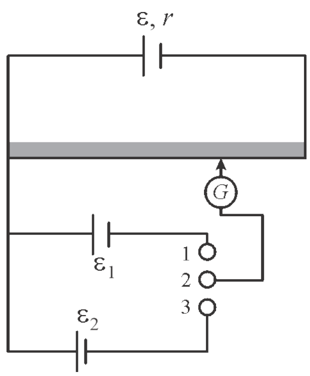
$$\varepsilon_1 = xl_1 \qquad \text{...(i)}$$

In case II,

In figure, (3) is joint to (2) then balance length = l_2

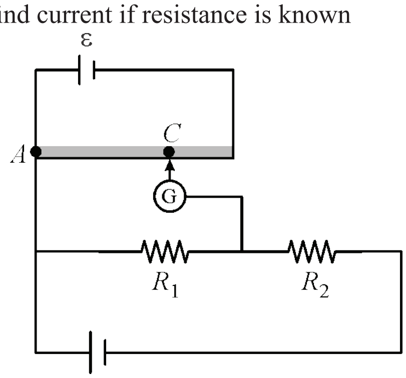
$$\varepsilon_2 = xl_2 \qquad \text{...(ii)}$$

$$\frac{\varepsilon_1}{\varepsilon_2}=\frac{l_1}{l_2}$$



If any one of ε_1 or ε_2 is known the other can be found. If x is known then both ε_1 and ε_2 can be found.

(*b*) To find current if resistance is known



$$V_A - V_C = xl_1$$

$$IR_1 = xl_1$$

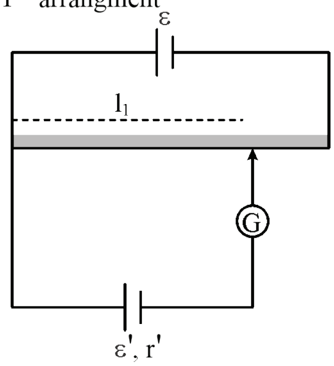
$$I=\frac{xl_1}{R_1}$$

Similarly, we can find the value of R_2 also.

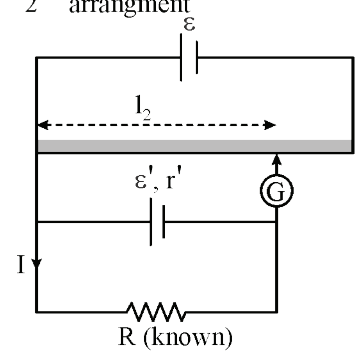
Potentiometer is ideal voltmeter because it does not draw any current from circuit at the balance point.

(*c*) To find the internal resistance of cell.

1[st] arrangment



2[nd] arrangment



By first arrangement $\quad \varepsilon' = xl_1 \quad$ ...(i)

By second arrangement $\quad IR = xl_2 \quad$ ...(ii)

$$I=\frac{xl_2}{R},\ \text{Also } I=\frac{\varepsilon'}{r'+R}$$

$$\therefore\ \frac{\varepsilon'}{r'+R}=\frac{xl_2}{R}\ \Rightarrow\ \frac{xl_1}{r'+R}=\frac{xl_2}{R}$$

$$r'=\left[\frac{l_1-l_2}{l_2}\right]R$$

Metre Bridge

It is used to measure unknown resistance.

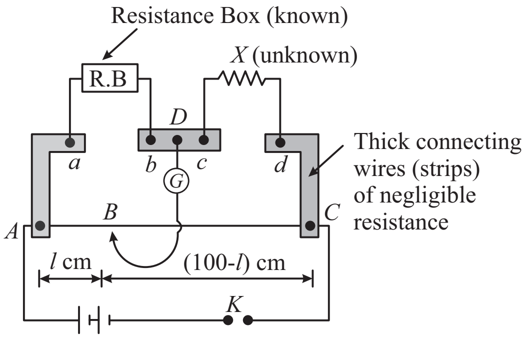
If $AB = l$ cm, then $BC = (100 - l)$ cm.

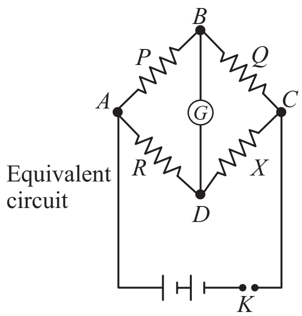
Resistance of the wire between A and B , $R \propto l$

[∵ Specific resistance ρ and cross-sectional area A are same for whole of the wire]

or $R = \sigma l$ ...(i)

where σ is resistance per cm of wire.





If P is the resistance of wire between A and B, then

$$P \propto l \quad \Rightarrow \quad P = \sigma (l)$$

Similarly, if Q is resistance of the wire between B and C, then

$$Q \propto 100 - l \quad \Rightarrow \quad Q = \sigma (100 - l) \quad \text{...(ii)}$$

Dividing (i) by (ii),

$$\frac{P}{Q} = \frac{l}{100 - l}$$

Applying the condition for balanced Wheatstone bridge, we get $R\,Q = P\,X$

$$\therefore \quad X = R\,\frac{Q}{P} \qquad \text{or} \qquad X = \frac{100 - l}{l} R$$

Since R and l are known, therefore, the value of X can be calculated.

CHAPTER 4

Moving Charges and Magnetism

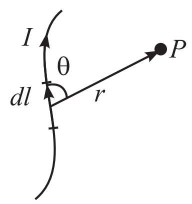
A static charge produces only electric field. A moving charge produces both electric field and magnetic field. A current carrying conductor produces only magnetic field.

Magnetic Field Produced by Current (Biot-Savart's Law)

The magnetic induction dB produced by an element dl carrying a current I at a distance r is given by:

$$dB = \frac{\mu_0\mu_r}{4\pi}\frac{I\,dl\sin\theta}{r^2} \Rightarrow \vec{dB} = \frac{\mu_o\mu_r}{4\pi}\frac{I(\vec{dl}\times\vec{r})}{r^3}$$

Here, the quantity Idl is called as current element.



μ = permeability of the medium = $\mu_0\mu_r$

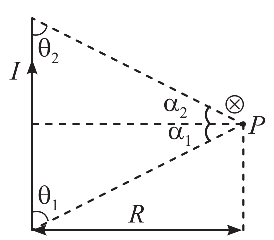
μ_0 = permeability of free space

μ_r = relative permeability of the medium (Dimensionless quantity)

Unit of μ_0 and μ is N A^{-2} or H m^{-1}; $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}

Magnetic Induction Due to a Straight Current Conductor

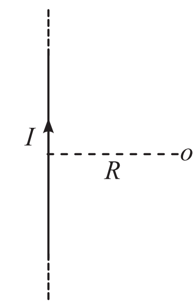
(i) Magnetic induction due to a finite wire.



$$B = \frac{\mu_0 I}{4\pi R}(\cos\theta_1 + \cos\theta_2) = \frac{\mu_0 I}{4\pi R}(\sin\alpha_1 + \sin\alpha_2)$$

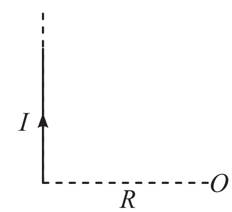
(ii) Magnetic induction due to an infinitely long wire

$$B = \frac{\mu_0 I}{2\pi R}\otimes \ (\alpha_1 = 90^o;\ \alpha_2 = 90^o)$$



(iii) Magnetic induction due to semi infinite straight conductor

$$B = \frac{\mu_0 I}{4\pi R}\otimes \ (\alpha_1 = 0^o;\ \alpha_2 = 90^o)$$



Magnetic Field Due to a Flat Circular Coil Carrying a Current

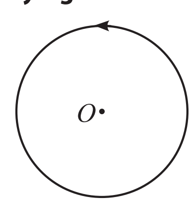
(i) At its centre: $B = \frac{\mu_0 NI}{2R}\odot$

where

N = total number of turns in the coil

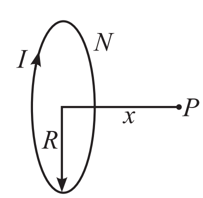
I = current in the coil

R = Radius of the coil



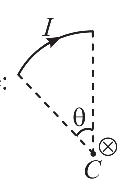
(ii) On the axis: $B = \frac{\mu_0 NIR^2}{2\left(x^2+R^2\right)^{3/2}}$

where x = distance of the point from the centre.



It is maximum at the centre, $B_C = \frac{\mu_0 NI}{2R}$

(iii) Magnetic field due to flat circular arc at its centre:



$$B = \frac{\mu_0 I\theta}{4\pi R}$$

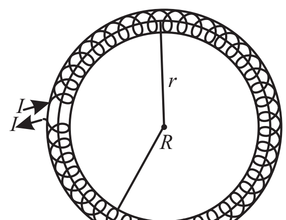
Magnetic Field Due to Infinite Long Solid Cylindrical Conductor of Radius *R*

- For $r \geq R : B = \dfrac{\mu_0 I}{2\pi r}$
- For $r < R : B = \dfrac{\mu_0 I r}{2\pi R^2}$

Magnetic Induction Due to a Solenoid

$B = \mu_0 nI$, where n is number of turns per meter and I is current. Direction is along the axis.

Magnetic Induction Due to Toroid



$B = \mu_0 nI$

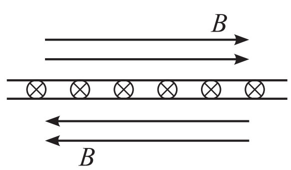
Where $n = \dfrac{N}{2\pi R}$

R is radius of toroid

N is total turns and $R \approx r$

Magnetic Induction Due to Current Carrying Sheet

$B = \dfrac{1}{2}\mu_0\lambda$($\lambda$ = Linear current density (A/m))



Ampere's Circuital Law

$\oint \vec{B}\cdot d\vec{l} = \mu_0 \Sigma I$ where ΣI = algebraic sum of all the enclosed current.

Motion of a Charge In Uniform Magnetic Field

(*a*) When $\vec{v} \,||\, \vec{B}$: Motion will be in a straight line and $\vec{F} = 0$

(*b*) When $\vec{v} \perp \vec{B}$: Motion will be in circular path with radius $R = \dfrac{mv}{qB}$ and angular velocity $\omega = \dfrac{qB}{m}$ and $F = qvB$.

(*c*) When $\vec{v}$ is at angle θ ($\theta \neq 0, \pi, \pi/2$) to $\vec{B}$: Motion will be helical with radius

$R = \dfrac{mv\sin\theta}{qB}$ and pitch $P_H = \dfrac{2\pi mv\cos\theta}{qB}$ and $F = qvB\sin\theta$.

Lorentz Force

An electric charge 'q' moving with a velocity $\vec{v}$ through a magnetic field of magnetic induction $\vec{B}$ experiences a force $\vec{F}$, given by $\vec{F} = q\vec{v}\times\vec{B}$. There fore, if the charge moves in a space where both electric and magnetic fields are superposed.

$\vec{F}$ = net electromagnetic force on the charge = $q\vec{E} + q\vec{v}\times\vec{B}$

This force is called the Lorentz Force

Motion of Charge in Combined Electric Field and Magnetic Field

- When $\vec{v} \parallel \vec{B}$ and $\vec{v} \parallel \vec{E}$, motion will be uniformly accelerated in straight line as $F_{\text{magnetic}} = 0$ and $F_{\text{electrostatic}} = qE$

  So the particle will be either speeding up or speeding down
- When $\vec{v} \perp \vec{B}$ and $\vec{v} \perp \vec{E}$, the particle will move undeflected and undeviated with same uniform speed if $v = \dfrac{E}{B}$ (This is called as velocity selector condition)

Magnetic Force on a Straight Current Carrying Wire

$\vec{F} = I\,(\vec{L}\times\vec{B})$

I = current in the straight conductor

$\vec{L}$ = displacement between end points of the conductor in the direction of the current in it

$\vec{B}$ = magnetic induction (Uniform throughout the length of conductor)

Note: In general, force is $\vec{F} = \int I(d\vec{l}\times\vec{B})$

Magnetic Interaction Force Between Two Parallel Long Straight Current wires

The interaction force between 2 parallel long straight wires is:

(*i*) Repulsive if the currents are anti-parallel.

(*ii*) Attractive if the currents are parallel.

This force per unit length on either conductor is given by

$F = \dfrac{\mu_0}{2\pi}\dfrac{I_1 I_2}{r}$.

Where r = perpendicular distance between the parallel conductors

Magnetic Torque on a Closed Current Loop Kept in a Uniform Magnetic Field

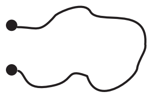
When a plane closed current circuit of 'N' turns and of area 'A' per turn carrying a current I is placed in uniform magnetic field, it experience a zero net force, but experiences a torque given by

$\vec{\tau} = NI\vec{A}\times\vec{B} = \vec{M}\times\vec{B} = BINA\sin\theta$

where $\vec{A}$ = area vector outward from the face of the loop where the current is anticlockwise, $\vec{B}$ = magnetic induction of the uniform magnetic field and

$\vec{M}$ = magnetic moment of the current loop = $NI\vec{A}$

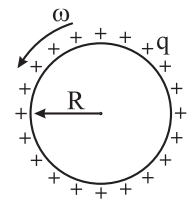
Force on a Random Shaped Conductor in a Uniform Magnetic Field



- Magnetic force on a closed loop in a uniform $\vec{B}$ is zero
- Force experienced by a wire of any shape is equivalent to force on a wire joining points *A* and *B* in a uniform magnetic field.

Magnetic Moment of A Revolving Charge

If a charge q is revolving at an angular velocity ω, its equivalent current is given as $I = \frac{q\omega}{2\pi}$ and its magnetic moment is $M = I\pi R^2 = \frac{1}{2}q\omega R^2$.



CHAPTER 5 Magnetism and Matter

Important terms

(*a*) Magnetic field – Total no. of lines of Magnetic force per unit area

(*b*) Magnetizing field

$$\vec{H} = \frac{\vec{B}}{\mu} A/m$$

$\vec{H}$ is independent of medium

(*c*) Intensity of magnetization

$$I = \frac{\text{Magnetic Moment}}{\text{Volume}}$$

$$\Rightarrow I = \frac{M}{V}$$

(*d*) Magnetic susceptibility,

$$\chi = \frac{I}{H}$$

(*e*) Magnetic permeability: The measure of the degree to which the lines of force can penetrate or permeate the medium

μ = permeability of medium

μ_0 = permeability of free space.

$$\boxed{|\vec{B}| = \mu_0 H + \mu_0 I = \mu H}$$

Relative Permeability.

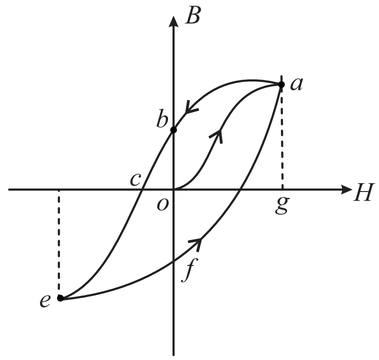
$$\mu_r = \frac{\mu}{\mu_0} = 1 + \chi$$

Magnetic Materials

Paramagnetic	Diamagnetic	Ferromagnetic
Feeble Magnetisation along $\vec{H}$	Feeble Magnetisation opposite of $\vec{H}$	Strong Magnetisation along $\vec{H}$
$0 < \chi < 1$	$-1 \le \chi < 0$	χ is of the order 10^3
$\mu_r > 1$	$0 < \mu_r < 1$	$\mu_r >> 1$
$\chi \propto \frac{1}{T}$	χ is Independent of T	$\chi = \frac{C}{T - T_C} (T > T_C)$

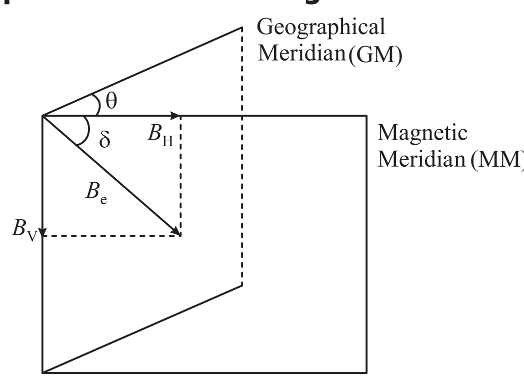
Here $T_C \rightarrow$ Curie temperature (above which a ferromagnetic material becomes paramagnetic.)

Hysteresis loop



- Retentivity = *ob*
- Coercivity = *oc*
- Area of loop = energy loss per unit volume per cycle.

Components of Earth's magnetic field



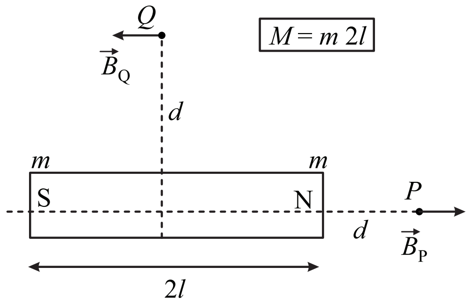
θ = Angle of declination (Angle between G.M and M.M)

δ = Angle of dip.

- $\tan\delta = \frac{B_V}{B_H}$
- $B_e = \frac{B_H}{\cos\delta}$
- $B_e = \sqrt{B_V^2 + B_H^2}$

Bar Magnet

(a) Magnetic field due to bar magnet.



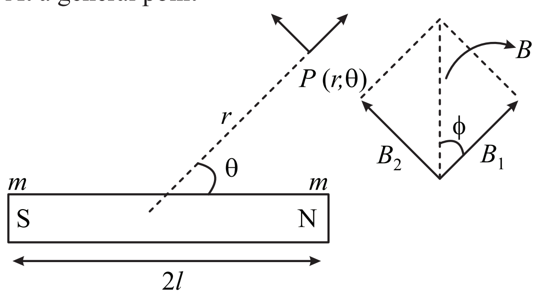
(i) $B_P = \left(\frac{\mu_0}{4\pi}\right)\frac{(4md)l}{\left(d^2 - l^2\right)^2} = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2 - l^2)^2}$

If $l^2 << d^2 \Rightarrow$; $B_P = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$

(ii) $B_Q = \frac{\mu_0}{4\pi}\frac{2ml}{\left(d^2 + l^2\right)^{3/2}} = \frac{\mu_0}{4\pi}\frac{M}{(d^2 + l^2)^{3/2}}$

If $l^2 << d^2$; $B_Q = \frac{\mu_0}{4\pi}\frac{M}{d^3}$

(iii) At a general point



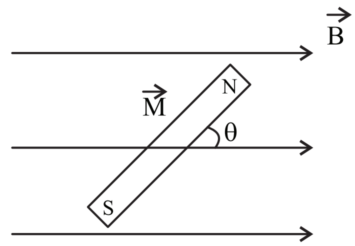
$$B_1 = \frac{\mu_o}{4\pi}\frac{2M\cos\theta}{r^3}$$

$$B_2 = \frac{\mu_o}{4\pi}\frac{M\sin\theta}{r^3}$$

$$B = \frac{\mu_0}{4\pi}.\frac{M}{r^3}\sqrt{1 + 3\cos^2\theta}$$

$$\tan\phi = \frac{\tan\theta}{2}.$$

(b) Bar magnet in uniform magnetic field.



(i) Torque on bar magnet $\vec{\tau} = \vec{M} \times \vec{B}$

(ii) Potential energy $U = -\vec{M}.\vec{B}$

(iii) Work done in rotating the bar magnet

$$W_{ext} = MB\left(\cos\theta_i - \cos\theta_f\right)$$

(iv) Time period of small oscillations

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

$I \rightarrow$ Moment of inertia of bar magnet

CHAPTER

6 Electromagnetic Induction

Magnetic Flux

$\phi = \vec{B} \cdot \vec{A} = BA\cos\theta$ weber for uniform $\vec{B}$.

$\phi = \int \vec{B} \cdot d\vec{A}$ for non uniform $\vec{B}$.

Faraday's Laws of Electromagnetic Induction

(i) An induced emf is setup whenever the magnetic flux linking that circuit changes.

(ii) The magnitude of the induced emf in any circuit is proportional to the rate of change of the magnetic flux linking the circuit, $\varepsilon \propto \frac{d\phi}{dt}$.

Lenz's Laws

The direction of an induced emf is always such as to oppose the cause producing it.

Law of EMI

$\varepsilon = -\frac{d\phi}{dt}$. The negative sign indicated that the induced emf opposes the change of the flux.

Motional EMF

When a conductor is moved across a magnetic field, an electromotive force (emf) is produced in the conductor. If the conductors forms part of a closed circuit then the emf produced causes an electric current to flow round the circuit. Hence an emf (and thus a current) is induced in the conductor as a result of its movement across the magnetic field. This is known as motional emf.

EMF Induced across a moving Straight Conductor in Uniform Magnetic Field

$E = BLv\sin\theta$ volt where $\left(\text{if } \vec{L} \perp \vec{v} \text{ and } \vec{B}\right)$

B = flux density in wb/m^2;

L = length of the conductor (m);

v = velocity of the conductor (m/s);

θ = angle between direction of motion of conductor & B.

Coil Rotation in Magnetic Field Such that Axis of Rotation is Perpendicular to the Magnetic Field

Instantaneous induced emf. $E = NAB\omega\sin\omega t = E_0\sin\omega t$, where

N = number of turns in the coil; A = area of one turn ;

B = magnetic induction; ω = uniform angular velocity of the coil;

E_0 = maximum induced emf.

Self Induction and Self Inductance

The property of the coil or the circuit due to which it opposes any change of the current coil or the circuit is known as **Self-Inductance**. It's unit is Henry.

Coefficient of Self inductance $L = \frac{\phi_s}{i}$ or $\phi_s = Li$

L depends only on;

(i) Shape of the loop and

(ii) Medium

i = current in the circuit.

ϕ_s = magnetic flux linked with the circuit due to the current i.

self induced emf $e_s = -\frac{d\phi_s}{dt} = -\frac{d}{dt}(Li) = -L\frac{di}{dt}$ (if L is constant)

Mutual Induction

If two electric circuits are such that the magnetic field due to a current in one is partly or wholly linked with the other, the two coils are said to be electromagnetically coupled circuits . Then any change of current in one produces a change of magnetic flux in the other and the latter opposes the change by inducing an emf within itself. This phenomenon is called **Mutual Induction.**

Induced emf in the latter circuit due to a change of current in the former is called **Mutually Induced EMF**.

The circuit in which the current is changed, is called the primary and the other circuit in which the emf is induced is called the secondary.

The co-efficient of mutual induction (mutual inductance) between two electromagnetically coupled circuit is the magnetic flux linked with the secondary per unit current in the primary.

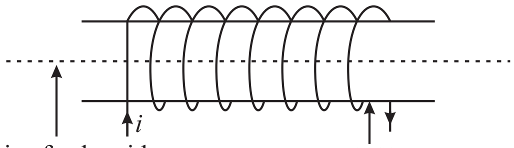
Mutual inductance $= M = \frac{\phi_m}{I_p} = \frac{\text{flux linked with secondary}}{\text{current in the primary}}$

Mutually induced emf $(E_m) = \frac{d\phi_m}{dt} = -\frac{d}{dt}(MI) = -M\frac{dI}{dt}$

M depends on

(1) geometry of loops (2) medium (3) orientation and distance between the loops.

Solenoid



There is a uniform magnetic field along the axis the solenoid (ideal : length >> diameter)

$B = \mu\, ni$ where;

μ = magnetic permeability of the core material;

n = number of turns in the solenoid per unit length;

i = current in the solenoid;

Self inductance of a solenoid $L = \mu n^2 Al$;

A = area of cross section of solenoid.

Super Conducting Loop in Magnetic Field

$R = 0$; $\varepsilon = 0$. Therefore ϕ_{total} = constant. Thus through a superconducting loop flux never changes.

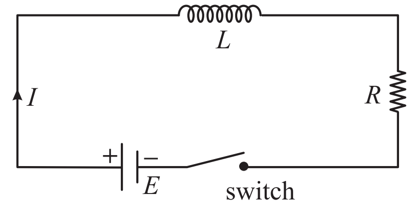
Energy Stored in an Inductor:

$$U = \frac{1}{2} LI^2.$$

Energy of interaction of two loops $U = I_1\phi_2 = I_2\phi_1 = MI_1I_2$, where M is mutual inductance.

Growth of a Current in an L−R Circuit

$I = \frac{E}{R}(1 - e^{-Rt/L})$. [If initial current = 0]

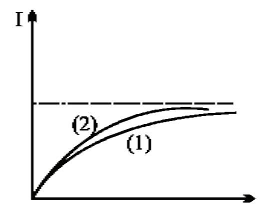


$\frac{L}{R}$ = time constant of the circuit.

$I_0 = \frac{E}{R}$.

(i) L behaves as open circuit at $t = 0$ [$i = 0$]

(ii) L behaves as short circuit at $t = \infty$.

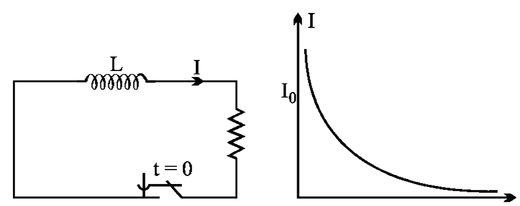


Curve (1) $\longrightarrow \frac{L}{R}$ Large

Curve (2) $\longrightarrow \frac{L}{R}$ Small

Decay of Current

Initial current through the inductor = I_0; Current at any instant $i = I_0 e^{-Rt/L}$



CHAPTER

7 Alternating Current

Instantaneous, RMS and Average Values

(i) Instantaneous value of alternating current

$I = I_0 \sin\omega t$ or $I = I_0 \cos\omega t$

(ii) Peak value of a.c. = I_0

(iii) Alternating emf. = $E = E_0 \sin\omega t$ or $E = E_0 \cos\omega t$

(iv) Mean or average value of a.c.

$$I_m \text{ or } I_a = \frac{2I_0}{\pi} = 0.637 I_0 \text{ for half cycle}$$

$= 0$ for full cycle.

(v) R.m.s. value of ac $I_{rms} = I_0/\sqrt{2} = 0.707 I_0$.

Phase Difference

(i) If the emf leads the current by $\pi/2$, the reactance is called purely inductive.

(ii) If the emf lags behind the current by $\pi/2$, the reactance is called purely capacitive.

(iii) If the emf is in phase with the current, the circuit is called purely resistive.

Sign Convension

Sign for phase difference (ϕ) between I and E for series LCR circuit:

ϕ is positive, when $X_L > X_C$

ϕ is negative, when $X_L < X_C$

ϕ is zero, when $X_L = X_C$

Resonance

(i) The LCR circuit is said to be resonance when

$X_L = X_C$ i.e., when $\omega L = \frac{1}{\omega C}$ and $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$ is called resonance frequency.

(ii) At series resonant frequency, $\omega_0 = \frac{1}{\sqrt{LC}}$, we have:

(*a*) $Z = R$ = minimum value of impedance.

(*b*) $I_0 = E_0/R$ = maximum value of peak current.

(*c*) $\phi = 0$ i.e., I and E are in phase with each other.

(*d*) V_L is equal and opposite to V_C.

(*e*) Potential drop across C and L together is zero.

(*f*) $E = V_R$

Energy Stored

(i) Energy stored in an inductor: $U = \frac{1}{2} L I_0^2$

(ii) Energy stored in a capacitor: $U = \frac{1}{2} C V^2 = \frac{1}{2}\frac{q_0^2}{C} = \frac{1}{2} q_0 V$

Power in AC Circuit

(i) The power in LCR circuit is given by

$P = EI = E_0 I_0 \sin\omega t \sin(\omega t - \phi)$.

Power in LCR circuit consists of two components

(*a*) Virtual power component = $\frac{1}{2} E_0 I_0 \cos(2\omega t + \phi)$.

It has frequency twice as that of A.C. Its value over the complete cycle is zero

(*b*) Real power component $= \frac{1}{2} E_0 I_0 \cos\phi$. It dissipates power and $\cos\phi$ is called power factor.

(ii) Inductive reactance: $X_L = \omega L$

Capacitive reactance: $X_C = \frac{1}{\omega C}$

Reactance: $X = X_L - X_C = \omega L - \frac{1}{\omega C}$

Impedance

(i) Impedance of LCR circuit: $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)}$$

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR}$$

$$= \frac{X_L - X_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R}$$

Power, $P = E_{rms} \times i_{rms} \times \frac{R}{Z}$

(ii) Band width = $\omega_2 - \omega_1 = 2\Delta\omega$

(iii) Sharpness of resonance $= \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R}$

(iv) Q factor: $Q = \frac{\text{Voltage across } L \text{ or } C}{\text{Applied voltage}}$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Transformer

(i) $\frac{E_s}{E_p} = \frac{N_s}{N_P} = k \text{ (say)}$ (transformer ratio)

(ii) For step up transformer, $k > 1$ and for step down transformer, $k < 1$

(iii) For step up transformer, $N_s > N_p$, therefore, $E_s > E_p$. And for the step down $N_s < N_p$ therefore, $E_s < E_p$.

(iv) The efficiency of the transformer is given by:

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

If I_p and I_s be the currents in the primary and secondary circuit, then

$$\eta = \frac{E_s I_s}{E_p I_p}$$

For ideal transformer $\eta = 1 = 100\%$. Therefore,

$$E_s I_s = E_p I_p \text{ or } \frac{I_s}{I_p} = \frac{N_p}{N_s} = \frac{1}{k}$$

Hence, for step up transformer, current in the secondary is less than that in the primary ($I_s < I_p$) and in a step down transformer, we have $I_s > I_p$.

CHAPTER 8

Electromagnetic Waves

Displacement Current

Displacement current is that current which appears in a region in which the electric field (and hence electric flux) is changing with time.

Note: We have

$$I_D = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d}{dt}(EA) = \varepsilon_0 \frac{d}{dt}\left(\frac{qA}{\varepsilon_0 A}\right) = \frac{dq}{dt} = I$$

[Charging of plate of a capacitor]

Modified Ampere's Circuital Law

$$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 \left(I + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

Electromagnetic Waves

$$\oint \vec{E} \cdot \overrightarrow{dl} = -\frac{d\phi_B}{dt} \text{ and } \oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt} \text{ (Maxwell's equations)}$$

These equations lead to the conclusion that, if either of the electric or magnetic field changes with time, the other field is induced in space. The net result of these interacting changing fields is the generation of electromagnetic disturbance, called electromagnetic waves which travel with the speed of light.

Mathematical Expression of EM Waves

$$E_y = E_0 \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right), B_Z = B_0 \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

Important Characteristics of EM Waves

(i) EM waves are produced by accelerated charged particles.

(ii) EM waves do not require any medium for their propagation. These waves can propagate in vacuum as well as in a medium.

Velocity of em waves in free space is given by

$$v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m/s}$$

Velocity of em waves in a medium is given by

$$v = \frac{c}{\sqrt{\mu_r k}}$$

[μ_r = relative permeability of medium]

[k = dielectric constant of medium]

(iii) EM waves are transverse in nature i.e. E and B are perpendicular to each other as well as perpendicular to the direction of propagation of the wave. E and B are related as follow:

$$\frac{E_0}{B_0} = c \text{ or } \frac{E}{B} = c$$

(iv) EM waves carry energy, which is shared equally by electric and magnetic fields.

The average energy density of an EM wave is given by

$$u = u_E + u_B = 2u_E = 2u_B$$

where $u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 (Bc)^2 \quad \left[\because \frac{E}{B} = c\right]$

$$= \frac{1}{2}\varepsilon_0 B^2 \left(\frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right)^2 = \frac{B^2}{2\mu_0} \quad \left[\because c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right]$$

(v) EM waves carry momentum and exert a radiation pressure

$$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} \text{ and momentum } p = \frac{U}{c}$$

(vi) EM waves transport energy. The rate of energy of EM wave transport per unit area is represented by a quantity called Poynting vector $(\vec{S})$ and is given by $\vec{S} = \frac{1}{\mu}(\vec{E} \times \vec{B})$

(vii) Electric vector of an em wave is responsible for optical effects, as $E_0 >> B_0$.

(viii) Intensity of an EM wave is given by

$$I = \frac{1}{2} c \varepsilon_0 E^2 = \frac{B^2 c}{2\mu_0}$$

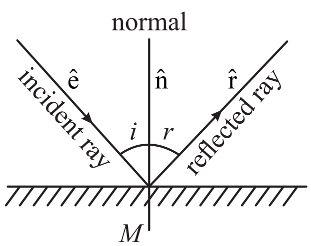
CHAPTER 9

Ray Optics and Optical Instruments

Reflection

Laws of Reflection

- The incident ray the reflected ray and normal to the surface of reflection at the point of incidence lie in the same plane, This plane is called the plane of incidence (also plane of reflection).
- The angle of incidence and the angle of reflection are equal $\angle i = \angle r$



In vector form $\boxed{\hat{r} = \hat{e} - 2(\hat{e}.\hat{n})\hat{n}}$

Object

Real: Point from which incident rays actually diverge.
Virtual: Point towards which incident rays appear to converge.

Image

Image is decided by reflected or refracted rays only. The point of image for a mirror is that point towards which the rays reflected from the mirror actually converge (real image).

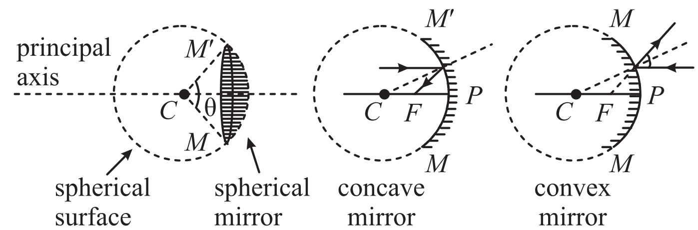
OR

From which the reflected rays appear to diverge (virtual image).

Characteristics of Reflection by a Plane Mirror

- The size of the image is the same as that of the object.
- For a real object the image is virtual and for a virtual object the image is real.
- For a fixed incident light ray, if the mirror is rotated through an angle θ the reflected ray turns through an angle 2θ in the same sense.
- Number of images (n) in inclined mirror. Find $\frac{360}{\theta} = m$
  - If m is even, then $n = m - 1$, for all positions of object.
  - If m is odd, then $n = m$, if object is not on bisector and $n = m - 1$, if object is bisector
  - If m is fraction then n = nearest even number

Spherical Mirrors



Mirror Formula: $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$.

f = focal length u = object distance

v = image distance

Note: Valid only for paraxial rays.

Transverse Magnification: $m_t = \frac{h_2}{h_1} = -\frac{v}{u}$

h_2 = height of image h_1 = height of object

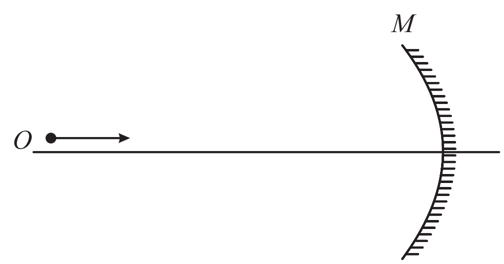
(both perpendicular to the principal axis of mirror)

Longitudinal magnification: $m_l = \frac{\text{Length of image}}{\text{Length of object}}$

For small object $m_l = -m_t^2$

Velocity of image of Moving Object (Spherical Mirror)

Velocity component along axis (Longitudinal velocity)



When an object is coming from infinity towards the focus of concave mirror

$\because \frac{1}{v}+\frac{1}{u}=\frac{1}{f} \quad \therefore -\frac{1}{v^2}\frac{dv}{dt}-\frac{1}{u^2}\frac{du}{dt}=0 \Rightarrow \vec{v}_{IM}=-\frac{v^2}{u^2}\vec{v}_{OM}=-m^2\vec{v}_{OM}$

- $V_{IM}=\frac{dv}{dt}$ = velocity of image with respect to mirror
- $V_{OM}=\frac{du}{dt}$ = velocity of object with respect to mirror.

Optical Power

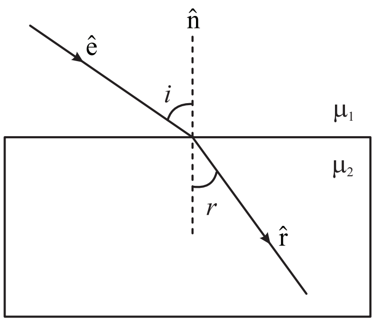
Optical power of a mirror (in Diopters) $=-\frac{1}{f}$

where f = focal length (in meters) with sign.

Laws of Refraction

(i) Incident ray, refracted ray and normal always lie in the same plane.

(ii) The product of refractive index and sine of angle of incidence at a point in a medium is constant. $\mu_1 \sin i = \mu_2 \sin r$ (Snell's law)



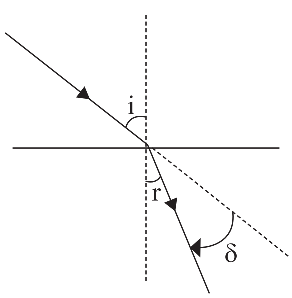
In vector form $(\hat{e}\times\hat{n}).\hat{r}=0$

Snell's Law

$$\boxed{\frac{\sin i}{\sin r}={}^1\mu_2=\frac{\mu_2}{\mu_1}=\frac{v_1}{v_2}=\frac{\lambda_1}{\lambda_2}}$$ In vector form $$\boxed{\mu_1|\hat{e}\times\hat{n}|=\mu_2|\hat{r}\times\hat{n}|}$$

Notes: Frequency of light does not change during refraction.

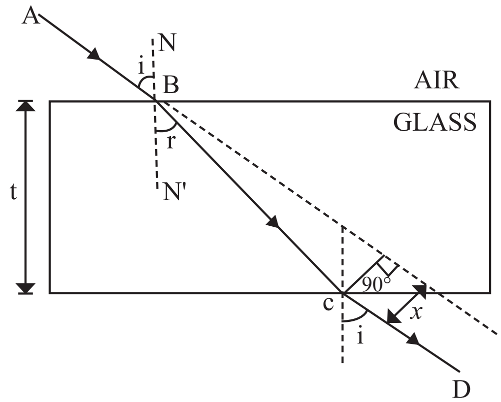
Deviation of a Ray Due to Refraction



$\boxed{\text{angle of deviation, } \delta = i - r}$ (clockwise)

Refraction Through a Parallel Slab

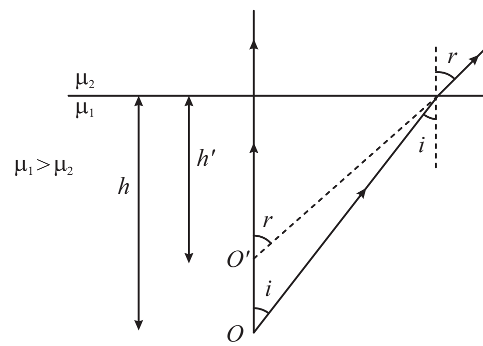
Emergent ray is parallel to the incident ray, if medium is same on both sides.



Lateral shift $x=\frac{t\sin(i-r)}{\cos r}$; t = thickness of slab

Notes: Emergent ray will not be parallel to the incident ray if the medium on two sides are different.

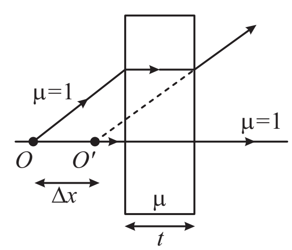
Apparent Depth of Submerged Object: (h' < h)



For near normal incidence $h'=\frac{\mu_2}{\mu_1}h$

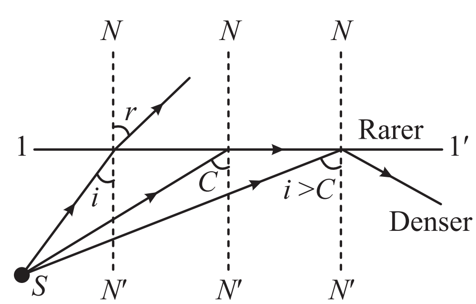
Δx = Apparent shift $=t\left(1-\frac{1}{\mu}\right)$

always in direction of incident ray.



Notes: h and h' are always measured from surface.

Critical Angle & Total Internal Reflection (TIR)

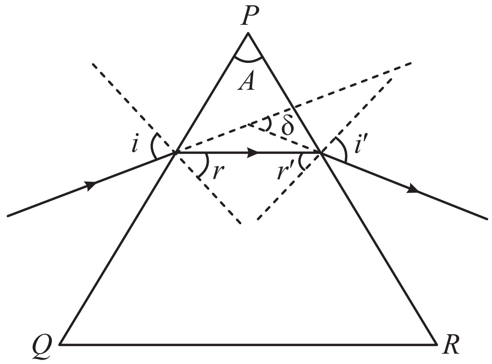


Conditions of TIR

- Ray is going from denser to rarer medium.
- Angle of incidence should be greater than the critical angle ($i > C$).

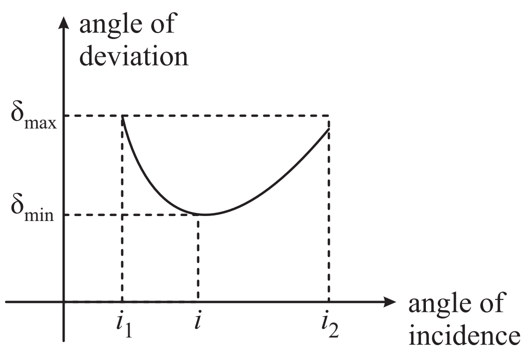
Critical angle $C = \sin^{-1}\frac{\mu_R}{\mu_D} = \sin^{-1}\frac{v_D}{v_R} = \sin^{-1}\frac{\lambda_D}{\lambda_R}$

Refraction Through Prism



- $\delta = (i + i') - (r + r')$
- $r + r' = A$
- $\delta = i + i' - A$

❖ Variation of δ versus i



❖ There is one and only one angle of incidence for which the angle of deviation is minimum. When $\delta = \delta_{min}$ then $i = i'$ and $r = r'$, the ray passes symmetrically about the prism, and then

$$\mu = \frac{\sin\left[\frac{A+\delta_{min}}{2}\right]}{\sin\left[\frac{A}{2}\right]},$$ where μ = absolute R.I. of glass.

Notes: When the prism is dipped in a medium then μ = R.I. of glass w.r.t. medium.

❖ For a thin prism ($A \le 10^o$) ; $\delta = (\mu - 1)A$

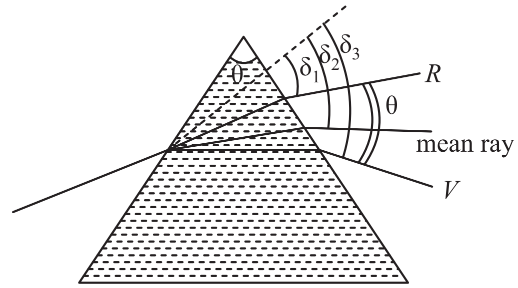
Dispersion of Light

❖ The angular splitting of a ray of white light into a number of components when it is refracted in a medium other than air is called **Dispersion of Light.**

❖ Angle between the rays of the extreme colours in the refracted (dispersed) light is called **Angle of Dispersion**. $\theta = \delta_v - \delta_r$

❖ **Dispersive power** (ω) of the medium of the material of prism.

$$\omega = \frac{\text{angular dispersion}}{\text{devitation of mean ray (yellow)}}$$



For small angled prism ($A \le 10^o$);

$$\omega = \frac{\delta_V - \delta_R}{\delta} = \frac{\mu_V - \mu_R}{\mu_y - 1}; \mu_y = \frac{\mu_V + \mu_R}{2}$$

μ_V, μ_R and μ_y are R.I. of material for violet, red and yellow colours respectively.

Refraction at Spherical Surface

(*a*) $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

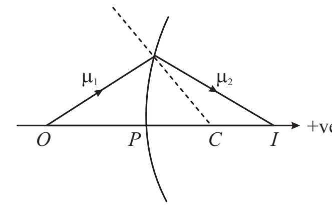
v, u and R are to be kept with sign as

$v = PI$

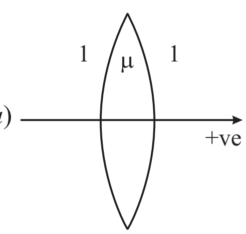
$u = -PO$

$R = PC$

(*b*) $m = \frac{\mu_1 v}{\mu_2 u}$



Lens Formula



(*a*) $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, (*b*) $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

(*c*) Magnification $m = \frac{v}{u}$

Power of Lenses

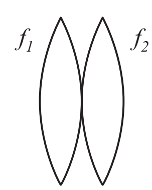
Reciprocal of focal length in meter is known as power of lens.

SI unit: Dioptre (*D*)

Power of lens: $P = \frac{1}{f\,(m)} = \frac{100}{f\,(\text{cm})}$ dioptre

Combination of Lenses

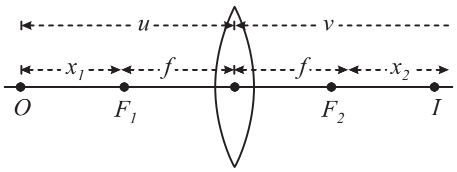
Two thin lens are placed in contact to each other



Power of combination. $P = P_1 + P_2 \Rightarrow \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

Use sign convention while solving numericals.

Newton's Formula



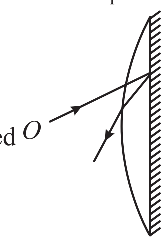
$$f = \sqrt{x_1 x_2}$$

x_1 = distance of object from first focus; x_2 = distance of image from second focus.

Silvering of Lens

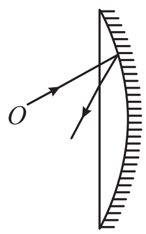
- Silvering of one surface of lens (use $P_{eq} = 2P_l + P_m$)

- When plane surface is silvered



$$f = \frac{R}{2(\mu - 1)}$$

- When convex surface is slivered



$$f = \frac{R}{2\mu}$$

Optical Instruments

For Simple Microscope

- Magnifying power when image is formed at D: $M = 1 + D/f$
- When image is formed at infinity $M = D/f$

For Compound Microscope

- Magnifying power when final image is formed at D, $M = -\frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$
- Tube length $L = v_0 + |u_e|$
- When final image is formed at infinity $M = -\frac{v_0}{u_0} \times \frac{D}{f_e}$ and $L = v_0 + f_e$

Astronomical Telescope

- Magnifying power when final image is formed at D: $M = -\frac{f_0}{u_e} = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$
- Tube length: $L = f_0 + |u_e|$
- When final image is formed at infinity: $M = \frac{f_0}{f_e}$ and tube length $L = f_0 + f_e$

Limit of resolution for microscope:

$$\frac{1.22\lambda}{2\mu \sin\theta} = \frac{1}{\text{Resolving power}}$$

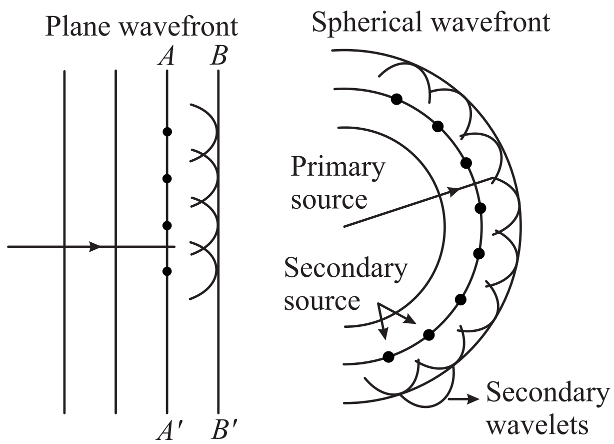
Limit of resolution for telescope:

$$\frac{1.22\lambda}{a} = \frac{1}{\text{Resolving power}}$$

CHAPTER 10
Wave Optics

Huygen's Wave Theory

- Each point source of light is a center of disturbance from which waves are emitted in all directions. The locus of all the particles of the medium oscillating in the same phase at a given instant is called a wavefront.
- Each point on a wave front is a source of new disturbance, called secondary wavelets. These wavelets are spherical and travel with speed of light in that medium.
- The forward envelope of the secondary wavelets at any instant gives the position of the new wavefront.
- In homogeneous medium, the wave front is always perpendicular to the direction of wave propagation.



Coherent Sources

Two sources are coherent if and only if they produce waves of same frequency (and hence wavelength) and have a constant initial phase difference.

Incoherent sources

Two sources are said to be incoherent if they have different frequency or initial phase difference varies with time.

Interference : YDSE

- Resultant intensity for coherent sources

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi_0$$

- Resultant intensity for incoherent sources

$$I = I_1 + I_2$$

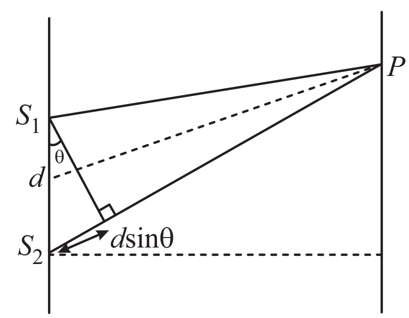
- Intensity $\propto$ width of slit $\propto$ (amplitude)^2

$$\Rightarrow \frac{I_1}{I_2} = \frac{W_1}{W_2} = \frac{A_1^2}{A_2^2}$$

$$\Rightarrow \frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2} = \left(\frac{A_1+A_2}{A_1-A_2}\right)^2$$

- Distance of nth bright fringe $y_n = \frac{n\lambda D}{d}$

  Path difference = $n\lambda$, where n = 0, 1, 2, 3, ...



- Distance of m^{th} dark fringe $y_m = \frac{(2m-1)\lambda D}{2d}$

  Path difference = $(2m - 1)\frac{\lambda}{2}$ where m = 1, 2, 3,.....

- Fringe width $\beta = \frac{\lambda D}{d}$
- Angular fringe width $\theta = \frac{\beta}{D} = \frac{\lambda}{d}$
- If a transparent sheet of refractive index μ and thickness t is introduced in one of the paths of interfering waves, optical path will become 'μt' instead of 't'. Entire fringe pattern shifts by $\frac{D[(\mu-1)t]}{d} = \frac{\beta}{\lambda}(\mu-1)t$ towards the side in which the thin sheet is introduced without any change in fringe width.

Diffraction

- In Fraunhoffer diffraction
  - For minima $a \sin\theta_n = n\lambda$
  - For maxima $a \sin\theta_n = (2n + 1)\frac{\lambda}{2}$

- Linear width of central maxima $W = \dfrac{2\lambda D}{a}$
- Angular width of central maxima $W_\theta = \dfrac{2\lambda}{a}$

Polarization

Brewster's law

$\mu = \tan\theta_p \Rightarrow \theta_p = \tan^{-1}\mu$, θ_p is polarization or Brewster's angle.

Here reflecting and refracting rays are perpendicular to each other.

Malus law

Malus law states that the intensity I of plane polarized that passes through an analyser varies as square of cosine of the angle between the plane of polariser and transmission axis of the analyser.

$I = I_0 \cos^2\theta$, $I_0 \rightarrow$ Intensity of incident polarized light

CHAPTER

11 Dual Nature of Radiation and Matter

Planck's Quantum Theory

- According to Planck's quantum theory, light is considered to be made up of small packets (or particles) of energy known as quanta of energy or radiation.

  Energy, $E = h\nu = \frac{hc}{\lambda} = \frac{12400}{\lambda(\text{Å})}$ eV

Photons

- Momentum of one photon is $\frac{h}{\lambda}$.
- When radiation interacts with matter, the radiation behaves as if it is made of particles like photons.
- Einstein proposed that electromagnetic radiation (or simply light) is quantized and exists in elementary amounts (quanta) that we now call photons.
- Photons are not deflected by electric and magnetic fields which shows that they are neutral and do not carry any charge.
- The energy of photon depends upon the frequency of radiation but is independent of the intensity of radiation.

Photoelectric Effect

- When light of suitable frequency illuminates a metal surface, electrons are emitted. This process of ejection of electrons using light is known as photoelectric emission. Photoelectrons ejected from metal have kinetic energies ranging from 0 to K_{max}.
- A certain minimum amount of energy is required for an electron to be pulled out from the surface of a metal. This minimum energy is called the work function (ϕ) of that metal. Work function is minimum for cesium (1.9 eV).
- Einstein equation for photoelectric effect is,

  $$h\nu = \phi + KE_{max} \quad \Rightarrow \frac{hc}{\lambda} = \frac{hc}{\lambda_0} + eV_s$$

- The minimum frequency of the incident light below which photoelectrons are not ejected from the metal surface is known as threshold frequency (ν_0).

  Work function, $\phi = h\nu_0 = \frac{hc}{\lambda_0}$

  λ_0 = threshold wavelength
- The minimum negative potential given to the metal plate with respect to the collector at which the photoelectric current becomes zero is known as stopping potential or cut-off potential. Here $KE_{max} = eV_s, V_s =$ stopping potential

  Stopping potential is independent of intensity of light used.
- The number of photoelectrons emitted per second is directly proportional to the intensity of the incident radiation.
- The maximum kinetic energy of the ejected electrons is independent of the intensity of incident radiation but depends upon the frequency of the incident radiation.

Radiation Pressure

Radiation pressure, $P = \frac{I}{c}(1+r)$. Here I is the intensity of incident radiation, c is the speed of light and r is the reflectivity of the surface.

For 100% reflection, $r = 1$ and for 100% absorption $r = 0$.

de-Broglie Hypothesis

- It says that a wave is associated with a moving material particle. The wavelength associated with a moving particle is given by $\lambda = \frac{h}{mv}$, where m is the mass of the particle moving with v velocity and h is Planck's constant. This wave is called de-Broglie wave.

Key Tips

- de-Broglie wavelength of a material particle, $\lambda = \frac{h}{mv}$
- de-Broglie wavelength in terms of energy of a particle (E), $\lambda = \frac{h}{\sqrt{2mE}}$
- de-Broglie wavelength of an electron accelerated through a potential V volt, $\lambda = \sqrt{\frac{150}{V}}$ Å $= \frac{12.27}{\sqrt{V}}$ Å
- de-Broglie wavelength of a particle in terms of temperature (T), $\lambda = \frac{h}{\sqrt{3mkT}}$

Heisenberg's Uncertainty Principle

According to Heisenberg's Uncertainty Principle, it is not possible to measure exactly both the position and momentum of a microscopic particle (say electron) at the same time. That is,

$$\Delta x \Delta p \geq \frac{\hbar}{2}, \text{ where } \hbar = \frac{h}{2\pi},$$

CHAPTER

12 Atoms

Rutherford Model

- Rutherford concluded that the majority of the space in an atom is empty. The entire positive charge and mass of an atom is concentrated in a small space known as the nucleus. Electrons revolve around the nucleus in circular orbits.
- The perpendicular distance between the initial velocity vector of the alpha-particle from a central line passing through the center of nucleus when the alpha-particle is far away from the nucleus is known as impact parameter.
- Distance of closest approach: Distance of a point from nucleus at which α-particles are nearest to the centre of nucleus.
- The centripetal force required to make an electron move in a circular orbit is provided by the electrostatic force of attraction between the negatively charged electron and the positively charged nucleus.
- Electrons are bound to the nucleus because the total energy of an electron in orbit is negative.
- Rutherford's atomic model could not explain the stability of the atom. It was also not able to explain the line spectra for various elements and the energies of electrons and their distribution around the nucleus.

Bohr's Model

- According to Bohr's first postulate, every atom consists of a central core called nucleus in which the entire mass and positive charge of the atom is concentrated. Electrons revolve around the nucleus in circular orbits. The centripetal force required for the circular motion of electrons is provided by the electrostatic force of attraction between the negatively charged electrons and the positively charged nucleus.

$$\frac{(Ze)e}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$$

- According to Bohr's second postulate, an electron can revolve only in certain fixed orbits known as stationary orbits. When an electron revolves in a stationary orbit its energy is constant and the angular momentum of the electron is an integral multiple of $h/2\pi$, where h is Planck's constant.

$$mvr = \frac{nh}{2\pi}$$

- According to Bohr's third postulate, when an electron jumps from an inner stationary orbit to an outer stationary orbit it absorbs energy and when it jumps back from an outer stationary orbit to an inner orbit it radiates energy.

$$E_i - E_f = hv = \frac{hc}{\lambda}$$

- **Bohr radius:** First orbit of hydrogen atom is called Bohr radius (a_0).
- **Ground state:** Lowest state of atom, called the ground state, is the state in which electron revolves in the orbit of smallest radius, the Bohr radius, a_0.
- **Ionization energy:** Minimum energy required to free an electron from the ground state of an atom is called the ionization energy.
- The stationary orbits are not equally spaced. The ratio of radii of the first three stationary orbits is $1^2 : 2^2 : 3^2 = 1 : 4 : 9$.

Spectral lines and Hydrogen Like Atoms

- In hydrogen like atoms,

  Radius of n^{th} orbit, $r_n = a_0\left(\frac{n^2}{Z}\right)$, $a_0 = 0.529\text{Å}$

  Orbital speed, $v_n = v_0\left(\frac{Z}{n}\right)$, $v_0 = 219\times10^6\, m/s$

  Energy in n^{th} orbit, $E_n = E_0\left(\frac{Z^2}{n^2}\right)$, $E_0 = -13.6\text{eV}$

- For attractive force inversely proportional to r^2 like in hydrogen like atoms, $TE = -KE = \frac{PE}{2}$
- Wavelength corresponding to spectral lines is given by the Rydberg's formula,

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

  R = Rydberg constant = $1.097 \times 10^7\ \text{m}^{-1}$

- Total number of possible transitions between any two states is $\frac{n(n+1)}{2}$, where n = difference between the two state numbers.
- The lines for hydrogen are said to be grouped into series, according to the level at which upward jump starts and downward jump ends.

- When an electron jumps from a higher energy state n to ground state, $n = 1$, the series of spectral lines emitted is called Lyman series. All of them lie in the ultraviolet region.
- When an electron jumps from a higher energy state n to state 2, the series of spectral lines emitted is called Balmer series. These lie in visible region.
- When an electron jumps from a higher energy state n to state 3, the series of spectral lines emitted is called Paschen series. All of them lie in near infrared region.
- When an electron jumps from a higher energy state n to state 4 the series of spectral lines emitted is called Brackett series. All of them lie in infrared region.
- When an electron jumps from a higher energy state n to state 5 the series of spectral lines emitted is called Pfund series. All of them lie in infrared region.

When Effect of motion of Nucleus is Considered,

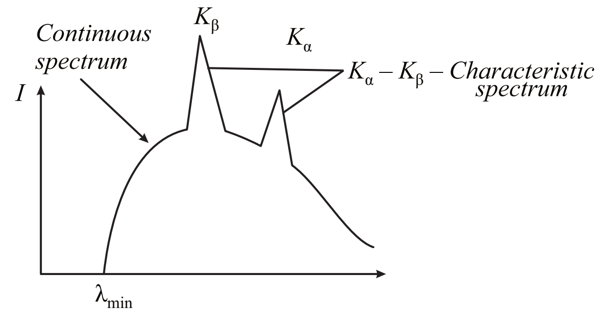
Radius of the orbit, $r_n = (0.529\ \text{Å})\dfrac{n^2}{Z}\cdot\dfrac{m}{\mu}$

Energy of the orbit, $E_n = (-13.6\,\text{eV})\dfrac{Z^2}{n^2}\cdot\dfrac{\mu}{m}$

Here μ is reduced mass, $\mu = \dfrac{Mm}{(M+m)}$,

where, M = mass of nucleus, m = mass of charge particle

X-Rays

- X-rays are produced by bombarding high speed electrons on a target of high atomic weight and high melting point.
- X-ray are electromagnetic radiations of short wavelength (0.1 Å to 10 Å).
- Are not affected by electric and magnetic fields.
- They cause photoelectric emission.
- 

  Characteristics for continuous X-rays, $eV = h\nu_{max} = \dfrac{hc}{\lambda_{min}}$

  e = electronic charge;

  V = accelerating potential

  ν_{max} = maximum frequency of X-Rays
- Intensity of X-rays depends on number of electrons hitting the target.
  - Cut off wavelength or minimum wavelength, $\lambda_{min} \cong \dfrac{12400}{V}$ Å

    Where V (in volts) is the potential difference applied to the tube
  - Continuous spectrum is due to retardation of electrons.
- Characteristic X-Rays

  For $K_\alpha, \lambda = \dfrac{hc}{E_K - E_L}$ For $K_\beta, \lambda = \dfrac{hc}{E_L - E_M}$
- Moseley's Law, $\sqrt{\nu} = a(Z-b)$, here a and b are positive constants for one type of X-rays (independent of Z).

CHAPTER

13 Nuclei

Characteristics of Nucleus

- The nucleus of an atom is its center. Most of the mass of an atom is concentrated in the nucleus.
- One atomic mass unit is defined as $(1/12)^{th}$ of the mass of the ${}^{12}_{6}C$ isotope. It is represented by the symbol u and it is the average mass of a nucleon.
- The stability of any nucleus depends on the number of protons and neutrons. For small nuclei to be stable, the number of protons must be roughly equal to the number of neutrons. As the number of protons increases, however, more neutrons are needed to maintain stability.
- Nuclei of different elements have different sizes because the mass number (A) for different elements is different. Density of nucleus is the almost same for all elements.
- Average radius of nucleus may be written as,

  $R = R_0 A^{1/3}$, $R_0 = 1.1 \times 10^{-15}$ m

Mass Defect

The sum of the individual masses of the separated protons and neutrons exceeds the mass of the stable nucleus by an amount ΔM. This difference in mass is known as the **mass defect** of the nucleus.

Mass-energy Relation

According to mass-energy relation, the mass of a body is a measure of its energy content.

Equivalence of mass and energy, $E = mc^2$

Note: 1 u = 1.66×10^{-27} kg ≡ 931.5 MeV

Binding Energy

- The binding energy of a nucleus is the energy that would have to be provided to split a nucleus into its individual nucleons. Binding energy of nucleus ${}_Z X^A$ of mass M, is given by $BE = (Zm_p + Nm_n - M)c^2$ and the mass defect, $\Delta m = Zm_p + Nm_n - M$.
- The average energy required to remove a nucleon from the nucleus to infinite distance is known as the **binding energy per nucleon**.
- Nuclei with high binding energies per nucleon are very stable as it takes a lot of energy to split them. Nuclei with lower binding energies per nucleon are easier to split.
- In order to become more stable, unstable nuclei tend to release some of their energy. Releasing this energy would decrease the amount of energy they contained, and therefore increase the amount of energy that must be added to them to split them apart.

Q Value

- Energy released in any nuclear reaction or collision is called Q value of the reaction.
- For nuclear reaction, $A + B \rightarrow C + Q$ (Energy)

  The energy of reaction Q is given by,

  $Q = (m_A + m_B - m_C)c^2 = \text{BE}_C - \text{BE}_A - \text{BE}_B$
- If Q is positive, energy is released and products are more stable in comparison to reactants.

  If Q is negative, energy is absorbed and products are less stable in comparison to reactants.

Nuclear Force

- The strong nuclear force is the force that holds nucleons together in the nucleus of an atom. It acts only over very short distances.
- Inside a nucleus, the nucleons are very close. The pull of the strong nuclear force is much greater than the push of the protons repelling each other, and therefore the nucleus remains intact.
- Adjacent neutrons experience no electrostatic repulsion between each other. There is only the strong force of attraction) between them.
- Two protons do repel each other when they are brought together, but in the nucleus they are so close to each other that the force of repulsion is overcame by the even stronger nuclear force.

Nuclear Fission

- By bombarding a particle on a heavy nucleus (A > 230), it splits into two or more light nuclei. In this process certain mass disappears which is obtained in the form of energy (enormous amount)

  $A + p \rightarrow B + C + Q$

- The energy released in a fission reaction comes from the difference between the mass of the original nucleus and the combined mass of the fission fragments.
- A chain reaction occurs when neutron, emitted from the decay of one atom, initiate fission in surrounding nuclei.
- An uncontrolled chain reaction occurs when every free neutron goes on to produce another fission reaction. It occurs in nuclear bombs and releases an enormous amount of energy.
- A controlled chain reaction occurs when only some of the emitted neutrons are able to induce further fission reactions, and the remaining neutrons are absorbed by a material that is not fissionable. In this situation energy can be released at a constant rate.
- Enrichment is the process of increasing the percentage of ^{235}U in a sample of uranium. Enrichment is important because naturally occurring uranium does not have a high enough percentage of ^{235}U to sustain a chain reaction.

Nuclear Fusion

- It is the phenomenon of fusing two or more light nuclei to form a single heavy nucleus.

  $A + B \rightarrow C + Q$ (Energy)
- The product (C) is more stable than reactants (A and B) and $m_C < (m_A + m_B)$.
- Mass defect, $\Delta m = [(m_A + m_B) - m_C]$ amu
- Energy released is $E = (\Delta m) \times 931$ MeV

Radioactivity

A nuclear phenomenon in which an unstable nucleus undergoes a decay is known as radioactivity. A decay equation is a representation of a decay reaction. It shows the changes occurring in nuclei undergoing decay and lists the products of the decay. The nucleus remaining after an atom undergoes radioactive decay is called a daughter nucleus. The daughter nucleus is more stable than the original nucleus. Generally, there are three types of radioactive decays,

(i) α decay

(ii) β^- and β^+ decay

(iii) γ decay

α decay

- An α-particle is a relatively slow-moving decay product consisting of two protons and two neutrons. It is the nucleus of helium and so can be written as ${}_2^4He$. α-particles carry positive charge.
- α-decay process: ${}_Z^AX \longrightarrow {}_{Z-2}^{A-4}Y + {}_2^4He$
- Q-value is, $Q = [m({}_Z^AX) - m({}_{Z-2}^{A-4}Y) - m({}_2^4He)]c^2$
- For the above shown decay kinetic energy of α-particle will be given by, $K_\alpha = \frac{m_Y}{m_X}Q$

β^- and β^+ decay

- A β^- particle is a fast-moving electron that is ejected from an unstable nucleus. The electron is produced when a neutron transforms into a proton and an electron.
- In β^- decay the resulting daughter nucleus has the same number of nucleons as the parent, but has one less neutron and one more proton.
- β^- particles are very light when compared to α particles. They travel at a large range of speeds-from that of an α- particle up to 99% of the speed of light.
- β^-decay

  ${}_Z^AX \longrightarrow {}_{Z+1}^{A}Y + \beta^- + v^-$

  Q – value $= [m({}_Z^AX) - m({}_{Z+1}^{A}Y)]c^2$
- β^+ decay

  ${}_Z^AX \longrightarrow {}_{Z-1}^{A}Y + \beta^+ + v$

  Q – value $= [m({}_Z^AX) - m({}_{Z-1}^{A}Y) - 2m_e]c^2$

  where $m({}_Z^AX)$, $m({}_{Z-1}^{A}Y)$ are atomic masses
- Electron capture (K-capture): When atomic electron is captured by nucleus, X-rays are emitted.

  ${}_Z^AX + {}_{-1}^{0}e \longrightarrow {}_{Z-1}^{A}Y + v$

  Q – value $= [m({}_Z^AX) - m({}_{Z-1}^{A}Y)]c^2$

  where $m({}_Z^AX) - m({}_{Z-1}^{A}Y)$ are atomic masses.

γ decay

A γ-ray is the packet of electromagnetic energy released when a nucleus is formed in excited state after α or β-decay which releases energy to come down to ground state. γ-rays travel at the speed of light, carry no mass or charge, and are not deflected by electric or magnetic fields.

Law of Radioactive Decay

The rate of disintegration is directly proportional to the number of radioactive atoms present at that time i.e., rate of decay $\propto$ number of nuclei.

Rate of decay = λ (number of active nuclei)

i.e., $\frac{dN}{dt} = -\lambda N.$

where λ is called the decay constant.

If N_0 is the number of parent nuclei at $t = 0$. The number that survives at time t is $N = N_0e^{-\lambda t}$

Probability of a nucleus for survival of time $t = \frac{N}{N_0} = \frac{N_0e^{-\lambda t}}{N_0} = e^{-\lambda t}$

Half-life

- A **half-life** is the time taken for half of a group of unstable nuclei to decay. In other words, it is the time during which the number of undecayed atoms in the sample becomes half the total number of atoms present initially in the sample. Half-life is represented by the symbol $T_{1/2}$.

  Half life: $T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$
- Number of nuclei present after n half lives i.e. after a time $t = nT_{1/2}$, $N = \frac{N_0}{2^n}$

- If a radioactive nucleus can decay by two different processes having half lives t_1 and t_2 then. Effective half-life of nucleus will be given by $\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$.

Mean or Average Life

The ratio of the total life time of all the atoms of the element to the total number of atoms present initially in the sample of the element is known as the **mean or average life**. Mean life is represented by the symbol τ. It is equal to the reciprocal of the decay constant (λ) of the element.

Average life: $\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693}$

Activity

- Activity of sample:
  $R = \lambda N = R_0 e^{-\lambda t}$
- Unit: 1 Bq = 1 decay/s, 1 curie = 3.7×10^{10} Bq, 1 rutherford = 10^6 Bq
- Activity per unit mass is called specific activity.
- Activity after n half lives: $\frac{A_0}{2^n}$

CHAPTER

14 Semiconductor Electronics: Materials, Devices and Simple Circuits

Semiconductors

(i) Semiconductors have conductivity between conductors and insulators. At low temperatures, their conductivity is low and with rise in temperature their conductivity increases.

(ii) In all solids, different energy levels combine to form two bands valence band and conduction band.

(iii) Valence band (*VB*) contains valence electrons. It may be partially or fully filled.

(iv) Conduction band (*CB*) contains free electrons. It may be empty or partially filled.

(v) The energy difference between valence band and conduction band is called forbidden energy gap.

(vi) The forbidden energy gap is less for conductors and large for insulators and in between for semiconductors.

(vii) Pure semiconductors are called intrinsic semi conductor Ex. Silicon and Germanium (Si and Ge).

P-N Junctions Diode

(i) A *p-n* junction is a single piece of semiconductor, one half of which is *p*-type and the other half is *n*-type.

(ii) The region near the junction is called depletion layer.

(iii) There are two types of connections of a diode

(*a*) Forward bias

(*b*) Reverse bias

(iv) When *p*-type is connected to positive and *n*-type connected to negative terminal then it is forward biased.

(v) In forward bias the diode offers minimum resistance and depletion region becomes narrowed. It is similar to 'ON' in an electrical switch.

(vi) In reverse bias *p*-type is connected to negative terminal and *n*-type is connected to positive terminal.

(vii) In reverse bias the diode offers maximum resistance, does not conduct and depletion region becomes widened. It is similar to 'OFF' in an electrical switch.

Zener Diode

(i) A properly and highly doped *p-n* junction diode which operates in reverse bias condition is called 'Zener diode'.

(ii) Silicon is preferred for making Zener diodes.

(iii) Zener diode is operated in reverse bias, which operates at a voltage called 'Zener voltage'.

(iv) Zener diode is used as a 'Voltage regulator'.

Transistors

(i) Transistor means transfer resistor.

(ii) There are two types of transistors called

(*a*) *npn* transistor

(*b*) *pnp* transistor.

(iii) In transistor there are three terminals called emitter, base and collector.

(iv) Transistor works as an amplifier and switch.

(v) Current gain of common emitter configuration is the ratio of small change in collector current to a small change in base current when collector-emitter voltage is constant.

(vi) Amplifier is a device which converts weak signals to strong signals and this process of converting weak signals to strong signals is called amplification.

(vii) Amplifiers are of two types

(*a*) Power amplifier

(*b*) Voltage amplifier

(viii) The amplifier which is used to raise the power level is known as 'Power amplifier'.

(ix) The amplifier which is used to raise voltage level is known as 'Voltage amplifier'.

Important Formulae

(i) Rectifier efficiency $(\eta) = \dfrac{\text{dc output power}}{\text{ac input power}}$

(ii) Half wave rectifier efficiency $(\eta) = \dfrac{0.406 \times R_L}{r_f + R_L}$

r_f = diode forward resistance, R_L = load resistance

(iii) Full wave rectifier efficiency $\eta = \frac{0.812 \times R_L}{r_f + R_L}$

(iv) Current gain $\beta = \left[\frac{\Delta I_C}{\Delta I_B}\right]_{V_{CC}}$, $\alpha = \left(\frac{\Delta I_C}{\Delta I_E}\right)$

Relation between α and β, $\beta = \frac{\alpha}{1-\alpha}$

ΔI_C= Change in collector current, ΔI_B= Change in base current

(v) Amplification factor $A = \frac{V_0}{V_i}$

V_0 = Output voltage; V_i = Input voltage

(vi) Voltage gain $= \frac{\Delta V_{CE}}{\Delta V_{BE}} = \beta \times \frac{R_L}{R_{in}}$

ΔV_{CE} = Change in output voltage,
ΔV_{BE} = Change in input voltage

(vii) Power gain, A_p = Current gain × voltage gain $= \beta^2 \times \frac{R_L}{R_{in}}$

CHAPTER

15 Communication Systems

1. Bandwidth of video signals is 4.2 MHz.
2. Coaxial cable is a widely used wire medium, which offers a bandwidth of approximately 750 MHz.
3. For transmission over long distances, signals are radiated into space using devices called antennas. The radiated signals propagate as electromagnetic waves and the mode of propagation is influenced by the presence of the earth and its atmosphere. Near the surface of the earth, electromagnetic waves propagate as surface waves. Surface wave propagation is useful up to a few MHz frequencies.
4. Communication through free space using radio waves takes place over a very wide range of frequencies from a few hundreds of KHz to a few GHz.
5. To radiate signals with high efficiency, the antennas should have a size comparable to the wavelength λ of the signal (at least $\sim \lambda/4$)
6. If an antenna radiates electromagnetic waves from a height h, then the range d is given by $\sqrt{2Rh}$ where R is the radius of earth.
7. The maximum line of sight distance d_M between the two antennas having heights h_T and h_R above the earth is given by $d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$
8. Low frequencies cannot be transmitted to long distances. Therefore, they are superimposed on a high frequency carrier signal by a process known as modulation.
9. The method in which the amplitude of carrier wave is varied in accordance with the modulating signal keeping the frequency and phase of carrier wave constant is called amplitude modulation (AM).
10. Amplitude modulated signal contains frequencies $(\omega_c - \omega_m)$, ω_c and $(\omega_c + \omega_m)$.
11. Amplitude modulated waves can be produced by application of the message signal and the carrier wave to a non-linear device, followed by a band pass filter.
12. AM detection, which is the process of recovering the modulating signal from an AM waveform, is carried out using a rectifier and an envelope detector.