Number System

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INTRODUCTION

A number system is a systematic way of expressing and representing numbers. It includes a set of symbols, rules, and **conventions for counting and performing arithmetic operations**. There are various number systems used in mathematics, with the most common being the **decimal system**, also known as the Hindu-Arabic numeral system.

TYPES OF NUMBER SYSTEM

- **Binary Representation:** The binary system (base-2) represents numbers using only two digits: 0 and 1.
- **Octal Representation:** The octal system (base-8) uses digits from 0 to 7.
- **Hexadecimal Representation:** The hexadecimal system (base-16) uses digits 0-9 supplemented by the letters A-F to represent values 10-15.

Decimal System

It is the most widely used number system in the world. It employs ten symbols, called digits, to represent numbers. These digits are: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Each digit has a specific value, and numbers are formed by combining these digits in various ways.

Representation of a Number

$ab = 10a + b$

$abc = 100a + 10b + c$

$abcd = 1000a + 100b + 10c + d$

Digits are classified in two categories:

(i) 0 is considered as insignificant digit because it represents the absence of a value or zero.

(ii) The other digits (1 through 9) are known as significant digits because they represent non-zero values.

Numerals

Numerals are mathematical symbols used to represent numbers in a systematic manner. They are composed of a set of digits taken from the available symbols in the chosen number system.

For example: the numeral "1234" in the decimal system represents the number: one thousand two hundred thirty-four.

In this numeral, "1" is the thousands digit, "2" is the hundreds digit, "3" is the tens digit, and "4" is the ones digit. The position of each digit within the numeral determines its value in the overall number.

Face Value

The face value of a digit in a numeral is the actual value of that digit, regardless of its position within the numeral. To put it simply, it's the inherent worth of the digit itself. For example

In the numeral 28746, the face value of:

The digit 8 is 8

The digit 7 is 7

The digit 6 is 6

The digit 4 is 4

Place Value (or Local Value)

The place value is a value assigned to a digit based on its position within the numeral.

The place value of a digit changes as its position changes within the numeral.

For example: 28746

The place value of 7, in the hundreds place, is 700.

The place value of 4, in the tens place, is 40.

The place value of 6, in the ones place, is 6.

Ten lakhs	One lakh	Ten thousands	Thousands	Hundreds	Tens	Ones
× 1000000	× 100000	× 10000	× 1000	× 100	× 10	× 1
		2	8	7	4	6

Note: 'Face value' remains the same for a digit. However, the 'place value' varies based on its position within the numeral, indicating its contribution to the overall numeric value of the entire number.

TYPES OF NUMBERS

Based on how the numbers can be represented on paper

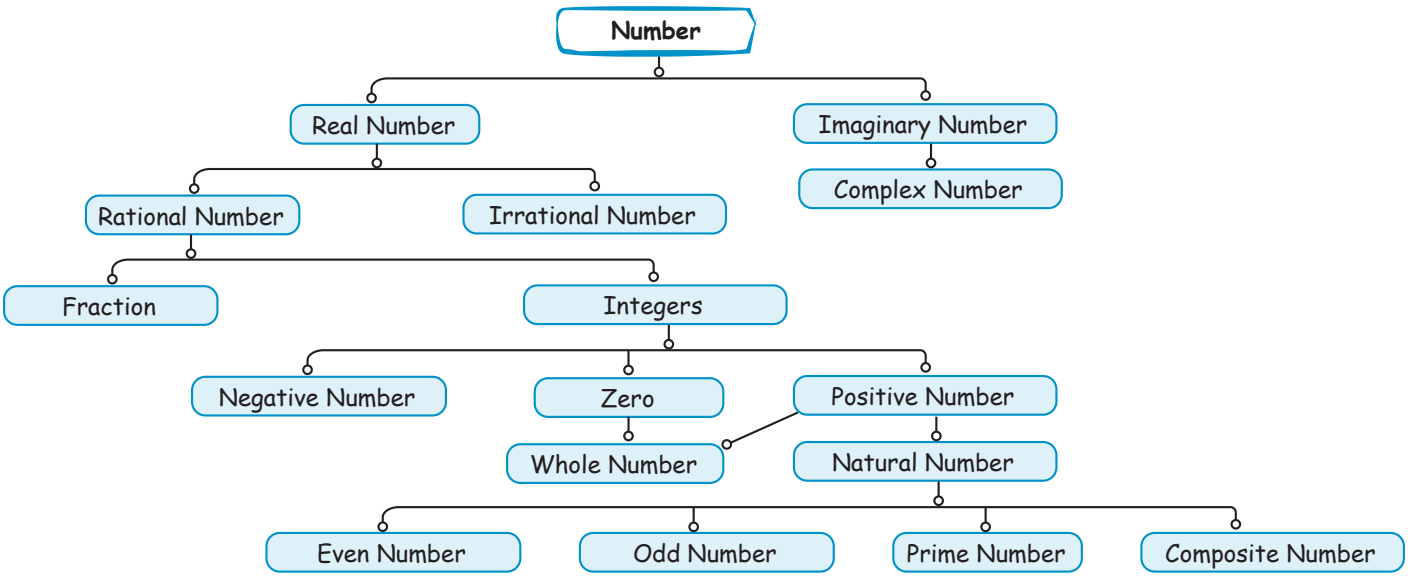


Fig. 1.1: Number

Real Numbers

Real numbers Include both Rational and Irrational numbers.

For example: $1, \frac{1}{5}, -5, \sqrt{3}, 1.732....$ etc.

Rational Numbers

A number which is in the form of p/q where p and q are co-prime integers and $q \neq 0$ is called rational numbers.

For example: $1, \frac{3}{5}, 7$ etc.

The smallest and largest rational numbers cannot be determined.

Irrational Numbers

A number which is not in the form of p/q or whose decimal expression is non-terminating and non-repeating is called as irrational numbers.

For example: $\sqrt{3}, \sqrt{5}, \sqrt{2}$ etc.

Note: Every positive irrational number has a negative irrational number corresponding to it.

- $\sqrt{3}-\sqrt{1} \neq \sqrt{2}, \quad \sqrt{5} \times \sqrt{2} = \sqrt{5 \times 2} = \sqrt{10}$

In some cases, the product of two irrational numbers is a rational number.

For example:

- $\sqrt{5} \times \sqrt{5} = 5$
- $(1+\sqrt{2}) \times (1-\sqrt{2}) = (1)^2 - (\sqrt{2})^2 = 1-2 = -1$

Imaginary Numbers

Imaginary numbers are the numbers obtained by taking the square root of the negative numbers.

General Form: ***a + ib***

Where "*a*" and "*b*" are real numbers and "*i*" is known as iota.

Value of i : $i = \sqrt{-1}, i^2 = -1, i^3 = -i, i^4 = 1$

It is mostly written in the form of real numbers multiplied by the imaginary unit called "i".

Integers

Integers are the whole numbers that include both positive numbers, negative numbers, and zero. They do not have fractional or decimal parts.

Integers are denoted by Z.

For example: Z = { .....,–3, –2, –1, 0, 1, 2, ...}.

Integers are Two Types

- **Positive Integers**

  Natural numbers are called positive integers. For example- { 1, 2, 3, ...}

- **Negative Integers**

  Negative Natural numbers are called negative integers. For example- { -1, -2, -3, ...}

Note: Zero is neither positive nor negative integer.

Every integer is a rational number. Zero is also a rational number.

Natural Numbers

A set of positive counting numbers which starts from 1 and go to infinity are called natural numbers.

Natural numbers are also called counting numbers. All natural numbers are positive.

Natural numbers are denoted by N and the **smallest natural number is 1.**

For example: N = { 1, 2, 3, 4, ... }

Properties of Natural Number

- **Commutative law**

  For any two natural numbers "a" and "b"

  $a + b = b + a$

  $a \times b = b \times a$

  For example: let 7 and 9 be two numbers, then

  $7 + 9 = 9 + 7 = 16$

  $7 \times 9 = 9 \times 7 = 63$

- **Associative law**

  For any three natural numbers "*a*" "*b*" and "*c*".

  $(a + b) + c = a + (b + c)$

  $(a \times b) \times c = a \times (b \times c)$

  For example: let 3, 4 and 5 be three numbers, then

  $(3 + 4) + 5 = 3 + (4 + 5) = 12$

  $(3 \times 4) \times 5 = 3 \times (4 \times 5) = 60$

- **Distributive law**

  For three natural numbers *a*, *b* and *c*

  $a \times (b + c) = (a \times b) + (a \times c)$

  $a + (b \times c) \neq (a + b) \times (a + c)$

  For example: let 2, 4 and 3 be numbers, then

  $2 \times (4 + 3) = (2 \times 4) + (2 \times 3)$

  $\Rightarrow 14 = 14$

  $2 + (4 \times 3) \neq (2 + 4) \times (2 + 3)$

  $\Rightarrow 24 \neq 30$

- **Cancellation law**

  For three natural numbers

  $(a + b) = (c + b)$

  $\Rightarrow a = c$

  $(a \times b) = (c \times b)$

  $\Rightarrow a = c$, where $b \neq 0$

Note:

1. There are 9 one-digit numbers. i.e., 9×10^0 and 90 two-digit numbers. i.e., 9×10^1.

   In general, the number of n digit numbers are $9 \times 10^{n-1}$

2. **Sum of first n natural numbers**

   $= 1 + 2 + 3 \ldots + n = \frac{n(n+1)}{2}$

3. **Sum of squares of first n natural numbers**

   $= 1^2 + 2^2 + 3^2 \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

4. **Sum of cubes of first n natural numbers**

   $= 1^3 + 2^3 + 3^3 \ldots\ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

5. **Sum of first n Even Numbers**

   $= 2 + 4 + 6 + 8 + \ldots + n = n\,(n + 1)$

6. **Sum of first n odd Numbers =** n^2

7. If a and b are two co-prime numbers and a given number is divisible by a and b, then it will also be divisible by $a \times b$.

   Example: 180 is divisible by 2 and 3, it will also be divisible by $2 \times 3 = 6$

Whole Numbers

Whole numbers are a set of numbers that include all the natural numbers along with zero.

All whole numbers are positive.

Smallest whole number is 0.

Whole numbers are denoted by W.

For example: W = { 0, 1, 2, 3, ...}

- **All natural numbers are whole numbers, but all whole numbers are not natural numbers.**

Even Numbers

Even numbers are the whole numbers that can be divided by 2 without leaving a remainder. In other words, an even number is any integer that is exactly divisible by 2.

Also, a number whose last digit is one of the following- 0, 2, 4, 6 or 8 is called an even number.

For example: 2, 4, 6, 8, 10, 12,..... etc.

Odd Numbers

A number which is not multiple of 2, is called an odd number. Also these numbers are not divisible by 2.

A number whose last digit is 1, 3, 5, 7 or 9 is called an odd number.

For example: 1, 3, 5, 7, 9, 11, 13, ... etc.

Note: Must learn for exams.

- Result is always an even number in case of following mathematical operations-
  - Even + Even = Even – Even = Even × Even = $(\text{Even})^{\text{even or odd}}$ = Even
  - Odd – Odd = Odd + Odd = Odd × Even = Even

- Result is always an odd number in case of following mathematical operations-
  - Even + Odd = Even – Odd = Odd × Odd = Odd/ Odd = $(Odd)^{even\ or\ odd}$ = Odd
  - Even/ Even = either even or odd number
  - Odd is never divisible by an even number.

Prime Numbers

A number which has only two factors (1 and the number itself) is called a prime number.

For example: 2, 3, 5, 7, 11,..... etc.

Smallest prime number is 2.

There is only one even prime number that is 2.

Consecutive Numbers

The series in which the next number is greater than 1 by the previous number is known as consecutive numbers.

For example:

15, 16, 17 are consecutive numbers.

Perfect Number

When the sum of all the factors including 1 but excluding the number itself of a given number is equal to the number itself then the number is a perfect number.

For example: 28 – Factors of 28 are 1, 2, 4, 7, 14, 28

Sum of all factors excluding 28 = 1 + 2 + 4 + 7 + 14 = 28

There are only 27 perfect numbers.

Composite Numbers

A number which has more than two factors or a number which is not prime, is called a composite number.

Smallest composite number is 4.

For example: 4, 6, 8, 9, 10, 12,..... etc.

1 is neither a prime nor a composite number.

Co-Prime Numbers

Numbers which have only one common factor (Digit 1 itself) are called co-prime numbers.

Hence, these are pairs of numbers whose HCF is 1.

Examples of such co-primes- (3, 5), (17, 21) etc.

Twin-Prime Numbers

The pair of prime numbers whose difference is 2 are called twin-primes.

For example: (5, 7), (17, 19), etc.

(3, 7) is not a twin prime.

QUESTIONS

Q1. Find four rational numbers between 2 and 7.

(a) $\frac{9}{2}, \frac{13}{4}, \frac{31}{8}, \frac{23}{4}$ (b) $\frac{9}{2}, \frac{4}{13}, \frac{31}{8}, \frac{23}{4}$

(c) $\frac{9}{2}, \frac{13}{4}, \frac{8}{31}, \frac{4}{23}$ (d) $\frac{9}{2}, \frac{4}{11}, \frac{8}{29}, \frac{23}{4}$

Ans: (a)

Explanation:

$\frac{9}{2} = \frac{8+1}{2} = 4\frac{1}{2}$

$\frac{13}{4} = \frac{12+1}{4} = 3\frac{1}{4}$

$\frac{31}{8} = \frac{24+7}{8} = 3\frac{7}{8}$

$\frac{23}{4} = \frac{20+3}{4} = 5\frac{3}{4}$

All these numbers lie between 2 and 7.

Trick lies in quickly identifying the nearest quotient through which the denominator divides the numerator.

Q2. Find an irrational number between 1/3 and 5/3.

(a) 0.532532532..... (b) 0.523523523.....

(c) 0.532533253312.... (d) 0.535353.....

Ans: (c)

Explanation:

$\frac{1}{3}$ = 0.33333333..... and $\frac{5}{3}$ =1.666666.....

Thus, all numbers in the four options lie between 1/3 and 5/3. However, an irrational number must be non-terminating and non-recurring as well. This number is 0. 532533253312.....

Q3. The average of all odd numbers up to 50 is

(a) 25 (b) 50

(c) 75 (d) 100

Ans: (a)

Explanation:

Odd numbers are 1, 3, 5, 7, 9, ... 47, 49, ...

Total number of odd values till 50 = 25

Sum of odd numbers = n^2

Sum of 25 odd numbers till 50 = $(25)^2$

Average = Sum of values/Number of values

Average = $(25)^2/25$

Average = 25

Q4. The number of times the digit 5 will appear while writing the integers from 1 to 1000 is

(UPSC CSAT 2019)

(a) 269 (b) 271

(c) 300 (d) 302

Ans: (c)

Explanation:

The frequency of occurrence of digit 5 in different series of numbers are as follows.

From 1 to 10 – 1 time

11 to 20 – 1 time

21 to 30 – 1 time

31 to 40 –1 time

41 to 50 – 2 times

51 to 60 – 10 times

61 to 70 – 1 time

71 to 80 – 1 time

81 to 90 – 1 time

91 to 100 – 1 time

So, occurrences of 5 from 1 to 100 = 20 times.

Similarly, occurrence of 5 from 1 to 1000

= 20 × 10 = 200 times

But, occurrence of 5 at hundredth place from 500 to 600 =100 times

So, total number of times 5 digit would appear = 200 + 100 = 300 times

Q5. Let *p*, *q*, *r* and *s* be natural numbers such that *p* – 2016 = *q* + 2017 = r – 2018 = *s* + 2019. Which one of the following is the largest natural number?

(UPSC CSAT 2020)

(a) p (b) q

(c) r (d) s

Ans: (c)

Explanation:

Given relationship is,

p – 2016 = *q* + 2017 = *r* – 2018 = *s* + 2019

Take, *p* – 2016 = *r* – 2018

$\Rightarrow r - p = 2 \Rightarrow r > p$ ...(i)

Again, *p* –2016 = *s* + 2019

$\Rightarrow p - s = 2019 + 2016$

$\Rightarrow p - s = 4035$

$\Rightarrow p > s$ ...(ii)

Again, *r* – 2018 = *q* + 2017

$\Rightarrow r - q = 2017 + 2018 = 4035$

$\Rightarrow r > q$ ...(iii)

From equation (i), (ii) and (iii) "r" is the largest natural number.

Q6. How many five-digit prime numbers can be obtained by using all the digits 1, 2, 3, 4 and 5 without repetition of digits? **(UPSC CSAT 2020)**

(a) Zero (b) One

(c) Nine (d) Ten

Ans: (a)

Explanation:

The sum of five-digits of any five-digit number by using 1,2, 3,4 and 5 without repetitions

= 1 + 2 + 3 + 4 + 5 =15

Since, 15 is divisible by 3, hence, a prime number cannot be formed according to the given conditions.

Q7. In the sum, ☒+1☒+5☒+☒☒+☒1=1☒☒; for which digit does the symbol ☒ stand?

(a) 2 (b) 3

(c) 4 (d) 5

Ans: (b)

Explanation:

This question can be solved by options easily.

Option (a): Take, ☒ = 2.

So, 2 + 12 + 52 + 22 + 21 ≠ 122

129 ≠ 122

Option (b): Take, ☒ = 3.

So, 3 + 13 + 53 + 33 + 31 = 133

133 = 133

Option (c): Take, ☒ = 4.

So, 4 + 14 + 54 + 44 + 41 ≠ 144

157 ≠ 144

Option (d): Take ☒ = 5.

So, 5 + 15 + 55 + 55 + 51 ≠ 155

181 ≠ 155

Q8. Consider two statements S1 and S2 followed by a question.

S1: *p* and q both are prime numbers.

S2: *p* + *q* is an odd integer

Question: Is *pq* an odd integer?

Which one of the following is correct?

(UPSC CSAT 2019)

(a) S1 alone is sufficient to answer the question

(b) S2 alone is sufficient to answer the question

(c) Both S1 and S2 taken together are not sufficient to answer the question

(d) Both S1 and S2 are necessary to answer the question

Ans: (b)

Explanation:

S1: *p* and *q* both are prime numbers, this shows that their product is an odd integer in all cases except when one of the numbers is 2 itself

S2: p + *q* is an odd integer. Thus one of the two integers is even as the sum of even and odd always make odd. This make the product of p and q divisible by 2 (thus p×q is even integer)

Hence, to answer the question, S2 alone is sufficient as S1 is inconclusive.

Q9. A printer numbers the pages of a book starting with 1 and uses 3089 digits in all. How many pages does the book have? **(UPSC CSAT 2019)**

(a) 1040 (b) 1048

(c) 1049 (d) 1050

Ans: (c)

Explanation:

Page number Number of digit

(1 – 9): 1 × 9 = 9

(10 – 99): 2 × 90 = 180

(100 – 999):3 × 900 = 2700

Total = 2889

Hence, digits left = 3089-2889 = 200.

Now from page number 1000 to above, we have 4 digits on each page.

So, number of pages covered in 200 digits $= \frac{200}{4} = 50$

So, required total number of pages = 999 + 50 = 1049

Q10. How many numbers are there between 100 and 300 which either begin with or end with 2? **(UPSC CSAT 2016)**

(a) 110 (b) 111

(c) 112 (d) 113

Ans: (a)

Explanation:

There are 10 numbers between 100 and 199 which either begin with or end with 2, i.e., 102, 112, 122, 132, 142, 152, 162, 172, 182, 192. And, the numbers with the said condition from 200 to 299 would be 100.

Hence, total numbers 100+ 10 = 110.

Q11. Four-digit numbers are to be formed using the digits 1, 2, 3 and 4, and none of these four digits are repeated in any manner. **(UPSC CSAT 2016)**

1. **Further, 1, 2 and 3 are not to immediately follow each other.**
2. **1 is not to be immediately followed by 3.**
3. **4 is not to appear at the last place.**
4. **1 is not to appear at the first place.**

How many different numbers can be formed?

(a) 6 (b) 9

(c) 8 (d) None of these

Ans: (a)

Explanation:

According to the given information, 6 different numbers can be formed, i.e., 2431, 2143, 3412, 3142, 3421, 4312.

Q12. If ABC × DEED = ABCABC; where *A*, *B*, *C*, *D* and *E* are different digits, what are the values of *D* and *E*? **(UPSC CSAT 2015)**

(a) *D* = 2 and *E* = 0 (b) *D* = 0 and *E* = 1

(c) *D* = 1 and *E* = 0 (d) *D* = 1 and *E* = 2

Ans: (c)

Explanation:

By multiplying any three-digit number with 1001, we get that number repeated two times, i.e.,

234 × 1001 = 234234

567 × 1001 = 567567

258 × 1001 = 258258

Thus, for each *ABC*, value of *DEED* will be 1001, i.e., *D* = 1 and *E* = 0

Q13. If x – y = 8, then which of the following must be true? **(UPSC CSAT 2018)**

1. **Both *x* and *y* must be positive for any value of *x* and *y*.**
2. **If *x* is positive, *y* must be negative for any value of *x* and *y*.**
3. **If *x* is negative, *y* must be positive for any value of *x* and *y*.**

Select the correct answer using the codes given below.

(a) Only 1 (b) Only 2

(c) Both 1 and 2 (d) Neither 1 nor 2 nor 3

Ans: (d)

Explanation:

Given, *x* – *y* = 8

Statement 1: Given *x* and *y* should be positive numbers for *x* – *y* = 8.

Assume, *x* = 2, *y* = –6

x – *y* = (2) – (–6) = 8

So, the given statement is wrong.

Statement 2: If x is positive, y must be negative for *x* – *y* = 8.

x = 16, *y* = 8

Let, *x* – *y* = 16 – 8 = 8

So, the given statement is wrong.

Statement 3: If *x* is negative, *y* must be positive for *x* – *y* = 8.

Let, *x* = –2, *y* = –10

So, *x* – y = –2 – (–10) = 8

So, the given statement is wrong.

Q14. What is the total number of digits printed, if a book containing 150 pages is to be numbered from 1 to 150? **(UPSC CSAT 2017)**

(a) 262 (b) 342

(c) 360 (d) 450

Ans: (b)

Explanation:

Total Digits from 1 to 9 = 9.

Total Digits from 10 to 99 = 90×2 = 180.

Total Digits from 100 to 150 = 51×3 = 153.

So, total number of digits = 9 + 180 + 153 = 342.

Q15. How many 3-digit natural numbers (without repetition of digits) are there such that each digit is odd and the number is divisible by 5? **(UPSC CSAT 2022)**

(a) 8 (b) 12

(c) 16 (d) 24

Ans: (b)

Explanation:

A three-digit number having all digits different, and which is divisible by 5 has to have the last digit as 0 or 5. It is an odd number thus the last digit will be 5.

The number of ways we can fill the first two digits from amongst 4 distinct digits = first digit in 4 ways × second digit in 3 ways = Total 12 ways.

Q16. How many pairs of natural numbers are there such that the difference of whose squares is 63? **(UPSC CSAT 2020)**

(a) 3 (b) 4

(c) 5 (d) 2

Ans: (a)

Explanation:

Let the required natural numbers be a and b As per statement, $a^2 - b^2 = 63$

Using formula $a^2 - b^2 = (a + b)(a - b)$

$(a + b)(a - b) = 63$

$63 = 9 \times 7$

Or $63 = 21 \times 3$

Or $63 = 63 \times 1$

There will be a total of three possible cases in which product of two numbers is 63

Case I: $(a + b) = 9$ and $(a - b) = 7$ Then $a = 8$ and $b = 1$

Case II: $(a + b) = 21$ and $(a - b) = 3$

Then $a = 12$ and $b = 9$

Case III: $(a + b) = 63$ and $(a - b) = 1$

Then $a = 32$ and $b = 31$

Total three such pair will be there

VBODMAS

The VBODMAS rule, which stands for Vinculum Brackets, Order (of powers or roots), Division, Multiplication, Addition, and Subtraction, provides a specific sequence for solving mathematical expressions with multiple mathematical operations.

According to this rule, in any mathematical expression this order should be followed while solving them from left to right. It is important to note that Division and Multiplication are of equal priority and are performed in the order they appear, as are Addition and Subtraction.

V	Vinculum or Bar	——
B	Brackets	( ), { }, []
O	Of	Of
D	Division	/ or ÷
M	Multiplication	×
A	Addition	+
S	Subtraction	–

Rules to follow while using VBODMAS rule are as follows:

- Simplify the Vinculum or bar — first.
- Simplify the Brackets
- Types of Brackets are:
  - Parenthesis or small or common brackets: ( )
  - Curly or middle brackets: { }
  - Square or big brackets: [ ]
- The order for solving brackets is ( ), { }, [ ] and if there is a minus (-) sign before the bracket then while removing the bracket, the sign of each term will change.
- Perform division or multiplication (go from left to right)
- Perform addition or subtraction (go from left to right)

QUESTIONS

Q17. Solve the following questions:

$5 + 8 - 7 \times 5$ of $9 - (35 \div \overline{8 - 3})$

(a) 390 (b) –390

(c) –309 (d) 309

Ans: (c)

Explanation:

Using BODMAS, solve vinculum,

$=5+8-7\times5 \text{ of } 9-(35\div5)$

Remove common brackets

$=5+8-7\times5 \text{ of } 9-7$

Solve "of", $=5+8-7\times45-7$

Solve multiplication,

$=5+8-315-7$

Solve addition, $=13-322$

Solve subtraction, = –309

Q18. Find the value of given expression

[76 - {90 ÷ 5 (24 - 36 ÷ 3) ÷ 3}]

(a) 75.5 (b) 4

(c) 77.5 (d) 71.5

Ans: (a)

Explanation:

First using small bracket

$(24-36\div3)=24-12=12$

Then the middle bracket [76 – {90 ÷ 5(12) ÷ 3}]

=[76 – {90 ÷ 60 ÷ 3}]

=[76 – {3/2} ÷ 3}]

=[76 – 1/2]

Then the big bracket 76 – 1/2 = 75.5

Q19. Find the value of given expression

[76-{90 ÷ 5 × (24 – 36 ÷ 3) ÷ 3}]

(a) 75.5 (b) 4

(c) 77.5 (d) 71.5

Ans: (b)

Explanation:

First using the small bracket

(24 – 36 ÷ 3)

= 24 – 12 = 12

Then the middle bracket

90 ÷ 5 × 12 ÷ 3 = 18 × 4 = 72

Then the big bracket

76 – 72 = 4

FACTORS & MULTIPLES

Two fundamental mathematical concepts closely related to each other are factors and multiples. A factor is a number that evenly divides another number, while multiples are numbers that result from multiplying a given number by another number to obtain various products. These concepts provide a foundational understanding of the relationships between numbers in mathematics.

What are Factors?

Factors are whole numbers that can be multiplied together to produce another number.

In other words, factors are the numbers that evenly divide into a given number without leaving a remainder. Factors come in pairs, with one factor multiplied by another factor equalling the original number.

For example: Consider the number 16:

The factors of 16 are the numbers that can multiply together to equal 16:

1 × 16 = 16

2 × 8 = 16

4 × 4 = 16

So, the factors of 16 are 1, 2, 4, 8 and 16.

What is a Multiple?

A multiple is a number that can be obtained by multiplying a given number by another whole number. In simpler terms, a multiple is the result of multiplying an integer by another integer.

For example: If we consider the number 6:

6 × 1 = 6 (6 is a multiple of 6)

6 × 2 = 12 (12 is a multiple of 6)

6 × 3 = 18 (18 is a multiple of 6)

6 × 4 = 24 (24 is a multiple of 6)

Note: Let *x* and *y* be two numbers then

1. If number *x* divides number *y*, then *x* is the factor of *y*.
2. If number *y* is divisible by number *x*, then *y* is the multiple of *x*.

Since the number 1 divides every integer, it is a common factor of every integer. Also, every number is divisible by 1 and every number is a multiple of 1.

Number of Factors (Divisors)

Write number N in terms of prime factors.

$N = 2^x \times 3^y \times 5^z \times .....$ (prime factorization)

Number of Factors of N = (x + 1) (y + 1) (z + 1) .....

Where x,y,z..... are the powers of respective prime factors.

For Example: Find the number of factors of 42

Prime factorization of $42 = 2^1 \times 3^1 \times 7^1$

Total number of factors (divisors)

= (1 + 1) × (1 + 1) × (1 + 1) = 8

Sum of Factors

Write number N in terms of prime factors.

Let N = $a^x \times b^y \times c^z \times \ldots$

Sum of all factors = $(a^0 + a^1 + a^2 \ldots + a^x)(b^0 + b^1 + b^2 \ldots + b^y)(c^0 + c^1 + c^2 \ldots + c^z)\ldots$

For Example: Calculate the sum of factors of 441.

Prime factorisation of 441 = $3^2 \times 7^2$

Sum of all factors = $(3^0 + 3^1 + 3^2)(7^0 + 7^1 + 7^2) = (1 + 3 + 9)(1 + 7 + 49)$

Sum of all factors = (13)(57) = 741

Product of Factors

Write number N in terms of prime factors.

Let N = $a^x \times b^y \times c^z \ldots\ldots$ (prime factorization)

Product of factors of N = $N^{(\text{number of factors}/2)} = N^{[(x+1)(y+1)(z+1)\ldots./2]}$

For Example: The Product of factors of 5472 is equal to:

Prime factorisation of 5472 = $2^5 \times 3^2 \times 19^1$

Product of factors of 5472 = $5472^{(\text{number of factors})/2}$

$= 5472^{[(5+1)(2+1)(1+1)/2]} = 5472^{[(6)(3)(2)/2]} = 5472^{36/2}$

Product of factors of 5472 = 5472^{18}

Sum of Reciprocal of all Factors

Express the number N in terms of prime factors.

Let $N = a^x \times b^y \times c^z \ldots\ldots$

Sum of all factors of $N = (a^0 + a^1 + a^2 \ldots + a^x)$

$(b^0 + b^1 + b^2 \ldots + b^y)(c^0 + c^1 + c^2 \ldots + c^z)\ldots$

Sum of reciprocal of all factors = Sum of all factors of N/ Number (N)

$= [(a^0 + a^1 + a^2 \ldots + a^x)(b^0 + b^1 + b^2 \ldots + b^y)(c^0 + c^1 + c^2 \ldots + c^z)\ldots]/N$

For Example: What is the value of the sum of reciprocal of all factors of 65025?

prime factorisation of 65025 = $3^2 \times 5^2 \times 17^2$

Sum of reciprocal of all factors

= (Sum of all factors of N)/N

Sum of reciprocal of all factors =

$[(3^0 + 3^1 + 3^2) + (5^0 + 5^1 + 5^2) + (17^0 + 17^1 + 17^2)]/65025$

$= [(1 + 3 + 9) + (1 + 5 + 25) + (1 + 17 + 289)]/65025 = (13 + 31 + 307)/65025$

Sum of reciprocal of all factors = 351/65025

Total Number of Prime Factors

To find the number of prime factors of a number first of all prime factorisation of a number is done.

For example: The number of prime factors of 484 is

Prime factorisation of 484 = $2^2 \times 11^2$

Since, prime factors are 2 and 11 therefore total number of prime factors = 2.

EXAMPLES

E1. All the factors of 81 are

(a) 3, 9, 27 (b) 3, 9, 27, 81

(c) 1, 3, 9, 27 (d) 1, 3, 9, 27, 81

Ans: (d)

Explanation:

The factors of 81 are 1, 3, 9, 27 and 81.

Because the number 81 is exactly divisible by these numbers leaving the remainder zero.

E2. Find the first three multiples of 13.

(a) 13, 26, 39 (b) 26, 39, 52

(c) 39, 52, 65 (d) 13, 65, 130

Ans: (a)

Explanation:

The multiples of 13 are 13, 26, 39, 52, 65, 78,...

$13 \times 1 = 13$

$13 \times 2 = 26$

$13 \times 3 = 39$

$13 \times 4 = 52$

$13 \times 5 = 65$, and so on

E3. Find the number of factors of 3780000.

(a) 60 (b) 240

(c) 100 (d) 150

Ans: (b)

Explanation:

Prime Factorisation of 3780000

$= 2^5 \times 3^3 \times 5^4 \times 7^1$

Number of factors = (5 + 1)(3 + 1)(4 + 1)(1 + 1)

$= 6 \times 4 \times 5 \times 2 = 240$

Multiplication Shortcuts:

Multiplying by 5 and 25:

- To multiply by 5, divide by 2 and then add a zero.

  For Example:

  $16 \times 5 \Rightarrow$ Divide 16 by 2 = 8 then add a zero = 80.

- To multiply by 25, divide by 4 and then add two zeros.

  For Example:

  $48 \times 25 \Rightarrow$ Divide 48 by 4 = 12 then add two zeros = 1200.

Multiplication of a Number by a Given Number which Contains all the Digits as 9

Split the given number (multiplier) in the form of $(10^n - 1)$.

Apply the distributive property of multiplication over subtraction.

For Example:

1. Multiply 234 by 9999.

   Step 1: 9999 can be written as (10000-1). Hence, the given problem is written as 234×(10000-1)

   Step 2: Now, apply the distributive property of multiplication over subtraction.

   234 ×(10000 – 1)

   = (234 × 10000) – (234 × 1)

   = 2340000 – 234 = 2339766

Therefore, the product of 234 and 9999 is 2339766.

THE DIGITAL SUM CONCEPT

Digital-sum method also known as Digit sum or add-up method is used to fasten the calculations involving different mathematical operations like multiplication, division, addition, subtraction, squares, square roots, cube roots, etc.

In this method, the given number is converted into a single digit by adding up all the digits of that number.

If the number obtained is two or more than two digit number then the same process is repeated till a single digit is obtained.

Shortcut Method

Since, a single digit is from 0 to 9, While calculating the digit-sum of a number, all the nines and all the digits that add up to nine can be eliminated.

Elimination of all the nines and all the digits that add up to nine will make the calculation faster. This elimination will have no effect on the final result.

Note:

- The digit-sum will always be a single digit, keep on adding the numbers until a single-digit is obtained.
- If the digit-sum of a number is 9, the digit-sum becomes 'zero.' 9 is the same as 0 in this method and can be used interchangeably as 9.
- If the digit sum value comes negative then just add 9 to it & the result will be the actual digit-sum value.

EXAMPLES

E4. Find the digit-sum of 534671234.

Explanation:

Digit-sum (add all the digits)

= 5 + 3 + 4 + 6 + 7 + 1 + 2 + 3 + 4 = 35

Again, add its digits 3 + 5 to get the answer 8.

Thus, Digit- sum of 534671234 is 8.

Another way is eliminating the numbers that add up to 9 (5 and 4, 3 and 6, 7 and 2)

1, 3, and 4 is left therefore digit sum is equal to 1+3+4 = 8

E5. The value of digit-sum of 44 x 97 x 67 x 89 is equal to

Explanation:

Write digit sum value of numbers,

8 × 7 × 4 x 8 = 56 × 4 × 8

= 2 × 4 × 8 = 8 x 8 = 64 = 1

Value of digit-sum of 44 × 97 × 67 × 89 = 1

E6. Find the value of the given expression:

$\sqrt{1+90\times91\times92\times93}$

(a) 8371 (b) 8721

(c) 8341 (d) 8231

Ans: (a)

Explanation:

Using Digital Sum Method and option elimination

Digit sum of question

= 1 + 9 × 10 × 11 × 12 = 1 + 0 × 1 × 2 × 3 = 1

Option (a) Digital Sum= 8 + 3 + 7 + 1 = 19 = 10 = 1

Option (b) Digital Sum= 8 + 7 + 2 + 1 = 18 = 9 = 0

Option (c) Digital Sum= 8 + 3 + 4 + 1 = 16 = 7

Option (d) Digital Sum= 8 + 2 + 3 + 1 = 14 = 5

DIVISIBILITY RULES

Divisibility by 2

If the unit's digit of a number is 0, 2, 4, 6 or 8, then the number is divisible by 2.

For example:

334, 762, 87640 etc.

Divisibility by 3

If the sum of the digits of a number is divisible by 3, then the number is also divisible by 3.

For example:

73533

7 + 3 + 5 + 3 + 3 = 21 which is divisible by 3.

73533 is divisible by 3.

For example:

2652

2 + 6 + 5 + 2 = 15, which is divisible by 3.

2652 is divisible by 3.

Divisibility by 4

(i) If the last 2 digits of a number is divisible by 4, then the number is divisible by 4.

For example:

67832 is divisible by 4, since 32 is divisible by 4.

(ii) If the last 2 digits of a number is zero, then the number is divisible by 4.

For example:

6700, 545000 are divisible by 4 as they have two zeros at the end.

Divisibility by 5

If the last digit of a number is either 0 or 5, then the number is divisible by 5.

For example:

115, 67520, 495 etc.

Divisibility by 6

If the number is divisible by 2 and 3 both then, the number is also divisible by 6.

For example:

378, 444 etc.

Divisibility by 7

If the difference between twice the last digit and the number formed by the remaining digits is either zero or a multiple of 7, then the number is divisible by 7.

For example: 637, $63 - (2 \times 7) = 63 - 14 = 49$, which is divisible by 7.

$\therefore$ 637 is divisible by 7.

Divisibility by 8

(i) If the last three digits of a number is divisible by 8, then the number is divisible by 8.

For example: 2368 - Last three digits = 368, which is divisible by 8.

$\therefore$ 2368 is divisible by 8.

(ii) If the last three digits of a number is zero, then the number is divisible by 8.

4354000, 54340000 are divisible by 8 as they have three zeros at the end.

Divisibility by 9

If the sum of the digits of a number is divisible by 9, then the number is also divisible by 9.

For example:

64737

Sum of digits = 6 + 4 + 7 + 3 + 7 = 27, which is divisible by 9.

$\therefore$ 64737 is divisible by 9.

Divisibility by 11

If the difference between the sum of its digits at odd places and the sum of its digits at even places is either 0 or a multiple of 11, then the number is divisible by 11.

For example:

9693926

= (9 + 9 + 9 + 6) – (6 + 3 + 2)

= 33 - 11 = 22, which is multiple of 11.

$\therefore$ 9693926 is divisible by 11.

Divisibility by 12

If the number is divisible by both 3 and 4, then the number is also divisible by 12.

For example:

2616 is divisible by both 3 and 4. Therefore, it is divisible by 12.

Divisibility by 13

To check whether a number is divisible by 13 or not, multiply its unit place digit by 4, then add the product to the number formed by the digits of the number apart from the unit's digit. If the result is 0 or a multiple of 13, then the number is divisible by 13.

For example:

Check whether 182 is divisible by 13 or not.

Last digit = 2

Multiply 2 by 4 = 8

Next step is to add this number to number formed by other digits i.e., 18+8 = 26

26 is divisible by 13.

Hence, 182 is also divisible by 13.

Divisibility by 14

If the number is divisible by both 2 and 7, then the number is also divisible by 14.

For example:

826 is divisible by both 2 and 7. Therefore, it is divisible by 14.

Divisibility by 15

If the number is divisible by both 3 and 5, then the number is also divisible by 15.

For example:

3735 is divisible by both 3 and 5. Therefore, it is divisible by 15.

Divisibility by 16

The number whose last four digits is divisible by 4, then the number is divisible by 16.

For example:

8675224 - Last 4-digits = 5224, which is divisible by 16.

∴ 8675224 is divisible by 16.

Divisibility by 17

To check whether a number is divisible by 17 or not, multiply its unit place digit by 5, then subtract the product from the number formed by the digits of the number apart from the unit's digit.

For example:

Check whether 663 is divisible by 17 or not.

Last digit = 3

Multiply 3 by 5 = 15

Next step is to subtract this number to number formed by other digits i.e., 66-15 = 51

51 is divisible by 17.

Divisibility by 18

If a number is divisible by both 2 and 9, then the number is also divisible by 18.

For example:

65436 is divisible by both 2 and 9. therefore it is also divisible by 18

Divisibility by 25

If the number formed by the last two digits are divisible by 25 or the last two digits are 0 then the number is divisible by 25.

For example:

10075 is divisible by 25

Divisibility by 125

If the number formed by the last three digits are divisible by 125 or the last three digits are 0 then the number is divisible by 125.

For example:

13250 is divisible by 125.

QUESTIONS

Q20. A digit n > 3 is divisible by 3 but not divisible by 6. Which one of the following is divisible by 4?

(UPSC CSAT 2020)

(a) 2n (b) 3n

(c) 2n + 4 (d) 3n+1

Ans: (d)

Explanation:

Such open ended questions can be attempted via trial and error to save time.

Let the digit be 9 which is greater than 3, divisible by 3 but not divisible by 6.

Now solve it by options.

Option (a); 2 × 9 = 18, not divisible by 4.

Option (b); 3 × 9 = 27, not divisible by 4.

Option (c); 2 × 9+ 4 = 22, not divisible by 4.

Option (d); 3 x 9+1 = 28, divisible by 4.

Q21. How many integers are there between 1 and 100 which have 4 as a digit but are not divisible by 4?

(UPSC CSAT 2020)

(a) 5 (b) 11

(c) 12 (d) 13

Ans: (c)

Explanation:

Integers between 1 and 100 having 4 as a digit are; 4, 14, 24, 34, 41, 42, 43, 44, 45, 46, 47, 48, 49, 54, 64, 74, 84 and 94, a total of 18.

In above, the integers divisible by 4 are; 4, 24, 44, 48, 64 and 84, a total of 6.

Hence, integers having digit 4 but not divisible by 4 = 18-6 = 12

Q22. Two statements are given followed by two conclusions.

Statement I: All numbers are divisible by 2.

Statement II: All numbers are divisible by 3.

Conclusion-I: All numbers are divisible by 6.

Conclusion-II: All numbers are divisible by 4.

Which of the above conclusions logically follows/ follow from the two given statements?

(UPSC CSAT 2020)

(a) Only conclusion-I

(b) Only conclusion-II

(c) Neither conclusion-I nor conclusion-II

(d) Both conclusion-I and conclusion-II

Ans: (a)

Explanation:

Criteria for divisibility of

(i) 2 - x number is divisible by 2 if the unit digit is zero or divisible by 2.

(ii) 3 - x number is divisible by 3 if the sum of digits is divisible by 3.

(iii) 6 - x number is divisible by 6 if the number is divisible by both 2 and 3.

(iv) 4 - x number is divisible by 4 if its last two digits are divisible by 4.

From above, we can draw the conclusion that the number which is divisible by 2 and 3 is divisible by 6 but we can't certainly conclude if it is divisible by 4.

Hence, conclusion-I follows only.

Q23. Let XYZ be a three-digit number, where (X + Y + Z) is not a multiple of 3. Then (XYZ + YZX + ZXY) is not divisible by **(UPSC CSAT 2020)**

(a) 3 (b) 9

(c) 37 (d) (X + Y + Z)

Ans: (b)

Explanation:

Given three-digit numbers is XYZ. Since, (X + Y + Z) is not a multiple of 3.

Taking an individual case, let's assume

X = 1

Y = 3

Z = 4

X + Y + Z = 8, which is not a multiple of 3.

XYZ + YZX + ZXY = 134 + 341 + 413 = 888

On checking by options, we get 888 is not divisible by 9.

Q24. If R and S are different integers both divisible by 5, then which of the following is not necessarily true? **(UPSC CSAT 2016)**

(a) R-S is divisible by 5 (b) R + S is divisible by 10

(c) RS is divisible by 25 (d) $R^2 + S^2$ is divisible by 5

Ans: (b)

Explanation:

Let, R = 10 and S = 5 then,

R+ S = 10 + 5 = which is not divisible by 10

Thus, option (b) is not necessarily true.

CYCLICITY AND UNIT DIGIT

The cyclical behavior of numbers depends upon their units (ones) digits. Each unit's digit shows a repetitive pattern which depends upon their powers.

For Example: Understanding the Cyclicity of 7

$7^1 = 7$, Unit Digit= 7

$7^2 = 49$, Unit Digit= 9

$7^3 = 343$, Unit Digit= 3

$7^4 = 2401$, Unit Digit= 1

$7^5 = 16807$, Unit Digit= 7

$7^6 = 117649$, Unit Digit= 9

$7^7 = 823543$, Unit Digit= 3

$7^8 = 5764801$, Unit Digit= 1

Hence, Number "7" has a cyclicity of 4 and Power Cycle(Pattern of last digits) of 7, 9, 3, 1,

Cyclicity cycle can be summarized as shown in the below table:

Number	Cyclicity	Power cycle
0	1	0
1	1	1
2	4	2, 4, 8, 6
3	4	3, 9, 7, 1
4	2	4, 6
5	1	5
6	1	6
7	4	7, 9, 3, 1
8	4	8, 4, 2, 6
9	2	9, 1

METHODS TO FIND UNIT DIGIT

1. If the unit digit of any number is 0, 1, 5 or 6, then the unit's digit remains the same.
2. If the unit's digit is 2, 3, 4,7, 8 and 9.

   Step1 : Divide the power by 4 and find the remainder.

   Step2 : Apply cyclicity rule on the power on unit digit by using the remainder

QUESTIONS

Q25. The digit in the unit place of 672345^{1874} is equal to

(a) 0 (b) 5

(c) 4 (d) 7

Ans: (b)

Explanation:

Any power raised to 5 the unit digit will be 5.

The unit place in $672345^{1874} = 5$

Q26. Find the unit digit of $(5767)^{35327}$

(a) 7 (b) 9

(c) 3 (d) none of these

Ans: (c)

Explanation:

$(5767)^{35327} = 7^{27} = 7^{27/4}$

On dividing 27 by 4 we get 3 as remainder,

Thus, unit digit of $(5767)^{35327}$ = unit digit of 7^3

$7^3 = 343$

Unit's digit is 3

Q27. The digit in the unit's place of the number represented by $(7652)^{1424} + (56778)^{2348}$

(a) 0 (b) 2

(c) 4 (d) 8

Ans: (b)

Explanation:

Unit's digit in $(7652)^{1424}$ = Unit's digit of 2^{24} = Unit's digit of 2^4

Since cyclicity of 2 is 4, the unit digit = 6

Unit's digit in $(56778)^{2348}$ = Unit's digit of 8^{48} = Unit's digit of 8^4

Since cyclicity of 8 is 4, the unit digit = 6

Unit's digit of $(7652)^{1424} + (56778)^{2348} = 6 + 6 = 12$

Unit's digit is 2.

Q28. The digit in the unit's place of the number represented by $(3^{94} - 6^{62})$ is

(a) 3 (b) 1

(c) 6 (d) 9

Ans: (a)

Explanation:

Unit digit in 3^4 is 1.

So, the unit digit in 3^{92} is 1.

Unit digit in 3^{94} is 9.

Unit digit in 6^1 is 6.

Unit digit in 6^{62} is 6.

Unit digit in $(3^{94} - 6^{62}) = (9 - 6) = 3$

Q29. Find the unit digit in the product $(26463)^{153} \times (2416)^{832}$.

(a) 0 (b) 5

(c) 4 (d) 8

Ans: (d)

Explanation:

unit digit in the given product

= unit digit in $3^{153} \times 6^{832}$.

Now, 3^4 gives unit digit 1.

So, the unit digit in 3^{152} is 1.

3^{153} gives unit digit = 3.

Unit digit in 6^1 is 6.

6^{832} gives unit digit = 6.

Hence, unit's digit in the product

= (3 × 6) = 18 = 8 (unit digit).

REMAINDER THEOREM

Dividend = (Divisor × Quotient) + Remainder

where,

- Dividend = The number which is being divided
- Divisor = The number by which division process is performed.
- Quotient = Greatest possible integer obtained as a result of division.
- Remainder = Left part of dividend which cannot be further divided by the divisor.

1. Remainder of $\left(\frac{a+b}{n}\right)$

= Remainder of $\left(\frac{a}{n}\right)$ + Remainder of $\left(\frac{b}{n}\right)$

If the remainder is more than n, it will be further divided by n.

For example:

Find Remainder of $\frac{13+27+38}{6}$.

According to Remainder Theorem

Remainder of $\frac{13+27+38}{6}$ = Remainder of $\left(\frac{13}{6}\right)$ + Remainder of $\left(\frac{27}{6}\right)$ + Remainder of $\left(\frac{38}{6}\right)$

$= 1 + 3 + 2 = 6$

Since 6 is remainder, remainder of $\left(\frac{6}{6}\right) = 0$.

2. The remainder of multiplication of a number by a value is the same as multiplication of the remainder of individual terms.

Remainder of $\left(\frac{a \times b}{n}\right)$

= Remainder of $\left(\frac{a}{n}\right)$ × Remainder of $\left(\frac{b}{n}\right)$

If the remainder is more than n, it will be further divided by n.

For example:

Find Remainder of $\frac{14 \times 17 \times 19 \times 21 \times 22}{5}$.

According to Remainder Theorem

Remainder of $\frac{14 \times 17 \times 19 \times 21 \times 22}{5}$

= Remainder of $\left(\frac{14}{5}\right) \times$ Remainder of $\left(\frac{17}{5}\right) \times$ Remainder of $\left(\frac{19}{5}\right) \times$ Remainder of $\left(\frac{21}{5}\right) \times$ Remainder of $\left(\frac{22}{5}\right) = 4 \times 2 \times 4 \times 1 \times 2 = 64$

Since 64 is more than 5, remainder of $\left(\frac{64}{5}\right) = 4$.

3. If the given expression can be written in the form $\frac{(ax+1)^n}{a}$ the remainder will be 1.

In such a case, no matter how large the value of the power n is, the remainder is always 1.

For example:

Remainder of $[(46)^{22655}/9]$

= Remainder of $[(9 \times 5+1)^{22655}/9] = 1$

4. If you can express the expression in the form $\frac{(ax-1)^n}{a}$ the remainder will be –1 for n = odd and remainder will be 1 for n = even .

For example:

Remainder of $[(44)^{22655}/9]$

= Remainder of $[(9 \times 5 - 1)^{22655}/9] = -1$

The remainder cannot be negative and so add 9 into it.

Thus, the remainder is 8.

5. If you can express the expression in the form $\frac{(a)^n}{a+1}$ the remainder will become "a" if "n" is odd and 1 if "n" is even.

6. **Fermat's Theorem:**

If the given equation can be expressed as $\frac{a^{P-1}}{P}$, then the remainder is 1.

where **P** is prime number and **a, P** are co-prime.

For example:

The remainder of $\frac{13^{56}}{9}$ is equal to:

$\frac{13^{56}}{9}$ can be rewritten as $\frac{(13^7)^8}{9}$

Since power of 13^7 is 8 which is one less than denominator (9) therefore the remainder is 1.

TYPES OF QUESTIONS

There are mainly two types of questions on Remainder Theorem

TYPE 1: QUESTION BASED ON BASICS OF REMAINDER THEOREM

QUESTIONS

Q30. The least number which must be added to 56434 to make it exactly divisible by 9 is:

(a) 2 (b) 3

(c) 4 (d) 5

Ans: (d)

Explanation: On dividing 56434 by 9,

Remainder = 4.

Required number to be added = 9-4 = 5

Q31. What least number must be subtracted from 432686 so that the remaining number is divisible by 12?

(a) 3 (b) 2

(c) 11 (d) 16

Ans: (b)

Explanation:

On dividing 432686 by 12,

Remainder = 2.

Required number to be subtracted = 2.

Q32. What is the least four-digit number when divided by 3, 4, 5 and 6 leaves a remainder 2 in each case? **(UPSC CSAT 2020)**

(a) 1012 (b) 1022

(c) 1122 (d) 1222

Ans: (b)

Explanation: LCM of (3, 4, 5 and 6) = 60

Least four-digit number divided by 3, 4, 5 and 6 is 60 × 17 = 1020

(Since, 60 × 16 = 960 is lower than one number which is three-digit).

Remainder in each case = 2 (given)

therefore the required four-digit number

= 1020 + 2 = 1022.

Q33. What is the remainder when 51 × 27 × 35 × 62 × 75 is divided by 100? **(UPSC CSAT 2020)**

(a) 50 (b) 25

(c) 5 (d) 1

Ans: (a)

Explanation:

Given product with division of 100 be,

$$\frac{51\times27\times35\times62\times75}{100}$$

$$=\frac{51\times27\times35\times(31\times2)\times(25\times3)}{2\times(2\times25)}$$

$$= \underbrace{\left(\frac{51\times27\times35\times31\times3}{2}\right)}_{\text{Remainders for this part}} \times \left(\frac{2\times25}{2\times25}\right)$$

$$= (1\times1\times1\times1\times1)\times \underbrace{(2\times55)}_{\text{common part in numerator and Denominator}} = 50$$

Q34. An 8-digit number 4252746B leaves the remainder 0 when divided by 3. How many values of B are possible? (UPSC CSAT 2019)

(a) 2 (b) 3

(c) 4 (d) 6

Ans: (c)

Explanation:

Given number = 4252746B

Sum of digits = 4 + 2 + 5 + 2 + 7 + 4 + 6 + B = 30 + B

A number whose sum of digits is divisible by 3 is also divisible by 3.

Hence, the possible values of B can be 0, 3, 6 or 9.

So, there are four possible values of B.

Q35. A number when divided by 185 gives a remainder 112. When the same number is divided by 37, then the remainder will be:

(a) 1 (b) 2

(c) 8 (d) 11

Ans: (a)

Explanation:

Number = (185 × Q) + 112

= (37 × 5Q) + (37 × 3) + 1

= 37 × (5Q + 3) + 1.

Remainder = 1.

Q36. The sum of digits of a three-digit number is subtracted from the number. The resultant number is always:

(a) divisible by 7 (b) divisible by 9

(c) not divisible by 7 (d) not divisible by 9

Ans: (b)

Explanation:

Let the 3-digit number be '*xyz*'

Then the number = 100*x* + 10*y* + *z* ... (i)

Sum of digits = *x* + *y* + *z* ... (ii)

99*x* + 9*y* = 9 (11*x* + *y*)

which is always divisible by 9

TYPE 2: QUESTION BASED ON FINDING THE NUMBER OF ZEROS

QUESTIONS

Q37. How many zeros are there at the end of the following product? (UPSC CSAT 2020)

1 × 5 × 10 × 15 × 20 × 25 × 30 × 35 × 40 × 45 × 50 × 55 × 60

(a) 10 (b) 12

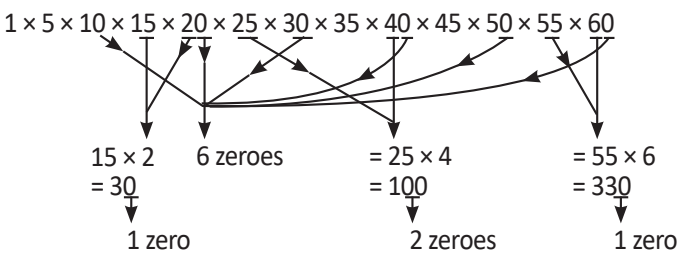
(c) 14 (d) 15

Ans: (a)

Explanation:

The given product is,

1 × 5 × 10 × 15 × 20 × 25 × 30 × 35 × 40 × 45 × 50 × 55 × 60 =



Hence, total zeroes = 1 + 6 + 2 + 1 = 10

Q38. How many zeros are there at the end of the following product?

1 × 5 × 15 × 25 × 35 × 45 x 55

(a) 5 (b) 0

(c) 4 (d) 3

Ans: (b)

Explanation:

In a product, zero is the result as the last digit if either there is **a pair of 2 and 5** in factors or zero at the end of the given number.

The given product is,

1 × 5 × 15 × 25 × 35 × 45 x 55

= 1 × 5 × 5 × 3 × 5 ×5 × 5 × 7 × 5 x 9 x 5 x 11

It neither contains any pair of 2 and 5 in the factors nor zero as a digit at the end of any given number.

PRACTICE QUESTIONS

1. **What is the difference between the place value of 8 and the face value of 8 in 348627?**
   (a) 7902 (b) 7992
   (c) 7992 (d) 7920

2. **Find the difference between the cube of the greatest single-digit prime number and the smallest 3-digit prime number.**
   (a) 1025648 (b) 1029958
   (c) 1059348 (d) 1025638

3. **Consider the following statements:**
   1. **All single-digit odd numbers are prime numbers**
   2. **No single-digit even number is a prime number**

   Which of the following statements is/are not correct?
   (a) Only 1 (b) Only 2
   (c) Both 1 & 2 (d) Neither 1 Nor 2

4. **Consider a 2-digit prime number AB where A is the square of the smallest odd prime number. Find the value of Cube of B.**
   (a) 125 (b) 27
   (c) 343 (d) 729

5. **Find the smallest value of P such that the number 282P85871 is divisible by 11.**
   (a) 1 (b) 3
   (c) 5 (d) 7

6. **Find the smallest value of P such that the whole number 17279P861 is divisible by 7.**
   (a) 1 (b) 3
   (c) 5 (d) 7

7. **If P24100Q is completely divisible by 88, find the value of P+Q (where P and Q are natural numbers).**
   (a) 10 (b) 15
   (c) 20 (d) 13

8. **How many of the following numbers are divisible by 6 but not by 9?**
   2133, 2343, 3474, 4131, 5286, 5340, 6336, 7347, 8115, 9276
   (a) 4 (b) 5
   (c) 6 (d) 3

9. **A 3-digit number 4p3 is added to another 3-digit number 984 to give a 4-digit number 13q7, which is divisible by 11. Then find the value of P + Q .**
   (a) 12 (b) 11
   (c) 10 (d) 15

10. **Find the unit digit in the expression: $(2468)^{2468}+(8642)^{8642}$**
    (a) 2 (b) 4
    (c) 6 (d) 0

11. **Find the number of zeroes in the following series: $3 \times 5 \times 7 \times 9 \times \ldots\ldots\ldots 55$**
    (a) 0 (b) 11
    (c) 22 (d) 33

12. **Find the number of zeros in $102! \times 152!$.**
    (a) 35 (b) 61
    (c) 152 (d) 102

13. **Find the unit digit in the expression: $(8!)^{8!^{8!^{8!\cdots\cdots\infty}}}$**
    (a) 0 (b) 2
    (c) 4 (d) 8

14. **How many digits are required to print a book of 1250 pages, with page numbers starting from 1?**
    (a) 2639 (b) 3893
    (c) 4694 (d) 4458

15. **How many numbers between 200 and 400 neither begin nor end with 3?**
    (a) 81 (b) 89
    (c) 101 (d) 105

16. **Find the possible number of positive integers x such that $6x + 960$ is completely divisible by x.**
    (a) 25 (b) 28
    (c) 32 (d) Infinite

17. **Find the approximate value of k in given equation:**
    $$0.25 \times \sqrt[3]{0.512} \div 0.1 + 215.978 \div \left(\frac{4}{9}\right) \text{ of } 80.988 = K^3$$
    (a) 5 (b) 3
    (c) 2 (d) 7

18. **Find approximate value in place of "?".**
    (2) of 453 + $(2400 \div 48.96 \times 80.98)^{1/2}$ = (?) × 17
    (a) 57 (b) 51
    (c) 89 (d) 76

19. **Find the remainder if 7^{2023} is divided by 10.**
    (a) 1 (b) 3
    (c) 5 (d) 7

20. **Find the remainder when 1! +2! +3! +4! +5! +....100! is divided by 5**
    (a) 0 (b) 1
    (c) 2 (d) 3

21. Find the largest number less than 3000 which is divided by 36, 54, and 60 leaving the remainder 24, 42, and 48 respectively.

(a) 2712 (b) 2688
(c) 2888 (d) 2856

22. Find the sum of the squares of the first 65 natural numbers.

(a) 91,665 (b) 92,665
(c) 93,665 (d) 94,665

23. Find the difference between the sum of the cube of the first ten natural numbers and the sum of the first twenty natural numbers.

(a) 2210 (b) 2625
(c) 2815 (d) 2830

24. Consider the following sum: \$8 + 2\$ + 31 + \$1 = 10\$. In the above sum, \$ stand for?

(a) 2 (b) 3
(c) 5 (d) 7

25. The least perfect square number which is divisible by 6, 8, and 15?

(a) 6400 (b) 1600
(c) 2500 (d) 3600

ANSWERS

1. (*b*)	2. (*b*)	3. (*c*)	4. (*c*)	5. (*a*)	6. (*c*)	7. (*a*)	8. (*d*)	9. (*c*)	10. (*d*)
11. (*a*)	12. (*b*)	13. (*a*)	14. (*b*)	15. (*b*)	16. (*b*)	17. (*c*)	18. (*a*)	19. (*b*)	20. (*d*)
21. (*b*)	22. (*c*)	23. (*c*)	24. (*a*)	25. (*d*)					

EXPLANATIONS

1. (b)

Explanation:

Here the digit 8 is at the thousands place so its place value is 8000

Face value of 8 = 8

Required difference between the place value and face value = 8000-8 = 7992

2. (b)

Explanation:

The greatest single-digit prime number is 7 and the smallest 3-digit prime number is 101

The difference between their cubes = $101^3 - 7^3$

1,030,301 – 343 = 1,029,958

3. (c)

Explanation:

1 and 9 are odd numbers but not prime numbers, hence statement 1 is not correct

2 is an even number but it's a prime number hence statement 2 is not correct

Therefore, both the statements are incorrect

4. (c)

Explanation:

Smallest odd prime number = 3

Then, A = 3×3 = 9

The number (A(b)) will be 9B

From 90 -100, there is only one prime number i.e., 97

Therefore, B = 7

Cube of B = 7×7×7 = 343

5. (a)

Explanation:

Divisibility rule for 11: For a number to be divisible by 11, the difference between the sum of digits at odd places and sum of digits at even places is either zero or multiple of 11.

2 8 2 P 8 5 8 7 1

Sum of digits at odd places = 2 + 2 + 8 + 8 + 1 = 21

Sum of digits at even places = 8 + P + 5 + 7 = 20 + P

Difference = 21 – (20 + P) = 1 – P

(1 – P) should be either 0 or a multiple of 11

Therefore, P = 1

6. (c)

Explanation:

Divisibility rule of 7: Divide the number into groups of 3 (from the left end). For the number to be divisible by 7, the difference between the sum of groups at odd places and the sum of groups at even places has to be either zero or a multiple of 7.

Grouping of the number = (172) (79P) (861)

The sum of groups at odd places = 172 + 861 = 1033

The sum of groups at even places = 79P

Difference = 1033 – (79P) = 1033 – (790+P) = 243 – P

(Since 238 is completely divisible by 7)

243 – P = 238 $\Rightarrow$ P = 5

7. (a)

Explanation: If the number is divisible by 88, it has to be divisible by both 11 and 8 (11 & 8 are co-prime)

Divisibility rule for 8: For a number to be divisible by 8, the last 3 digits should be divisible by 8 or the last three digits should be 0

'00Q' must be divisible by 8

Thus, Q = 8 (Since Q is a natural number, Q can't be 0)

Now the number can be written as **P241008**

Divisibility rule for 11: For a number to be divisible by 11, the difference between the sum of digits at odd places and the sum of digits at even places is either zero or multiple of 11.

$\underline{P}\ 2\ \underline{4}\ 1\ \underline{0}\ 0\ \underline{8}$

Sum of digits at odd places = P + 4 + 0 + 8 = P + 12

Sum of digits at even places = 2 + 1 + 0 = 3

Difference = (P + 12) – 3 = P + 9

(P + 9) can't be 0 for any natural number P

(P + 9) is divisible by 11 for P = 2

We have, P = 2 and Q = 8

P + Q = 2 + 8 = 10

8. (d)

Explanation:

Divisibility rule for 9: To be divisible by 9, the sum of digits must be divisible by 9.

Divisibility rule for 6: To be divisible by 6, a number must be divisible by both 2 and 3.

Divisibility rule for 3: To be divisible by 3, the sum of digits must be divisible by 3.

We can find the divisibility of 2 by checking if its last digit is even.

Check the divisibility of all even numbers by 3

1. 3474 $\Rightarrow$ sum of digits = 3 + 4 + 7 + 4 = 18 (divisible by 3)
2. 5286 $\Rightarrow$ sum of digits = 5 + 2 + 8 + 6 = 21 (divisible by 3)
3. 5340 $\Rightarrow$ sum of digits = 5+3+4+0 = 12 (divisible by 3)
4. 6336 $\Rightarrow$ sum of digits = 6+3+3+6 = 18 (divisible by 3)
5. 9276 $\Rightarrow$ sum of digits = 9+2+7+6 = 24 (divisible by 3)

So, the numbers divisible by 6 are: 3474, 5286, 5340, 6336, 9276

Check the divisibility of above numbers by 9

1. 3474 $\Rightarrow$ Sum of digits = 3 + 4 + 7 + 4 = 18 (divisible by 9)
2. 5286 $\Rightarrow$ Sum of digits = 5 + 2 + 8 + 6 = 21 (not divisible by 9)
3. 5340 $\Rightarrow$ Sum of digits = 5 + 3 + 4 + 0 = 12 (not divisible by 9)
4. 6336 $\Rightarrow$ Sum of digits = 6 + 3 + 3 + 6 = 18 (divisible by 9)
5. 9276 $\Rightarrow$ Sum of digits = 9 + 2 + 7 + 6 = 24 (not divisible by 9)

So, the numbers which are divisible by 6 but not 9 are: 5286, 5340, 9276

9. (c)

Explanation:

$$\begin{array}{r} 4p3 \\ +984 \\ \hline 13q7 \end{array}$$

The equation shows that $p + 8 = q$, and there is no carry forward digit at 100^{th} place. So the P will be either 1 or 0.

Divisibility rule for 11: For a number to be divisible by 11, the difference between the sum of digits at odd places and the sum of digits at even places is either zero or multiple of 11.

$13q7$ is divisible by 11 (given)

Difference = (1 + q) – (3 + 7) = q – 9 (either 0 or multiple of 11)

We get, $q = 9$

P + 8 = q (given)

$\Rightarrow P = 1$.

So, $p + q = 1 + 9 = 10$

10. (d)

Explanation:

In multiplication (or in power terms) the unit digit of the resultant will depend on the unit digit of the given number and we know that the unit digit for the numbers 2,3,7 and 8 follow the cyclicity of 4

$2^1=2$	$8^1=8$
$2^2=4$	$8^2=4$
$2^3=8$	$8^3=2$
$2^4=6$	$8^4=6$

$\frac{2468}{4} = 0(remainder)$ & $\frac{8642}{4} = 2(remainder)$

Unit digit of $(2468)^{2468}$ will be similar to that of $8^4 = 6$

Unit digit of $(8642)^{8642}$ will be similar to that of $2^2 = 4$

Unit digit of $(2468)^{2468} + (8642)^{8642} = 6+4= 10 = 0$ (unit digit)

11. (a)

Explanation:

In order to get a zero in the product there must be at least one pair of 2 and 5.

Since in the equation, all the numbers are odd there will not be a zero

12. (b)

Explanation:

In order to get a zero in the product there must be a pair of 2 and 5.

The number of zeroes in the product = sum of the number of zeroes present in each number.

Number zeroes in 102! and 152! = total number of possible pairs of 2's and 5's

Check only number of 5's (numbers of 2's will be anyway higher than number of 5's)

5	102
	20
	4
	24

5	152
	30
	6
	1
	37

There are 24 (= 20 + 4) and 37 (= 30 + 6 + 1) fives (5's) in 102! and 152! respectively,

Number of zeros in 102! ×152! = 24 + 37 = 61

13. (a)

Explanation:

In multiplication (or in power terms), the unit digit of the resultant will depend on the unit digit of the given number.

The unit digit for 0 will be 0 (for any value of power term)

Here 8! is having unit digit of 0 (all numbers after 5! will end with 0)

Anything to the power 0 will have the unit digit as 0 only

14. (b)

Explanation:

Digits required for page 1- 9 = 9×1 = 9

Digits required for page 10-99 = 90×2 = 180

Digits required for page 100-999 = 900×3 = 2700

Digits required for page 1000-1250 = 251×4 = 1004

Total number of digits required = 9 + 180 + 2700 + 1004 = 3893

15. (b)

Explanation:

Total numbers between 200 and 400 = 199 (excluding 200 & 400)

Numbers beginning with 3 from 300 to 399 = 100

Numbers ending with 3 are 203, 213, 223, 233, 243, 253, 263, 273, 283, 293 = 10

(Not counting 303, 313.....393 because they are already covered in numbers beginning otherwise it will lead to double counting)

Numbers between 200 to 400 which neither begin nor end with 3 = 199 – 100 – 10 = 89

16. (b)

Explanation:

6x + 960 is divisible by x. It means 'x' is a factor of 960.

Prime factorization of 960 is:

$960 = 2^6 \times 3 \times 5$

Total number of factors = (6+1) ×(1+1)×(1+1)

= 7×2×2= 28

Note: Total number of factors of a composite number = $(P_1+1)\times(P_2+1)\times(P_3+1)$ .....(P_n+1).

Where P_1, P_2, P_3... P_n are the powers of respective prime factors.

So, x can be 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24, 30, 32, 40, 48, 60, 64, 80, 96, 120, 160, 192, 240, 320, 480 and 960.

17. (c)

Explanation:

We can approximate the given expression using VBODMAS rule as:

$$0.25\times\sqrt[3]{0.512}\div 0.1+216.978\div\left(\frac{4}{9}\right)\text{ of }81=K^3$$

$$0.25\times 0.8\div 0.1+216\div\left(\frac{4}{9}\right)\text{ of }81=K^3$$

$0.25 \times 8 + 216 \div 36 = K^3$

$2 + 6 = K^3$

$k = 2$

18. (a)

Explanation:

We can approximate the given expression as:

$2 \times 453 + (2401 \div 49 \times 81)^{1/2} = (?) \times 17$

$2 \times 453 + (7^4 \div 7^2 \times 9^2)^{1/2} = (?) \times 17$

$906 + (7^2 \times 9^2)^{1/2} = (?) \times 17$

$906 + 7 \times 9 = (?) \times 17$

$969 = (?) \times 17$

$(?) = 57$

19. (b)

Explanation:

Remainder after dividing with 10 is nothing but the unit digit of the number

In multiplication (or in power terms), the unit digit of the resultant will depend on the unit digit of the given number

We know that the unit digit for the numbers 2,3,7 and 8 follow the cyclicity of 4

$7^1 = 7$ $7^5 = 7$

$7^2 = 9$ $7^6 = 9$

$7^3 = 3$ $7^7 = 3$

$7^4 = 1$ $7^8 = 1$

and so on

Remainder of $\frac{2023}{4} = 3$

Then, unit digit of 7^{2023} = unit digit of 7^3

Therefore, the unit digit of $7^{2023} = 3$ (since $7^3 = 343$)

Thus, the remainder of $\frac{7^{2023}}{10} = 3$

20. (d)

Explanation:

The unit digit of 5! and onwards is 0.

Thus, all the numbers are completely divisible by 5 and hence will not leave any remainder.

The remainder will be obtained only due to (1! +2! +3! +4!)

$\frac{1!+2!+3!+4!}{5} = \frac{1+2+6+24}{5} = \frac{33}{5}$

Thus, remainder = 3

21. (b)

Explanation:

The least number which is divided by a, b, and c leaving remainder p, q, and r respectively is the LCM of $(a, b, c) - k$;

where $k = a - p = b - q = c - r$.

LCM of (36, 54, 60) = 540

Here, $k = 36 - 24 = 54 - 42 = 60 - 48 = 12$

So, the general form of the desired number

$= 540N - 12$

To obtain the largest such number less than 3000, put N=5

Therefore, the required number $= 540 \times 5 - 12 = 2688$

22. (c)

Explanation:

Sum of the square of first n natural numbers

$= n(n+1)\frac{(2n+1)}{6}$

Given, $n = 65$

$\therefore$ Required sum

$= \frac{(65 \times 66 \times 131)}{6} = 65 \times 11 \times 131 = 93,665$

23. (c)

Explanation:

Sum of first 'n' natural numbers $= n\frac{(n+1)}{2}$

n = 20 (given)

Sum of first 20 natural numbers

$= \frac{(20 \times 21)}{2} = 10 \times 21 = 210$

Sum of cube of first 'n' natural numbers $= \left[\frac{n(n+1)}{2}\right]^2$

$n = 10$ (given)

Sum of cube of first 10 natural numbers

$= \left[\frac{n(n+1)}{2}\right]^2 = \left[\frac{10(11)}{2}\right]^2 = (55)^2 = 3025$

Required difference = 3025 – 210 = 2815

24. (a)

Explanation:

Let's denote '$' with x

Then we have,

$10x + 8 + 20 + x + 31 + 10x + 1 = 100 + x$

$21x - x = 100 - 8 - 20 - 31 - 1$

$20x = 100 - 60 = 40$

$x = 2$

25. (d)

Explanation:

The prime factorization of 6 is 2×3.

The prime factorization of 8 is $2 \times 2 \times 2$.

The prime factorization of 15 is 3×5.

The least perfect square number divisible by 6, 8, and 15 = $2^4 * 3^2 * 5^2$

$= 16 \times 9 \times 25 = 3600$.

Fractions

2

INTRODUCTION

A fraction is a quantity that is expressed in terms of a ratio.

$$\text{Fraction}=\frac{\text{Numerator}}{\text{Denominator}}=\frac{p}{q}$$

Where p is the numerator, q is the denominator. q is not equal to zero and p, q are co-prime.

TYPES OF FRACTIONS

1. **Simple Fraction**

   If the numerator and the denominator are both integers, it is called a simple fraction.

   For example: $\frac{2}{3},\frac{9}{5}$ etc.

2. **Proper Fraction**

   In a proper fraction, the numerator is less than its denominator.

   Therefore, Fractions that are less than 1, are called Proper Fractions.

   For example: $\frac{3}{11},\frac{17}{29}$

3. **Improper Fraction**

   In an improper fraction, the numerator is greater than or equal to its denominator.

   For example: $\frac{7}{5},\frac{21}{19},\frac{11}{11}$

4. **Mixed Fraction**

   It consists of an integer and a proper fraction in this form $x\frac{p}{q}$, where x is an integer.

   In expanded form, $x\frac{p}{q}=\frac{(qx+p)}{q}$.

   For example: $3\frac{2}{7}=\frac{7(3)+2}{7}=\frac{23}{7}$

5. **Equivalent Fractions/Equal fractions**

   Fractions that have different numerators and denominators but the fractions are equal to the same numerical value, are called as equivalent fractions.

   For example: $\frac{2}{3}=\frac{8}{12}=\frac{20}{30}=\frac{26}{39}$

   Value of a fraction is not changed by multiplying or dividing the numerator and denominator by the same number.

6. **Continued Fraction**

   It consists of the fractional denominator.

   For example:

$$1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\frac{1}{5}}}}$$

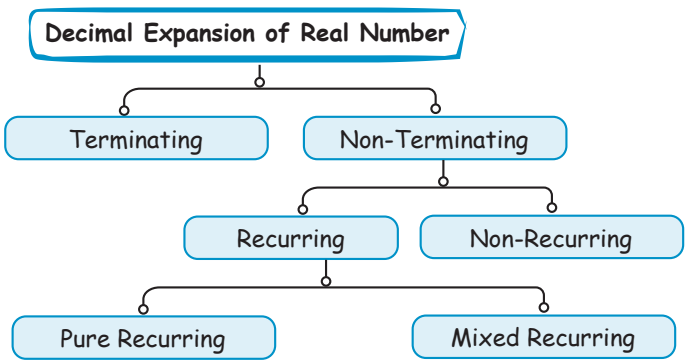


Fig. 1.1: Decimal Expansion of Real Number

Terminating Decimal Fraction

It is defined as the fraction in which the number of digits after a decimal point is countable (finite).

For example: 3.73, 7.89, 152.765

Non-Terminating Decimal Fraction

If the number of digits after decimal point in the decimal form of fraction is not finite or is uncountable then it is known as non-terminating decimal fraction.

THERE ARE TWO TYPES OF NON-TERMINATING FRACTIONS

1. **Non-Terminating Recurring (Repeating) Decimal Fractions**

   It is defined as a decimal that has infinite digits after decimal point, but repeats same one or more numbers in a particular pattern.

For example: $\frac{1}{3}$ or 0.3333333....., $\frac{5}{6}$ = 0.833333333.....

There are two types Non-terminating recurring decimal fractions:

(i) Non-Terminating Pure Recurring Decimal Fractions

Repetition of the same one or more digit after the decimal point.

For example: $\frac{11}{3}$ = 3.666666.....

Method to Convert Non-terminating Pure Recurring Decimals into Simple Fractions:

Step I: Write the non-terminating recurring decimal in bar notation.

Step II: Write the given number in the form of addition of non decimal and decimal parts.

Step III: Express the digit in the form of fraction with denominators as many nines (digit 9) as the number of digits recurring in the given number after decimal.

For Example:

Convert 0.32323232....into a simple fraction.

Fraction = 0.323232.....can be written as $0.\overline{32}$

Put as many 9 in the denominator as the number of repeating digits after the decimal,

Therefore, $0.\overline{32} = \frac{32}{99}$.

(ii) Non-terminating mixed recurring decimal fractions:

When one or more digits after the decimal point do not repeat, but the remaining digits or set of digits (after the decimal point) do repeat, then decimal is said to be mixed recurring.

For example: 0.83333333...., 0.7692222222....

Method to Convert Mixed Recurring Decimal Fraction into Simple Fraction

Step 1: Write the given number in the form of addition of non decimal and decimal parts.

Step 2: Subtract the number formed by all the digits in the decimal part and the number formed by digits that are not repeated. The number obtained in this step will be the numerator of the fraction.

Step 3: Write the number nines (digit 9) as many times as the number of recurring digits and write zero (digit 0) as many as the number of non-recurring digits after nines in the denominator.

For Example: Convert 5.25191919..... into a simple fraction.

Fraction = 5.25191919..... can be written as $5.25\overline{19} = 5 + 0.25\overline{19}$

$= 5 + \frac{2519 - 25}{9900} = 5 + \frac{2494}{9900} = 5\frac{2494}{9900} = \frac{51994}{9900}$

2. Non-terminating Non-Recurring Fractions

General form - x. *abc.....def.....xyz.....*

For example: $\sqrt{2}$ = 1.41421356237309504880...

MATHEMATICAL OPERATIONS ON FRACTIONS

1. Addition

In addition of fractions, two cases arises which are:

- **Case-I:** If denominators of the fractions are the same, then simply add the numerators. Numerator of the resulting fraction will be obtained by adding numerators of each fractions and the denominator of the resulting fraction will be the same as that of the given fraction.

  Example: Add $\frac{2}{7} + \frac{3}{7} + \frac{5}{7} + \frac{13}{7}$

  $\Rightarrow \frac{2}{7} + \frac{3}{7} + \frac{5}{7} + \frac{13}{7} = \frac{2+3+5+13}{7} = \frac{23}{7}$

- **Case-II:** If denominators of the fractions are different,

  Step 1: Take the L.C.M of denominators.

  Step 2: Make denominators of every fraction the same as L.C.M by multiplying the numerator and denominator of the fraction by required number .

  Step 3: Add the new fractions obtained.

  Step 4: Numerator will be obtained by adding numerators and the denominator will be the L.C.M of the given fractions.

  For Example: Add $\frac{1}{7} + \frac{5}{21} + \frac{2}{9}$

  L.C.M of 7, 21, 9 = 63

  $\frac{2}{7} \times \frac{9}{9} + \frac{3}{21} \times \frac{3}{3} + \frac{5}{9} \times \frac{7}{7}$

  $= \frac{18}{63} + \frac{9}{63} + \frac{35}{63} = \frac{18+9+35}{63} = \frac{62}{63}$

2. Subtraction

In subtraction of fractions, two cases arises which are:

Case I: If denominators of the fractions are the same, then simply subtract the numerators. Numerator will be obtained by subtracting numerators and the denominator will be the same as that of the given fraction.

Example: Subtract $\frac{29}{17}-\frac{13}{17}$.

$$\frac{29}{17}-\frac{13}{17}=\frac{29-13}{17}=\frac{16}{17}$$

Case II: If denominators of the fractions are different,

Step 1: Take the L.C.M of denominators.

Step 2: Make denominators of every fraction the same as L.C.M by multiplying the numerator and denominator of the fraction by the required number.

Step 3: Subtract the new fractions obtained.

Step 4: The Numerator will be obtained by subtracting numerators and the denominator will be the L.C.M of the given fractions.

For Example: Add $\frac{9}{5}-\frac{1}{15}$

L.C.M of 5, 15 = 15

$$\frac{9}{5}\times\frac{3}{3}-\frac{1}{15}\times\frac{1}{1}=\frac{27}{15}-\frac{1}{15}=\frac{27-1}{15}=\frac{26}{15}$$

Note: While subtracting numerators, if the number with negative sign is greater than that of positive sign then the resulting number will be negative.

For Example: $\frac{27}{17}-\frac{29}{17}=\frac{27-29}{17}=\frac{-2}{17}$

3. Multiplication:

Fractions are multiplied by multiplying numerators and denominators to numerators and denominators of other fractions respectively.

For Example: $\frac{1}{5}\times\frac{7}{3}\times\frac{13}{9}=\frac{1\times7\times13}{5\times3\times9}=\frac{91}{135}$

4. Division:

Fractions are divided by multiplying numerators and denominators to denominators and numerators of other fractions respectively.

For Example: $\frac{19}{3}\div\frac{17}{5}=\frac{19}{3}\times\frac{5}{17}=\frac{19\times5}{3\times17}=\frac{95}{51}$

COMPARISON RULES OF FRACTIONS

1. If fractions have the same denominator then the fraction having the greater numerator is the greater fraction.
2. If fractions have the same numerator then the fraction having the greater denominator is the smaller fraction.
3. If the numerators and denominators of the given fractions are different then different methods are used.

Methods

(a) Cross-Multiplication Method:

Let two fractions be $\frac{a}{b},\frac{c}{d}$ where a and b are positive co-prime numbers and similarly c and d are positive co-prime numbers, then

(i) If ad > bc then $\frac{a}{b}>\frac{c}{d}$

(ii) If ad = bc then $\frac{a}{b}=\frac{c}{d}$

(iii) If ad < bc then $\frac{a}{b}<\frac{c}{d}$

For Example: Which of the following fractions is greater: $\frac{29}{13},\frac{21}{17}$?

Using cross multiplication method and Comparing,

$29\times17=493$

$13\times21=273$

Since, $493>273$

therefore, $\frac{29}{13}>\frac{21}{17}$.

(b) Changing fraction into decimal form:

Step 1: Change the given fraction into decimal form.

Step 2: Compare the numbers in decimal form. Greatest number will be the greatest fraction or vice-versa.

For Example: Arrange the given fractions in ascending order: $\frac{1}{5},\frac{3}{7},\frac{2}{9},\frac{5}{8}$

$\frac{1}{5}=0.2$

$\frac{3}{7}=0.4286$

$\frac{2}{9}=0.2222$

$\frac{5}{8}=0.625$

Ascending order is:

$\frac{1}{5}<\frac{2}{9}<\frac{3}{7}<\frac{5}{8}$

(c) Equating denominators:

Step 1: Take L.C.M of the denominators of given fractions.

Step 2: Make the denominator of all fractions equal by multiplying the numerator and denominator of the fraction by the same number so that denominator of fraction is equal to L.C.M of denominators.

Step 3: Fraction with greatest numerator is greatest.

For **Example:**

Arrange the following fractions in descending order:

$\frac{13}{11}, \frac{9}{33}, \frac{21}{22}, \frac{11}{9}$

L.C.M of 11, 33, 22, 9 = 198

$\frac{13}{11} \times \frac{18}{18} = \frac{234}{198}$, ...(i)

$\frac{9}{33} \times \frac{6}{6} = \frac{54}{198}$, ...(ii)

$\frac{21}{22} \times \frac{9}{9} = \frac{189}{198}$, ...(iii)

$\frac{11}{9} \times \frac{22}{22} = \frac{242}{198}$ ..(iv)

Comparing equations (i), (ii), (iii) and (iv), we get

$\frac{242}{198} > \frac{234}{198} > \frac{189}{198} > \frac{54}{198}$

Therefore, the fractions in descending order is

$\frac{11}{9} > \frac{13}{11} > \frac{21}{22} > \frac{9}{33}$.

(d) Equating numerators:

Step 1: Take L.C.M of the numerators of given fractions.

Step 2: Make the numerator of all fractions equal by multiplying the numerator and denominator of the fraction by the same number so that the numerator of the fraction is equal to L.C.M of numerators.

Step 3: Fraction with the greatest denominator is least.

For Example: Arrange the following fractions in ascending order:

$\frac{13}{5}, \frac{39}{7}, \frac{26}{21}, \frac{1}{9}$

L.C.M of 13, 39, 26, 1 = 78

$\frac{13}{5} \times \frac{6}{6} = \frac{78}{30}$, ...(i)

$\frac{39}{7} \times \frac{2}{2} = \frac{78}{14}$, ...(ii)

$\frac{26}{21} \times \frac{3}{3} = \frac{78}{63}$, ...(iii)

$\frac{1}{9} \times \frac{78}{78} = \frac{78}{702}$ ...(iv)

Comparing equations (i), (ii), (iii) and (iv), we get

$\frac{78}{702} < \frac{78}{63} < \frac{78}{30} < \frac{78}{14}$

Therefore, $\frac{1}{9} < \frac{26}{21} < \frac{13}{5} < \frac{39}{7}$.

ROUNDING OFF DECIMALS

In some decimal expansion of fractions, the number of digits is large after the decimal point.

For example: 7.3265789, 29.459940008897.

For calculations and in some cases, decimal numbers up to a certain number of places is required. In such cases, rounding off decimal expansion is done.

Rules:

1. If the digit of the decimal place is five or more than five, then the digit in the preceding decimal place is increased by one.

   For example: 2.6754321 is rounded off up to two decimal places then the result will be 2.68 since the third digit after the decimal point is 5 so the preceding digit will be increased by 1.

2. If the digit in the last place is less than five, then the digit in the preceding place remains unchanged.

   For example: 6.67354 is rounded off up to four decimal places then the result will be 6.6735 since the fifth digit after decimal point is 4 so there will be no change in the preceding digit.

TYPES OF QUESTIONS

There are mainly three types of questions on Fractions

TYPE 1: QUESTIONS BASED ON BASICS OF FRACTIONS

QUESTIONS

Q1. What are the least fractions which, when added to and subtracted from $\frac{31}{15}+\frac{19}{45}$ so that the result is a whole number?

(a) $\frac{23}{45},\frac{22}{45}$ (b) $\frac{23}{45},\frac{32}{45}$

(c) $\frac{32}{45},\frac{23}{45}$ (d) $\frac{23}{54},\frac{22}{45}$

Ans: (a)

Explanation:

$\frac{31}{15}+\frac{19}{45}=\frac{93+19}{45}=\frac{112}{45}=2\frac{22}{45}$

Least fraction to be added, $\frac{112}{45}+\frac{23}{45}=\frac{135}{45}=3$

Least fraction to be subtracted, $\frac{112}{45}-\frac{22}{45}=\frac{90}{45}=2$

Q2. Which one of the following will have a minimum change in its value if 5 is added to both numerator and the denominator of the fraction. Solve this question considering given options.

(UPSC CSAT 2020)

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$

(c) $\frac{4}{5}$ (d) $\frac{5}{6}$

Ans: (d)

Explanation:

Option (a): Fraction is,

$\frac{2}{3}=0.6666....or\ 0.67$

$\frac{2+5}{3+5}=\frac{7}{8}=0.875 or\ 0.88$

Difference = 0.88 – 0.67 = 0.21

Option (b): Fraction is,

$\frac{3}{4}=0.75$

$\frac{3}{4}=\frac{3+5}{4+5}=\frac{8}{9}=0.88888...or\ 0.89$

Difference = 0.89 – 0.75 = 0.14

Option (c): Fraction is,

$\frac{4}{5}=0.8$

$\frac{4+5}{5+5}=\frac{9}{10}=0.9$

Difference = 0.9 – 0.8 = 0.1

Option (d): Fraction is,

$\frac{5}{6}=0.83333...or\ 0.83$

$\frac{5+5}{6+5}=\frac{10}{11}=0.909090....or\ 0.91$

Difference = 0.91 – 0.83 = 0.08

Hence, required fraction is $\frac{5}{6}$.

Note: a fraction which is closest to the whole number '1' in value undergoes the smallest change when common values are added (or subtracted) from it. This way, $\frac{5}{6}$ will undergo the least change

Q3. The recurring decimal representation 1.272727..... is equivalent to **(UPSC CSAT 2020)**

(a) $\frac{13}{11}$ (b) $\frac{14}{11}$

(c) $\frac{127}{99}$ (d) $\frac{137}{99}$

Ans: (b)

Explanation: Given representation 1.272727... is equivalent

$=1.\overline{27}=1+\frac{27}{99}$

$=\frac{126}{99}$

$=\frac{14}{11}$

Q4. What is the largest number among the following? **(UPSC CSAT 2020)**

(a) $\left(\frac{1}{2}\right)^{-6}$ (b) $\left(\frac{1}{4}\right)^{-3}$

(c) $\left(\frac{1}{3}\right)^{-4}$ (d) $\left(\frac{1}{6}\right)^{-2}$

Ans: (c)

Explanation:

Option (a): $\left(\frac{1}{2}\right)^{-6} = \frac{(1)^{-6}}{(2)^{-6}} = \frac{(2)^6}{(1)^6} = 64$

Option (b): $\left(\frac{1}{4}\right)^{-3} = \frac{(4)^3}{(1)^3} = 64$

Option (c): $\left(\frac{1}{3}\right)^{-4} = \frac{(3)^4}{(1)^4} = 81$

Option (d): $\left(\frac{1}{6}\right)^{-2} = \frac{(6)^2}{(1)^2} = 36$

Therefore, the largest number is $\left(\frac{1}{3}\right)^{-4}$.

Q5. If the numerator and denominator of a proper fraction are increased by the same positive quantity which is greater than zero, the resulting fraction is **(UPSC CSAT 2019)**

(a) always less than the original fraction

(b) always greater than the original fraction

(c) always equal to the original fraction

(d) such that nothing can be claimed definitely

Ans: (b)

Explanation:

Such questions are better solved with a specific example and extrapolating that to all general cases.

Let the fraction be $\frac{1}{2}$

Suppose we add a number 3 to both numerator and denominator, we get

$\frac{1+3}{2+3} = \frac{4}{5}$

$\frac{1}{2} = 0.5$

$\frac{4}{5} = 0.8$

Therefore,

$\frac{4}{5} > \frac{1}{2}$

Hence, the new fraction is always greater than the original fraction.

Q6. X and Y are natural numbers other than 1, and Y is greater than X. Which of the following represents the largest number? **(UPSC CSAT 2018)**

(a) XY (b) $\frac{X}{Y}$

(c) $\frac{Y}{X}$ (d) $\frac{(X+Y)}{XY}$

Ans: (a)

Explanation:

Given, *X* and *Y* are natural numbers other than 1, i.e.,, 2, 3,...., N.

Given X, $Y \neq 1$ and $Y > X$. In such cases it is prudent to come to an answer by assuming specific values for the variables.

Let us assume, $X = 2$, $Y = 4$.

For these values, we get from options

(a) $XY = 2 \times 4 = 8$

(b) $\frac{X}{Y} = 2/4 = 0.5$

(c) $\frac{Y}{X} = 4/2 = 2$

(d) $\frac{(X+Y)}{XY} = \frac{6}{8}$ 6/8 = 0.75

XY represents the largest number.

TYPE 2: QUESTIONS BASED ON OPERATIONS ON DECIMAL NUMBERS

QUESTIONS

Q7. The value of $5.7\overline{31} + 8.\overline{253}$ is equal to:

(a) $14.\overline{983367}$ (b) $13.\overline{9845663}$

(c) $19.\overline{384566}$ (d) $18.\overline{3944432}$

Ans: (b)

Explanation:

Given equation, $5.7\overline{31} + 8.\overline{253}$

$5.7\overline{31} = 5.731313131.....$

$8.\overline{253} = 8.253253253.....$

$5.7\overline{31} + 8.\overline{253}$ = 5.731313131 + 8.253253253

= 13.984566384 = $13.\overline{9845663}$

Therefore $5.7\overline{31} + 8.\overline{253} = 13.\overline{9845663}$

Q8. A student appeared in 6 papers. The maximum marks are the same for each paper. His marks in these papers are in the proportion 5 : 6 : 7 : 8 : 9 : 10. Overall he scored 60%. In how many papers did he score less than 60% of the maximum marks? (UPSC CSAT 2021)

(a) 2 (b) 3

(c) 4 (d) 5

Ans: (b)

Explanation:

Let total marks in each subject be 100.

Therefore, total marks for all 6 subjects = 600

Overall marks scored = 60% of 600 = 360

According to the question, $5x + 6x + 7x + 8x + 9x + 10x = 360$ or $45x = 360$ or $x = 8$

So, marks in the given 6 subjects must be: $5 \times 8 = 40$, $6 \times 8 = 48$, $7 \times 8 = 56$, $8 \times 8 = 64$, $9 \times 8 = 72$, $10 \times 8 = 80$

Therefore, in 3 subjects the student has scored less than 60% marks.

Note: Such application based questions from fractions require application of other domains as well (like percentages in this case).

Q9. An amount of money was distributed among A, B and C in the ratio p : q : r

Consider the following statements:

1. **A gets the maximum share if p is greater than (q+r).**
2. **C gets the minimum share if r is less than (p+q).**

Which of the above statements is/are correct? (UPSC CSAT 2021)

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

Ans: (a)

Explanation:

Ratio of distribution of money among A, B and C is p : q : r.

Statement 1 is correct: If $p > (q + r)$, then p is definitely the largest number.

So, A must have got the maximum share.

Statement 2 is incorrect: If $r < (p + q)$, then r may or may not be the smallest number. For example, $5 < (2 + 4)$ So, C may or may not have got the minimum share.

TYPE 3: MISCELLANEOUS QUESTION ON FRACTIONS

QUESTIONS

Q10. The value of $1-\cfrac{1}{3-\cfrac{1}{5+\cfrac{1}{7+\cfrac{1}{9+\cfrac{1}{11}}}}}$ **is equal to**

(a) $\frac{6599}{10254}$ (b) $\frac{6959}{10524}$

(c) $\frac{6995}{12054}$ (d) $\frac{6599}{10425}$

Ans: (a)

Explanation:

$$1-\cfrac{1}{3-\cfrac{1}{5+\cfrac{1}{7+\cfrac{1}{9+\cfrac{1}{11}}}}} = 1-\cfrac{1}{5+\cfrac{1}{7+\cfrac{11}{100}}}$$

$$= 1-\cfrac{1}{3-\cfrac{100}{711}} = 1-\cfrac{1}{3-\frac{711}{3655}}$$

$$= 1-\frac{3655}{10254} = \frac{10254-3655}{10254} = \frac{6599}{10254}$$

PRACTICE QUESTIONS

1. **What is the difference between the largest and smallest fraction among $\left(\frac{5}{9}\right),\left(\frac{7}{11}\right),\left(\frac{8}{15}\right),\left(\frac{11}{17}\right)$?**

   (a) $\left(\frac{29}{255}\right)$ (b) $\left(\frac{8}{99}\right)$

   (c) $\left(\frac{1}{45}\right)$ (d) $\left(\frac{17}{165}\right)$

2. **Which of the following fractions does not lie between $\left(\frac{5}{11}\right)$ & $\left(\frac{9}{14}\right)$?**

   (a) $\left(\frac{1}{2}\right)$ (b) $\left(\frac{4}{7}\right)$

   (c) $\left(\frac{2}{3}\right)$ (d) $\left(\frac{3}{5}\right)$

3. **If 'P5P110' is a six digit number which is divisible by 11, then P is equal to:**

   (a) 2 (b) 4

   (c) 6 (d) 8

4. **Consider the following question.**

   Question: 'P' is a natural number greater than 1.

   Quantity I: Remainder left when 125^P is divided by 100

   Quantity II: Remainder left when 130^P is divided by 100

   Statements regarding two Quantities I and II are given above with a Question.

   What is the correct relationship between Quantity I and Quantity II.

   (a) Quantity I > Quantity II

   (b) Quantity I < Quantity II

   (c) Quantity I ≥ Quantity II

   (d) Quantity I = Quantity II or No relation

5. **For what value of 'x', the sum of the digits in the number $(6^x + 1)$ is 1 (keep on adding the digits till single digit is achieved)?**

   (a) For x = 0 only

   (b) For any whole number

   (c) For any natural number

   (d) For any natural number greater than 1

6. **If 'p' is a perfect square integer such that $43 \le (3p - 33) \le 75$, then the value of p is:**

   (a) 16 (b) 25

   (c) 36 (d) 49

7. **How many pairs of positive integers 'p' and 'q' satisfy the following equation:**

   $$\left(\frac{1}{p}\right)+\left(\frac{4}{q}\right)=\left(\frac{1}{12}\right)$$

   Where q is an odd integer less than 58?

   (a) One (b) Two

   (c) Three (d) Five

8. **A fraction is in the form of $\left(\frac{m}{n}\right)$. Value of 'n' is 3 more than 'm'. If 'm' is increased by 7 and 'n' is decreased by 2, then the value of the fraction becomes 2. What is the original fraction?**

   (a) $\left(\frac{8}{11}\right)$ (b) $\left(\frac{1}{4}\right)$

   (c) $\left(\frac{3}{8}\right)$ (d) $\left(\frac{5}{8}\right)$

9. **If $\frac{1}{p}=\frac{1}{q}+\frac{1}{r}$ then which of the following is true, with respect to value of 'r'?**

   (a) $\frac{pq}{p-q}$ (b) $\frac{pq}{q-p}$

   (c) $\frac{p-q}{pq}$ (d) None of these

10. **Which of the following is the least possible fraction that will get divided by each one of $\left(\frac{5}{18}\right),\left(\frac{8}{9}\right)$, and $\left(\frac{10}{27}\right)$?**

    (a) $\left(\frac{25}{9}\right)$ (b) $\left(\frac{40}{9}\right)$

    (c) $\left(\frac{25}{3}\right)$ (d) $\left(\frac{81}{4}\right)$

11. **If the numerator of a fraction is increased by 'P%' while denominator of the fraction is increased by 'Q%', then what will be the ratio of original fraction to new fraction?**

    (a) $\left(\frac{P}{Q}\right)$ (b) $\left\{\left(\frac{100+P}{100+Q}\right)\right\}$

    (c) $\left\{\frac{(Q-1)}{P}\right\}$ (d) $\left\{\frac{(100+Q)}{(100+P)}\right\}$

12. **Two statements I and II are given below with regard to two numbers followed by question.**

Statement-I: $p > q^3$

Statement-II: $p > q^6$

Question: If the product of 'p' and 'q' is a positive number. Is $(p + q) > 0$?

Which one of the following is correct in respect of the Statements and the Question?

(a) Statement-I alone is sufficient to answer the Question.

(b) Statement-II alone is sufficient to answer the Question.

(c) Either Statement-I alone or Statement-II alone is sufficient to answer the question.

(d) Statement-I and II together are sufficient to answer the question.

13. **Which is the greatest among the following?**

(a) $\left(\frac{1}{2}\right)^{-4}$ (b) $\left(\frac{1}{5}\right)^{-3}$

(c) $\left(\frac{1}{3}\right)^{-5}$ (d) $\left(\frac{1}{12}\right)^{-2}$

14. **Amol purchased cloth for curtains to decorate his home. He divided it into 3 parts. The first part is $\left(\frac{1}{5}\right)^{th}$ of the whole length and the second is half of the first part. Find the ratio of the length of the longest to the shortest piece of curtain.**

(a) 5 : 3 (b) 5 : 2

(c) 7 : 1 (d) 3 : 2

15. **A pipe is cut into 3 pieces such that length of the medium part is a cube of a number. Lengths of each piece and that of the whole pipe is a multiple of 25. Length of the largest and the smallest part is 160% and 60% of medium part, respectively. What is difference between length of the largest part and the smallest part of the pipe.**

(a) 100 units (b) 125 units

(c) 150 units (d) 175 units

16. **If value of denominator of each of the following fractions is added to the value of its numerator while the value of numerator is subtracted from the value of its denominator, then the fraction thus obtained will have maximum value for which of the following fraction.**

(a) $\left(\frac{8}{13}\right)$ (b) $\left(\frac{9}{17}\right)$

(c) $\left(\frac{10}{19}\right)$ (d) $\left(\frac{11}{23}\right)$

ANSWERS

1. (*a*)	2. (*c*)	3. (*d*)	4. (*a*)	5. (*d*)	6. (*c*)	7. (*c*)	8. (*d*)	9. (*b*)	10. (*b*)
11. (*d*)	12. (*b*)	13. (*c*)	14. (*c*)	15. (*b*)	16. (*a*)				

EXPLANATIONS

1. (a)

Explanation:

$\left(\frac{5}{9}\right) \sim 0.55$

$\left(\frac{7}{11}\right) \sim 0.63$

$\left(\frac{8}{15}\right) \sim 0.53$

$\left(\frac{11}{17}\right) \sim 0.64$

Largest Fraction = $\left(\frac{11}{17}\right)$

Smallest fraction = $\left(\frac{8}{15}\right)$

Required difference = $\left(\frac{11}{17}\right) - \left(\frac{8}{15}\right)$

$= \frac{29}{255}$

2. (c)

Explanation:

$\left(\frac{5}{11}\right) \sim 0.45$

$\left(\frac{9}{14}\right) \sim 0.64$

Option (a) $\left(\frac{1}{2}\right) = 0.5$ lies between $\left(\frac{5}{11}\right)$ and $\left(\frac{9}{14}\right)$

Option (b) $\left(\frac{4}{7}\right) = 0.57$ lies between $\left(\frac{5}{11}\right)$ and $\left(\frac{9}{14}\right)$

Option (c) $\left(\frac{2}{3}\right) = 0.67$ does not lies between $\left(\frac{5}{11}\right)$ and $\left(\frac{9}{14}\right)$

3. (d)

Explanation:

Divisibility rule of 11: If the difference of the alternating sum of digits of the number is a multiple of 11

Now, P – 5 + P – 1 + 1 – 0 = 0 or 11

Or, 2P – 5 = 0 or 11

When 2P – 5 = 0

$P = \frac{5}{2} = 2.5$ (value of P cannot be in decimal)

When 2P – 5 = 11

2P = 16

$P = \frac{16}{2} = 8$

4. (a)

Explanation:

Quantity I:

Any power of 25 will always have 5 at the end and hence will leave a positive remainder when divided by 100.

Quantity II:

Any power of 30 will always have 00 at the end and hence will leave a 0 remainder when divided by 100.

So, Quantity I > Quantity II

5. (d)

Explanation:

Let Given number (Y) = $6^x + 1$

On putting x = 0, we get:

$Y = 6^0 + 1 = 1 + 1 = 2$ {sum of digits is 2}

On putting x = 1, we get:

$Y = 6^1 + 1 = 6 + 1 = 7$ {sum of digits is 7}

On putting x = 2, we get:

$Y = 6^2 + 1 = 36 + 1 = 37$ {sum of digits is 1}

On putting x = 3, we get:

$Y = 6^3 + 1 = 216 + 1 = 217$ {sum of digits is 1}

On putting x = 4, we get:

$Y = 6^4 + 1 = 1296 + 1 = 1297$ {sum of digits is 1}

Hence, sum of digits of number 'Y' will always be 1 if x = 2, 3, 4 .....

This means that, sum of digits of number 'Y' will always be 1 if x is any natural number greater than 1.

6. (c)

Explanation:

Taking p = 16

$3p - 33 = 3 \times 16 - 33 = 48 - 33 = 15$

$\Rightarrow 43 \le 15 \le 75$ (not true)

Taking p = 25

$3p - 33 = 3 \times 25 - 33 = 75 - 33 = 42$

$\Rightarrow 43 \le 42 \le 75$ (not true)

Taking p = 36

$3p - 33 = 3 \times 36 - 33 = 108 - 33 = 75$

$\Rightarrow 43 \le 75 \le 75$

Hence, Condition is satisfied.

So, p = 36

7. (c)

Explanation:

$\left(\frac{1}{p}\right) + \left(\frac{4}{q}\right) = \left(\frac{1}{12}\right)$

Or, $\left(\frac{1}{p}\right) = \left(\frac{1}{12}\right) - \left(\frac{4}{q}\right)$

Or $p = \left\{\frac{12q}{(q-48)}\right\}$ ...(i)

So, here minimum value of 'q' to be greater than 48 i.e. 49

Also, given that 'q' is an odd integer with value less than 58.

So, possible values of 'q' will be 49, 51, 53, 55 and 57

When q = 49, from equation 1, p = 588 (Integer)

When q = 51, p = 204 (Integer)

When q = 53, p = 127.2 (Not an integer)

When q = 55, p = 90.85 (Not an integer)

When q = 57, p = 76 (Integer)

So, three pairs of positive integers 'p' and 'q' satisfies the given equation.

8. (d)

Explanation:

From the question, it is given that

n = m + 3

Also, $\left\{\frac{(m+7)}{(n-2)}\right\}=2$

Or m + 7 = 2n – 4

Or m + 7 = 2(m + 3) – 4

Or m + 7 = 2m + 6 – 4

Or m = 5

So, n = m + 3 = 5 + 3 = 8

So, the original fraction is $\left(\frac{5}{8}\right)$

9. (b)

Explanation:

$\left(\frac{1}{p}\right)=\left(\frac{1}{q}\right)+\left(\frac{1}{r}\right)$

Or, $\left(\frac{1}{r}\right)=\left(\frac{1}{p}\right)-\left(\frac{1}{q}\right)=\frac{(q-p)}{pq}$

Or, $r=\frac{pq}{(q-p)}$

10. (b)

Explanation:

We have to find the L.C.M of the given fractions: $\left(\frac{5}{18}\right),\left(\frac{8}{9}\right)$ and $\left(\frac{10}{27}\right)$

L.C.M of fractions = $\frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$

$=\frac{\text{L.C.M of (5, 8, 10)}}{\text{H.C.F. of (18, 9, 27)}}$

$=\left(\frac{40}{9}\right)$

11. (d)

Explanation:

Let the numerator of the fraction be 'M' and the denominator be 'N'.

Hence, the original fraction $=\left(\frac{M}{N}\right)$

New numerator = M + P% of M

$=\left\{\frac{M(100+P)}{100}\right\}$

New denominator = N + Q% of N

$=\left\{\frac{N(100+Q)}{100}\right\}$

Hence, new fraction

$=\frac{\left\{\frac{M(100+P)}{100}\right\}}{\left\{\frac{N(100+Q)}{100}\right\}}$

$=\left(\frac{M}{N}\right)\times\left\{\frac{(100+P)}{(100+Q)}\right\}$

Ratio of original fraction to new fraction

$=\frac{\left(\frac{M}{N}\right)}{\left[\frac{M}{N}\right]}\times\left\{\frac{(100+P)}{(100+Q)}\right\}$

$=\left\{\frac{(100+Q)}{(100+P)}\right\}$

Thus, the required ratio $=\left\{\frac{(100+Q)}{(100+P)}\right\}$

12. (b)

Explanation:

If the product '*p*' and '*q*' is greater than 0, i.e. $pq > 0$, then both '*p*' and '*q*' must have the same sign.

It means either $p > 0, q > 0$ OR $p < 0, q < 0$.

Statement I: $p > q^3$

Now, there are two possibilities:

Either (i) both '*p*' and '*q*' are positive. In that case, $p + q > 0$;

(ii) Both *p* and *q* are negative.

Assume, $p = -5$ and $q = -10$

So, $p + q = -15 < 0$

Hence, Statement I alone is not sufficient.

Statement II: Given that $p > q^6$

So, q^6 is positive and 'p' is greater than it.

So, $p > 0$

It will be possible when both p and q will be positive, because their product is a positive number.

So, $p + q > 0$.

Hence, question can be answered by using Statement II alone.

13. (c)

Explanation:

$\left(\frac{1}{2}\right)^{-4} = 2^4 = 16$

$\left(\frac{1}{5}\right)^{-3} = 5^3 = 125$

$\left(\frac{1}{3}\right)^{-5} = 3^5 = 243$

$\left(\frac{1}{12}\right)^{-2} = 12^2 = 144$

Greatest Value is 243

14. (c)

Explanation:

Let the complete length of the curtain be 'L' units.

Length of the first part = $\frac{1}{5} \times L = \left(\frac{L}{5}\right)$ units

Length of the second part = $\frac{1}{2} \times \frac{1}{5} \times L = \left(\frac{L}{10}\right)$ units

So, length of the third part = $L - \left(\frac{L}{5} + \frac{L}{10}\right) = \frac{7L}{10}$ units

Thus, the length of the longest part = $\frac{7L}{10}$ units

And length of the shortest part = $\frac{L}{10}$ units

Therefore, the required ratio = $\frac{\frac{7L}{10}}{\frac{L}{10}}$ = 7:1

15. (b)

Explanation:

Length of the medium part can be 125 units (= 5^3), 1000 units (= 10^3), 3375 units (= 15^3) and so on

When the length of medium part = 125 units

Length of smallest part = 0.60 × 125 = 75 units

Length of largest part = 1.60 × 125 = 200 units

Required difference = 200 – 75 = 125 units

When the length of medium part = 100 units

Length of smallest part = 0.60 × 1000 = 600 units

Length of largest part = 1.60 × 1000 = 1600 units

Required difference = 1600 – 600 = 1000 units (not in option)

16. (a)

Explanation:

For option 'a':

Required value $= \left\{\frac{(8+13)}{(13-8)}\right\} = \left(\frac{21}{5}\right) = 5.2$ {largest}

For option 'b':

Required value $= \left\{\frac{(17+9)}{(17-9)}\right\} = \left(\frac{26}{8}\right) = 3.25$

For option 'c':

Required value $= \left\{\frac{(10+19)}{(19-10)}\right\}$

$= \left(\frac{29}{9}\right) \sim 3.22$

For option 'd':

Required value

$= \left\{\frac{(23+11)}{23-11}\right\} = \left(\frac{34}{12}\right) \sim 2.833$

LCM and HCF

3

INTRODUCTION

In this chapter, we explore two key math ideas: LCM (Least Common Multiple) and HCF (Highest Common Factor). LCM helps us find the smallest number that two or more numbers can divide. It's very useful in solving real-life problems like scheduling events. HCF helps us find the biggest number that can divide two or more numbers without leaving anything behind, helping in things like sharing sweets equally. We will learn how to easily find LCM and HCF and see how they help us in everyday maths problems.

LEAST COMMON MULTIPLE (LCM)

The LCM of numbers is the least possible number that is completely divisible by each of the numbers. For example, 15 is least possible number which is divisible by 3 and 5. Then the LCM of 3 and 5 is 15.

METHODS TO FIND LCM

There are two methods to find the LCM of given numbers which are

1. **Prime Factorization Method**

   Follow the steps below to find LCM of given numbers.

   Step 1: Write the given numbers as a product of their prime factors.

   Step 2: Choose the prime factors with the highest index in all the given numbers.

   Step 3: Multiply all the prime factors with the respective highest indices to get the LCM of given numbers.

   Example: LCM of 48, 76, 162.

   Prime Factorisation of:

   $48 = 2^4 \times 3$

   $76 = 2^2 \times 19$

   $162 = 2 \times 3^4$

   LCM = $2^4 \times 3^4 \times 19 = 24624$.

2. **Division Method**

   Follow the steps below to find LCM of given numbers.

   Step 1: On LHS write a common prime factor while on RHS write given numbers.

   Divide all the numbers with the common prime factors.

   Step 2: The division process continues until there is no common factor to the numbers.

   Step 3: The product of divisors and remaining numbers with no common factor is the LCM of given numbers.

   Example: LCM of 48, 76, 162.

2	48, 76, 162
2	24, 38, 81
3	12, 19, 81
	4, 19, 27

   Since there are no common factors between 4, 19, 27.

   Therefore, LCM = $2 \times 2 \times 3 \times 4 \times 19 \times 27 = 24624$

LCM OF POLYNOMIALS

The product of the factors with highest powers when two or more polynomials are factorised is known as the lowest common multiple (LCM) of the polynomials.

For example: Find LCM of $(x^2 - 9)$, $(x^2 - 5x + 6)$ and $(x^2 - 4)$.

FACTORISATION OF:

- $(x^2 - 9) = (x - 3)(x + 3)$
- $(x^2 - 5x + 6) = (x - 3)(x - 2)$
- $(x^2 - 4) = (x - 2)(x + 2)$

LCM = $(x - 3)(x + 3)(x - 2)(x + 2) = (x^2 - 9)(x^2 - 4) = x^4 - 13x^2 + 36$

LCM OF FRACTIONS

Firstly write the given fractions in their lowest terms then,

$$\text{L.C.M} = \frac{\text{L.C.M of numerators}}{\text{H.C.F of denominators}}$$

For example: Find LCM of $\frac{3}{4}, \frac{9}{24}, \frac{11}{12}, \frac{21}{20}$.

$$\text{L.C.M} = \frac{\text{L.C.M of } (3, 9, 11, 21)}{\text{H.C.F of } (4, 24, 12, 20)} = \frac{693}{4}$$

LCM OF DECIMAL NUMBERS

Step 1: Write the numbers without decimals

Step 2: Find the LCM of the numbers without decimal.

Step 3: Find the position of the decimal point from right to left according to the minimum decimal places among the given numbers.

For example: Find LCM of 0.45, 0.315, 6.3, and 0.27.

LCM of 45, 315, 63, and 27 = 1890

In the given decimal numbers, the minimum decimal place is 0.1

Therefore, LCM of 0.45, 0.315, 6.3, and 0.27 = 189.0 = 189

PROPERTIES OF LCM

- The smallest number which is exactly divisible by x, y and z is LCM of x, y, z.
- The LCM of two or more numbers is greater than or equal to the greatest number of given numbers.
- The numbers which when divided by x, y and z leaves a remainder R in each case.
  Then the required numbers = (LCM of x, y, z) × K + R, where K is a constant
- The numbers which when divided by x, y and z leaves the remainder of a, b, c such that common difference(d) = $x - a = y - b = z - c$.
  Then the required numbers = (LCM of x, y and z) × $K - d$, where K is a constant

TYPES OF QUESTIONS

There are two types of questions on LCM

TYPE 1: QUESTIONS BASED ON BASICS OF LCM

QUESTIONS

Q1. When "x" is increased by 9 it is completely divisible by 7, 13, and 17. What is the smallest possible value of "x"?

(a) 1583 (b) 1538

(c) 1853 (d) 1358

Ans: (b)

Explanation:

Required no. = LCM of (7, 13, 17) – 9

= 1547 – 9 = 1538

Q2. Find the greatest number which when subtracted from 5432 is exactly divisible by 5, 14, 17.

(a) 4442 (b) 4422

(c) 4224 (d) 4242

Ans: (d)

Explanation:

Required number = 5432 – LCM of 5, 14, 17 = 5432 – 1190 = 4242.

TYPE 2: QUESTIONS BASED ON APPLICATIONS OF LCM

QUESTIONS

Q3. The alarm of three different clocks ring after every 24 sec., 48 sec., and 144 sec. respectively. If they all ring simultaneously at 10:15:00 hrs, then at what time will they again ring simultaneously?

(a) 10 : 17 : 24 hrs (c) 10 : 17 : 42 hrs

(b) 10 : 12 : 24 hrs (d) 10 : 17 : 12 hrs

Ans: (a)

Explanation:

LCM of 24, 48, 144 = 144

The alarm will ring simultaneously after 144 seconds or 2 min 24 secs.

Clock will ring simultaneously at 10: 17: 24 hrs

Q4. Seeta and Geeta go for a swim after a gap of every 2 days and every 3 days, respectively. If on 1st January both went for a swim together, when will they go together next? **(UPSC CSAT 2019)**

(a) 7th January (b) 8th January

(c) 12th January (d) 13th January

Ans: (d)

Explanation:

Given that, Sita and Gita go swimming after a gap of every 2 days and every 3 days intervals, respectively. Thus, Sita swims every 3rd day while Geeta every 4th Day.

They will go together again at

= LCM of (2 + 1) and (3 + 1)

= LCM of 3 and 4

= 12 days

Hence, 12 days after 1st January or they will go together on 13th January.

Q5. In a school, every student is assigned a unique identification number. A student is a football player if and only if the identification number is divisible by 4, whereas a student is a cricketer if and only if the identification number is divisible by 6. If every number from 1 to 100 is assigned to the students, then how many of them play cricket as well as football? (UPSC CSAT 2019)

(a) 4 (b) 8

(c) 10 (d) 12

Ans: (b)

Explanation:

If a student must play both cricket and football, then his/her identification number must be divisible by both 4 and 6.

So, LCM of (4, 6) = 12.

So, there are 8 multiples of 12 between 1 to 100 which are 12, 24, 36, 48, 60, 72, 84, 96.

Hence, there are 8 students who play both cricket and football.

Q6. Five persons fire bullets at a target at an interval of 6, 7, 8, 9 and 12 s, respectively. The number of times they would fire the bullets together at the target in an hour is

(a) 6 (b) 7

(c) 8 (d) 9

Ans: (b)

Explanation:

The five persons will together fire at the target at an interval of 't' second, where

t = LCM of (6, 7, 8, 9, 12) = $2^3 \times 3^2 \times 7 = 504$ s

Number of seconds in an hour = 3600.

= [3600/504]= $7 \times 504 + 72$.

So, number of times the five persons would fire the bullets together at the target in an hour is 7.

Q7. Three persons start walking together and their steps measure 40 cm, 42 cm and 45 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps? (UPSC CSAT 2011)

(a) 25 m 20 cm (b) 50 m 40 cm

(c) 75 m 60 cm (d) 100 m 80 cm

Ans: (a)

Explanation:

LCM of 42, 45 and 40 will give the minimum distance each should walk so that each can cover the same distance in complete steps. So required distance,

= LCM of (42, 45, 40) = 2520 cm

= 25 m 20 cm

HIGHEST COMMON FACTOR (HCF)

The highest number that exactly divides two or more given numbers leaving zero remainder is known as Highest common factor (HCF) of the given numbers.

For example: HCF of 24 and 36, is "12" as it is the highest number that divides 24 and 36 completely.

METHODS TO FIND HCF

1. Prime Factorisation Method.

Step 1: Write the given numbers in the form of the product of their prime factors.

Step 2: Choose all common prime factors with the lowest power of each common prime factor in the given numbers.

Step 3: Product of all common prime factors with their respective lowest power is the HCF.

For Example: Find the HCF of 729, 828 and 5625:

Prime Factorisation of:

$729 = 3^6$

$828 = 2^2 \times 3^2 \times 23$

$5625 = 3^2 \times 5^4$

Therefore, HCF = $3^2 = 9$.

2. Division method

Follow the steps below to find HCF of two numbers by division method

Step 1: Divide higher number with the smaller number

Step 2: Divide smaller number with the remainder of above division

Step 3: Then the second remainder divides the first remainder and the process of division continues till the remainder is zero. The last divisor is the HCF

For example: Find the HCF of 48, 120

$$\begin{array}{r} 2 \\ 48\overline{)120} \\ 96 \\ \hline 24 \end{array} \Rightarrow \begin{array}{r} 2 \\ 24\overline{)48} \\ 48 \\ \hline 0 \end{array}$$

HCF of 48 and 120 is 24.

HCF OF POLYNOMIALS

The product of common factors when two or more polynomials are factored is known as HCF of the polynomials.

For example: Find the HCF of $25x^2 (x-1) (x+1)^2$ and $5xy (x+1)^3 (x-3)$

- $25x^2 (x-1) (x+1)^2$

  $=5^2x^2(x-1)(x+1)(x+1)$

- $5xy (x+1)^3 (x-3)$

  $= 5xy(x+1)(x+1)(x+1)(x-3)$

HCF = $5x(x+1)(x+1) = 5x (x+1)^2$

HCF OF FRACTIONS

Express the given fractions in their lowest terms.

$$\text{H.C.F} = \frac{\text{H.C.F. of numerators}}{\text{L.C.M. of denominators}}$$

For Example: Find HCF of $\frac{7}{13}, \frac{28}{65}, \frac{35}{39}, \frac{63}{52}$.

$$\text{H.C.F of fraction} = \frac{\text{H.C.F of numerators}}{\text{L.C.M of denominators}}$$

$$= \frac{\text{H.C.F of (7, 28, 35, 63)}}{\text{L.C.M of (13, 65, 39, 52)}} = \frac{7}{780}$$

HCF OF DECIMAL NUMBERS

STEP 1: Find the HCF of given numbers without decimals.

STEP 2: Place the decimal point from right to left according to the maximum decimal places in the given numbers.

For Example: Find HCF of 0.5, 0.15,12.5 and 0.625.

HCF of 5, 15, 125 and 625 = 5

In the given decimal numbers, the maximum decimal place is 0.001.

Therefore, HCF of 0.5, 0.15,12.5 and 0.625 = 0.005

PROPERTIES OF HCF

- The HCF of two or more numbers is smaller than or equal to the smallest number of given numbers.
- Let "K" be the largest number which when divide a,b,c gives the same remainder 'r' & quotients are x, y, z respectively then **K = HCF of (a–b, b–c, c–a)**

  For example: What is the greatest number which leaves the same remainder in each case when divides 265, 580, and 825

  Explanation: Difference of the numbers 580 - 265 = 315, 825 – 580 = 245 and 825 – 265 = 560

  Factor of 315= 3 × 3 × 5 × 7

  Factor of 245= 5 × 7 × 7

  Factor of 560= 2 × 2 × 2 × 2 × 5 × 7

  Hence, HCF. of 315, 245, 560 = 5 × 7= 35

- The greatest number which divides x, y and z to leave the remainder R is

  HCF of $(x-R)$, $(y-R)$ and $(z-R)$.

  For Example: The greatest number which divides 16, 42 and 55 to leave the remainder 3 is equal to:

  Given numbers: 16, 42 and 55

  Remainder = 3

  HCF of 16 – 3, 42 – 3, 55 – 3 = HCF of 13, 39, 52

  HCF of 13, 39, 52 = 13

  Therefore, HCF of 16, 42, 55 is 13.

- The greatest number which divides x, y, z to leave remainders a, b, c is

  HCF of $(x-a)$, $(y-b)$ and **$(z-c)$**

  For Example: Calculate the greatest number which divides 17, 19, 21 to leave remainder 2, 4, and 6 respectively.

  Given numbers: 17, 19 and 21

  Respective remainder = 2, 4 and 6

  HCF of 17-2, 19-4, 21 – 6 = HCF of 15, 15, 15

  HCF of 15, 15, 15 = 15

  Therefore, the greatest number is 15.

PRODUCT RULE OF LCM AND HCF

If p and q are two numbers then,

Product of numbers $(p \times q)$ = (HCF. of p and q) × (LCM of p and q)

TYPES OF QUESTIONS

There are two types of questions on HCF

TYPE 1: QUESTIONS BASED ON BASICS OF HCF

QUESTIONS

Q8. If the sum of two numbers is 40 and the HCF. and LCM of these numbers are 5 and 75 respectively. Find the sum of reciprocal of the two numbers.

(a) $\frac{1}{8}$ (b) 8

(c) $\frac{8}{75}$ (d) $\frac{8}{15}$

Ans: (c)

Explanation:

Let the two numbers be A and B

$A + B = 40$

HCF = 5 and LCM = 75

We know that, Product of two numbers

= LCM * HCF

A*B = 5*75

Sum of the reciprocals

$= \frac{1}{A}+\frac{1}{B}=\frac{A+B}{AB} = \frac{40}{5\times75}=\frac{8}{75}$

Q9. The greatest number that will divide 137, 345 and 770 leaving remainders 5, 9 and 2 respectively is

(a) 9 (b) 5

(c) 12 (d) 14

Ans: (c)

Explanation:

Required number

= HCF of (137 – 5), (345 –9) and (782 – 14)

= HCF of 132, 336 and 768

Prime factorization of:

132 = 2 × 2 × 3 × 11

336 = 2 × 2 × 2 × 2 × 3 × 7

768 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 3

HCF of 132, 336 and 768 = 2 × 2 × 3 = 12

TYPE 2: QUESTIONS BASED ON APPLICATIONS OF HCF

QUESTIONS

Q10. Four metal rods of lengths 78 cm, 104 cm, 117 cm, and 169 cm are to be cut into parts of equal length. Each part must be as long as possible. What is the maximum number of pieces that can be cut?

(UPSC CSAT 2009)

(a) 27 (b) 36

(c) 43 (d) 400

Ans: (b)

Explanation:

Since each rod must be cut into parts of equal length and each part must be as long as possible, HCF should be taken.

HCF of 78, 104, 117 and 169 = 13.

Number of parts from 78 cm rod = $\frac{78}{13}$ = 6

Number of parts from 104 cm rod = $\frac{104}{13}$ = 8

Number of parts from 117 cm rod = $\frac{117}{13}$ = 9

Number of parts from 169 cm rod = $\frac{169}{13}$ = 13

So, maximum number of pieces = 6 + 8 + 9 + 13 = 36

Q11. A rectangular room of length 4.25 meters and breadth 6.25 meters is to be covered with square tiles of the largest possible size without any gaps or overlaps. Length of tile is equal to

(a) 42 cm (b) 62 cm

(c) 25 cm (d) None of these

Ans: (c)

Explanation:

Largest size of the tile = HCF. of 4.25 meters and 6.25 meters

Largest size of the tile = HCF. of 425 cm and 625 cm = 25 cm

Q12. What is the greatest length *x* such that 3½ and 8¾ m are integral multiples of *x*?

(UPSC CSAT 2022)

(a) $1\frac{1}{2}$m (b) $1\frac{1}{3}$m

(c) $1\frac{1}{4}$m (d) $1\frac{3}{4}$m

Ans: (d)

Explanation:

$3\frac{1}{2} = \frac{7}{2}$ and $8\frac{3}{4} = \frac{35}{4}$

x = HCF of $\frac{7}{2}$ and $\frac{35}{4}$

$= \frac{\text{(LCM of 2 and 4)}}{\text{(HCF of 7 and 35)}} = \frac{7}{4}$

Hence, $\frac{7}{4}$ or $1\frac{3}{4}$m.

PRACTICE QUESTIONS

1. **If the sum of two numbers is 40 and the HCF. and LCM of these numbers are 5 and 75 respectively. Find the sum of reciprocal of the two numbers.**

   (a) $\frac{1}{8}$ (b) 8

   (c) $\frac{8}{75}$ (d) $\frac{8}{15}$

2. **Find the least number which when divided by 5, 9, 12 and 15 leaves a remainder of 4 in each case.**

   (a) 176 (b) 184

   (c) 124 (d) 146

3. **Find the least number which divided by 6, 8, 14 leaves a remainder of 2, 4, 10 respectively.**

   (a) 340 (b) 332

   (c) 164 (d) 172

4. **A rectangular hall 6.5 meters long and 4.25 meters wide is paved exactly with square tiles of the same size. Find the largest size of the tiles which can be used for this purpose.**

   (a) 15 cm (b) 5 cm

   (c) 25 cm (d) 35 cm

5. **The LCM of two numbers is 96. The numbers are in the ratio 3 : 4. Then the sum of the two numbers is:**

   (a) 56 (b) 36

   (c) 66 (d) 46

6. **A boy's hostel has 3 halls of area measuring 70 m^2, 98 m^2, and 126 m^2. Spaces for students are allotted such that each student is getting the same rectangular space if each Student is getting a length of 7 m. Find the breadth of rectangular space each student will get.**

   (a) 2m (b) 4m

   (c) 6m (d) 5m

7. **Find the greatest number which divides 93, 140 and 187 leaves a remainder of 3, 5 and 7 respectively.**

   (a) 10 (b) 25

   (c) 15 (d) 45

8. **Five bells commence tolling together and toll at intervals 2, 4, 6, 8 and 10 seconds respectively. How many times do they toll together in one hour?**

   (a) 30 (b) 31

   (c) 26 (d) 60

9. **Three persons Raju, Shyam and Baburao started running together from the same point on a 600 metres long circular track with the speed of 50 m/min, 40 m/min and 25 m/min respectively. After how much time will they meet again at the starting point?**

   (a) 60 min (b) 90 min

   (c) 100 min (d) 120 min

10. **The product of two numbers is 2420 and their HCF is 11. The number of possible such pairs is:**

    (a) 4 (b) 3

    (c) 2 (d) 5

11. **Three numbers are in the ratio of 2 : 3 : 5 and their LCM is 1500. Their HCF is:**

    (a) 30 (b) 25

    (c) 50 (d) 100

12. **When a group of students is divided into groups of 3, 4 and 5, two students are left every time. Find the least possible value of the number of students in the group.**

    (a) 32 (b) 52

    (c) 62 (d) 42

13. **The number of pairs of positive integers whose sum is 91 and HCF is 7 are:**

    (a) 3 (b) 4

    (c) 5 (d) 6

14. **Find the least perfect square which is completely divisible by 14, 18 and 24.**

    (a) 7056 (b) 4032

    (c) 3024 (d) 5544

15. **Sum of two numbers x, y = 1350. What is the maximum value of the HCF of x and y?**

    (a) 1350 (b) 900

    (c) 675 (d) 450

16. **The Sum of two numbers x and y is 1650. Find the maximum value of the HCF of x and y where $x \neq y$.**

    (a) 550 (b) 650

    (c) 825 (d) 750

17. **There are 2 numbers x and y such that $x > y$. HCF (x, y) = M and LCM (x, y) = N. What is the LCM of ($x - y$, y)?**

    (a) MN (b) $\frac{M}{(x-y)x}$

    (c) $\frac{M}{Ny(x-y)}$ (d) $\frac{(x-y)y}{M}$

18. Consider the sitting arrangement of students in an annual day function. If the teachers make a row of students of 15 each, there will be 12 students left. If they make rows of 24 each, then there will be 21 students left and if they make rows of 30 each, there will be 27 students left. What is the minimum number of students present in the school?

(a) 87 (b) 93
(c) 117 (d) 123

19. 4 logs of wood of length 5 $\frac{1}{4}$ m, 2 $\frac{12}{15}$ m, 3 $\frac{1}{2}$ m and 4 $\frac{9}{10}$ m are cut into small pieces of equal length . Each piece of wood is as lengthy as possible. If seven pieces are given to one carpenter, how many carpenters are there at the work?

(a) 15 (b) 17
(c) 19 (d) 21

ANSWERS

1. (*c*)	2. (*b*)	3. (*c*)	4. (*c*)	5. (*a*)	6. (*a*)	7. (*d*)	8. (*b*)	9. (*d*)	10. (*c*)
11. (*c*)	12. (*c*)	13. (*d*)	14. (*a*)	15. (*c*)	16. (*a*)	17. (*d*)	18. (*c*)	19. (*c*)	

EXPLANATIONS

1. (c)

Explanation:

Let the two numbers be *A* and *B*

$A + B = 40$

HCF = 5 and LCM = 75

We know that, Product of two numbers = LCM × HCF

$A \times B = 5 \times 75$

Sum of the reciprocals = $\frac{1}{A}+\frac{1}{B}=\frac{A+B}{AB}=\frac{40}{5\times75}=\frac{8}{75}$

2. (b)

Explanation:

The least number which is divisible by 5,9,12,15 is their LCM

LCM of 5,9,12,15 =180

∴ The number which will give a remainder of 4 when divided by each number is 180 + 4 = 184

3. (c)

Explanation:

The least number which divided by *X*, *Y* and *Z* leaves a remainder of *a*, *b* and *c* respectively is

LCM of (*X*, *Y*, *Z*) – *K*

Where, $K = (X - a) = (Y - b) = (Z - c)$

So, the value of K = (6-2) = (8-4) = (14-10) = 4

The least number which when divided by 6, 8, 14 leaves a remainder of 2, 4, 10 respectively

= LCM of (6, 8, 14) – 4

LCM of (6, 8, 14) = 168

The required number will be 168 – 4 = 164

4. (c)

Explanation:

Since the area has to be paved with same size of tiles, the maximum size of each side of tile which can be selected is the HCF of 6.5m and 4.25m

HCF of 650 cm and 425 cm = 25 cm

Therefore, the largest tile will be a square with each side measuring 25cm

5. (a)

Explanation:

Let the Numbers be 3*x* : 4*x*

Their LCM will be 12*x*

$12X = 96 \Rightarrow X = 8$

The sum of the numbers will be $3X + 4X = 3 \times 8 + 4 \times 8 = 56$

6. (a)

Explanation:

We need to divide the space available in each hall into rectangles of the same size i.e., HCF of 70, 98, 126

HCF of (70,98,126) = 14

Each student will get a space of $14m^2$. Since the length of the rectangle has to be 7m, the breadth of the rectangular space will be 2m

7. (d)

Explanation:

The number which gives remainder of 3 after dividing 93 means it completely divides 90

The number which gives remainder of 5 after dividing 140 means it completely divides 135

The number which gives remainder of 7 after dividing 187 means it completely divides180

Therefore, the required number will be HCF of (90,140,180)

HCF of (90, 135, 180) = 45

8. (b)

Explanation:

LCM of 2, 4, 6, 8, 10 = 120

Hence all bells toll together every 120 seconds i.e. every 2 minutes

Number of times they toll together in one hour (60 min) = $\frac{60}{2}+1$ = 30+1 = 31 times

(+1 to include the first time of tolling together)

9. (d)

Explanation:

Time taken by each to complete the circular track:

Speed of Raju is 50m/min ∴ Time taken by him

$=\frac{600}{50}=12\text{min}$

Speed of Shyam is 40m/min ∴ Time taken by him

$=\frac{600}{40}=15\text{min}$

Speed of Baburao is 25m/min ∴ Time taken by him

$=\frac{600}{25}=24\text{min}$

Time at which they meet again = LCM of (12, 15, 24)

LCM of (12, 15, 24) =120

They will meet after 120 minutes

10. (c)

Explanation:

HCF of two numbers is 11 (given)

Let the numbers be 11*x* and 11*y*

11*x* × 11*y* = 2420

xy = 20

Now, the co-primes with product 20 are (1, 20) and (4, 5).

So, the required numbers are (1×11 and 20×11) and (4×11 and 5×11) i.e.,

(11, 220) and (44, 55)

2 such possible pairs

11. (c)

Explanation:

Let the numbers be 2*x*, 3*x*, 5*x*

Then, their LCM = 30*x*

So, 30*x* = 1500 $\Rightarrow$ *x* =50

Therefore, the numbers are (2×50), (3×50), (5×50) i.e., 100, 150, 250

Thus, required HCF = 50

12. (c)

Explanation:

The number which is completely divisible by 3,4,5 is their LCM

LCM of 3,4,5 = 60

Since each time 2 students are left, the number of students = 60 + 2 = 62

13. (d)

Explanation:

The HCF of numbers is 7

Let the numbers be 7*x* and 7*y*

7*x* + 7*y* = 91

x + y = 13

co-primes with the sum equals to13 are (1,12) (2,11) (3,10) (4,9) (5,8) (6,7)

Total possible pairs are 6

14. (a)

Explanation:

The least number which is divisible by 14,18 and 24 = LCM of 14,18 and 24

LCM of 14,18 and 24 = 504

504 = 2×<u>2×2</u>×<u>3×3</u>×7

To make it a perfect square of a number, we will have to multiply 504 by 2 and 7

504 × 2 × 7 = 7056

Thus, the least perfect square which is completely divisible by 14, 18 and 24 is 7056.

15. (c)

Explanation:

The maximum value of HCF will be possible when all the factors of the numbers(x & y) are same i.e.,

1350 = 675+675 (Since there is no mention that numbers are different)

So, the maximum HCF value will be 675

16. (a)

Explanation:

The maximum value of HCF will be possible when all the factors of the numbers are same i.e., when the sum is exactly divided into two.

Since $x \neq y$ we cannot divide sum into 2, next maximum HCF can be obtained by dividing the sum into 3 parts

$\frac{1650}{3} = 550$

So numbers can be x = 550 and y = 1100

Here, HCF = 550

17. (d)

Explanation:

Product of numbers = LCM * HCF

Given $x > y$, HCF = M, LCM = N

When two numbers x, y are there such that $x > y$ then HCF of (x, y) = HCF of $(x - y, y)$

From the above we can say, HCF of $(x - y, y)$ = M

LCM * HCF = Product of 2 numbers

LCM*M = (x – y)*y

LCM = $(x - y)\frac{y}{M}$

Thus, the LCM of x – y and y = (x - y) y / M

18. (c)

Explanation:

When rows of 15 is made 12 are remaining

When rows of 24 are made 21 are remaining

When rows of 30 are made 27 are remaining

The least number which when divided by X, Y and Z leaves a remainder of a, b and c respectively is LCM of $(X, Y, Z) - K$

Where K = (X – a) = $(Y - b)$ = (Z – c)

The value of K = (15-12) = (24-21) = (30-27) = 3

[LCM of (X, Y, Z)] -K = (LCM of 15,24,30) – 3 = 120 – 3 = 117

Minimum number of students = 117

19. (c)

Explanation:

Lengths of the logs : $5\frac{1}{4}$ m, $2\frac{12}{15}$ m, $3\frac{1}{2}$ m and $4\frac{9}{10}m$

i.e., lengths $\frac{21}{4}$ m, $\frac{42}{15}$ m, $\frac{7}{2}$ m and $\frac{49}{10}$ m

$$\text{HCF of fraction} = \frac{\text{HCF of numerator}}{\text{LCM of Denominator}}$$

$$\text{HCF} = \frac{\text{HCF of}(21,42,7,49)}{\text{LCM of}(4,15,2,10)} = \frac{7}{60}$$

Number of pieces = $(\frac{17}{4} + \frac{43}{15} + \frac{9}{2} + \frac{39}{10})/(7 \div 60)$

$= \frac{931}{60} * \frac{60}{7} = 133 \text{piece}$

Number of carpenters required = $\frac{133}{7} = 19$

Algebra

4

INTRODUCTION

The branch of mathematics in which arithmetical operations like addition, subtraction etc., are applied to symbols rather than specific numbers is known as Algebra. The key elements in algebra are: **Constant** (whose value does not change), **Variable** (unknown quantity), and **Equation** (Mathematical Expression equating two statements)

TYPES OF ALGEBRAIC EXPRESSION

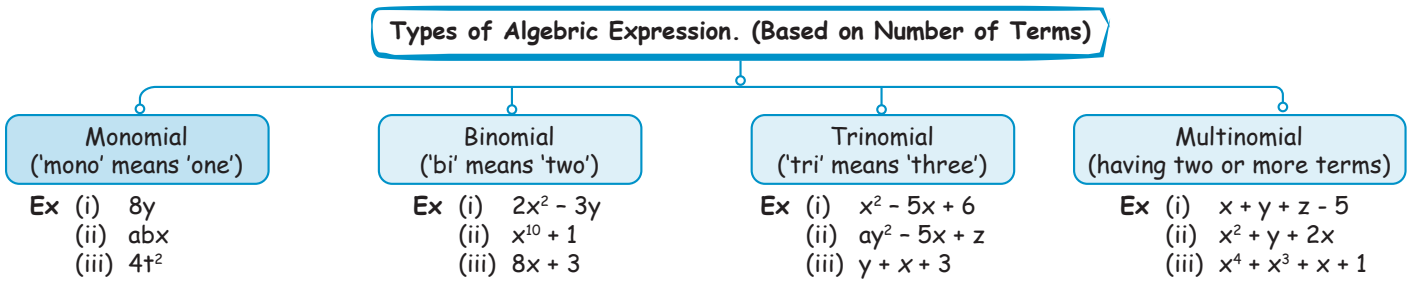


Fig. 4.1: Types of Algebric Expression. (Based on Number of Terms)

POLYNOMIALS

It is an algebraic expression consisting of terms which are combinations of variables and constants.

General form of polynomial

$P(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + a_3x^{n-3} + \ldots\ldots\ldots$ ...(i)

Power of x in each term must be a non-negative integer.

$a_0, a_1, a_2, a_3, \ldots\ldots\ldots$ are real numbers.

Degree of a polynomial

Highest power of variables in an algebraic expression is known as the degree of the polynomial.

In equation (i), the highest power of "x" is "n". Hence, "n" is the degree of the given polynomial.

For Example: In polynomial $9x^8 - 17x^5 - 11x^2 + 23x - 57$

Since, the greatest power of x in the given polynomial is 8, therefore the degree of the polynomial is 8.

Types of Polynomials

Zero Polynomial: $P(x) = 0$ is called zero polynomial and its degree is not defined.

Constant Polynomial: A polynomial of zero degree is known as constant polynomial.

For Example: $P(x) = 8$

$P(x)$ can be written as $P(x) = 8x^0$.

Note: Any non-zero digit or variable raised to power zero is equal to 1.

Linear Polynomial: A polynomial of degree one is known as linear polynomial.

General form: $p(x) = ax + b$, where $a \neq 0$.

For Example: $p(x) = 5x + 2$, $p(x) = 3x$ etc.

Quadratic Polynomial: A polynomial of degree two is called a quadratic polynomial.

General form: $P(x) = ax^2 + bx + c$, where $a \neq 0$.

For Example: $p(x) = 3x^2 - 5x + 7$

Cubic Polynomial: A polynomial of degree three is called a cubic polynomial.

General form: $P(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$.

For Example: $-2x^3 + 3x^2 + 5x + 8$, $6x^3 + 2x + 1$, $6x^3 - 8$ etc.

Biquadratic Polynomial: A polynomial of degree four is called a biquadratic polynomial.

General form: $P(x) = ax^4 + bx^3 + cx^2 + dx + e$, where $a \neq 0$.

For Example: $23x^4 - 13x^3 + 7x^2 + 5x - 9$

ZEROS OF A POLYNOMIAL

If for x = a, the value of the polynomial p(x) is 0 i.e., p(a) = 0; then x = a is a zero of the polynomial p(x).

For Example:

(i) For the polynomial $p(y) = y^2 - 5y - 14$;

$p(7) = 7^2 - 5 \times 7 - 14 = 49 - 35 - 14 = 0$

$y = 7$ is a zero of the polynomial

Also, $p(-2) = (-2)^2 - 5 \times (-2) - 14$

$= 4 + 10 - 14 = 0$

$y = -2$ is also a zero of the polynomial $p(y)$

Note:

(i) Every linear polynomial has one and only one zero.

(ii) If the degree of a polynomial is n; the largest number of zeros it can have is also n which implies that if the degree of a polynomial is 5, the polynomial can have at most 5 zeros.

REMAINDER THEOREM

Let $P(x)$ be a polynomial which is divided by $(x - a)$ where "a" is a constant (number) then if $Q(x)$ is the quotient and $P(a)$ be the remainder then according to remainder theorem:

$$P(x) = (x-a)Q(x) + P(a)$$

EXAMPLES

E1. Find the remainder when $2x^4 + x^3 - 14x^2 - 19x - 6$ is divided by $x^2 + 3x + 2$?

Explanation:

To find remainder we can use long $2x^2 - 5x - 3$ division Method

$$\begin{array}{r} 2x^2 - 5x - 3 \\ x^2 + 3x + 2 \overline{)\, 2x^4 + x^3 - 14x^2 - 19x - 6} \\ \underline{2x^4 \pm 6x^3 \pm 4x^2} \\ -5x^3 - 18x^2 - 19x - 6 \\ \underline{-5x^3 - 15x^2 - 10x} \\ -3x^2 - 9x - 6 \\ \underline{-3x^2 - 9x - 6} \\ + \quad + \quad + \\ \hline 0 \end{array}$$

Dividend = Divisor × Quotient + Remainder

$2x^2 + x^3 - 14x - 19x - 6 = (x^2 + 3x + 2)(2x^2 - 5x + 3) = 0$

$\Rightarrow$ Reminder is 0

E2. Find the value of 'a' and 'b' when $P(x) = x^3 - 6x^2 + 11x - 6b$ is divided by $x^2 + x + a$ Secures remainder zero.

Explanation:

$$\begin{array}{r} x - 7 \\ x^2 + x + a \overline{)\, x^3 - 6x^2 + 11x - 6b} \\ \underline{x^2 \pm x^2 \pm ax} \\ -7x^2 + (11 - a)x - 6b \\ \underline{\mp 7x^2 \mp 7x \quad \mp 7a} \\ (18 - a)x + 7a - 6b \end{array}$$

Remainder = $(18 - a = 0)x + 7a - 6b$

But remainder is zero

$\Rightarrow 18 - a = 0$ and $7a - 6b = 0$

$a = 18$ and $b = \frac{7a}{6}$

$$b = \frac{7 \times 18}{6}$$

$b = 21$

$a = 18, b = 21$

FACTOR THEOREM

The factor theorem states that if $f(x)$ is a polynomial of degree $n (n \geq 1)$ and 'a' is any real number, then $(x - a)$ is a factor of $f(x)$ if $f(a) = 0$.

Note:

(i) Remainder obtained on dividing polynomial $p(x)$ by $x - a$ is equal to $p(a)$.

(ii) If a polynomial $p(x)$ is divided by $(x + a)$ the remainder is the value of $p(x)$ at $x = -a$.

(iii) A polynomial $p(x)$ is divided by $(ax - b)$, the remainder is the value of $p(x)$ at $x = \frac{b}{a}$.

(iv) If a polynomial $p(x)$ is divided by $(b - ax)$, the remainder is equal to the value of $p(x)$ at $x = \frac{b}{a}$.

(v) $(x - a)$ is a factor of polynomial $p(x)$ if $p(a) = 0$ whereas $(x + a)$ is a factor of polynomial $p(x)$ if $p(-a) = 0$.

(vi) $(x - a)$ $(x - b)$ is a factor of polynomial $p(x)$, if $p(a) = 0$ and $p(b) = 0$.

EXAMPLES

E3. Find the remainder when $2x^3 - x^2 + 3x - 9$ is divided by $x - 1$.

Explanation:

Let $p(x) = 2x^3 - x^2 + 3x - 9$ is divided by $x - 1$ then the remainder is equal to $p(1)$.

Putting $x = 1$ in $p(x)$, we get

$p(1) = 2(1)^3 - (1)^2 + 3(1) - 9$

$p(1) = 5 - 10$ or $p(1) = -5$

Hence, the remainder is -5.

E4. Find the value of a and b where $x + 2$ and $x - 1$ are the factors of $x^3 - ax^2 + bx + 1$.

Explanation:

$x + 2$ is a factor of $p(x) \Rightarrow p(-2) = 0$

$(-2)^3 - a(-2)^2 + b(-2) + 1 = 0$

$\Rightarrow -8 - 4a - 2b + 1 = 0$

$\Rightarrow -4a - 2b = 7$ ...(i)

$x - 1$ is a factor of $p(x)$. $\Rightarrow p(1) = 0$

$1 - a(1)^2 + b(1) + 1 = 0$

$\Rightarrow -a + b = -2$ ...(ii)

By solving (1) and (2)

$\Rightarrow a = -\frac{1}{2} \qquad \Rightarrow b = -\frac{5}{2}$

DIFFERENT TYPES OF EQUATIONS

LINEAR EQUATIONS

Linear equation is defined as the algebraic equation in which the highest power of a variable is one.

In other words, the degree of linear equation is one.

These equations are called linear because the graph of such equations on the $x - y$ cartesian plane is a straight line.

If a linear equation consists of one variable then it is known as a linear equation in one variable. On the other hand, if an equation consists of two variables with degree of one then it is known as a linear equation in two variables and so on.

General form of linear equations: $ax + by = c$

where a, b and c are constants and "x" and "y" are variables.

The values of "x" and "y" which satisfy this equation are called its solution.

For example:

Linear equation in one variable: $3x + 2 = 5$

Linear equation in two variables: $2x + 5y = 8$

Linear equation in three variables: $x + 3y + 6z = 15$

CONSISTENCY OF A SYSTEM OF LINEAR EQUATIONS IN TWO VARIABLES

If there are two numbers of variables then minimum two equations are needed to find out the value of the variable.

Let two linear equations be:

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

where x and y are variables and $a_1, b_1, c_1, a_2, b_2, c_2$ are constants.

Any pair of (x, y) which will satisfy above equations are known as solutions.

There are two types of system of equation

Consistent System:

A system of linear equations which is said to be consistent if it has at least one solution.

Inconsistent system:

A system of linear equations which has no solutions is known as an inconsistent system.

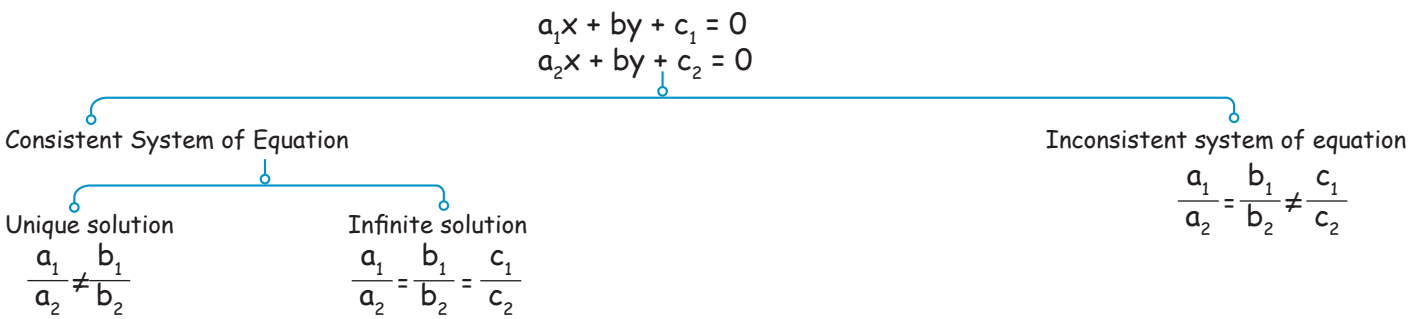


Fig. 4.3

METHODS OF SOLVING LINEAR EQUATION IN TWO VARIABLES

The methods to solve equations are given below:

1. Substitution
2. Elimination

Method 1: Method of Substitution

Following steps are followed:

Step 1: Solve one equation by and find one variable in terms of the other variable.

Step 2: Put the value of the variable in the second equation to obtain the equation in the single variable.

Step 3: Solve the equation in one variable and find out the value of the variable.

Step 4: Substitute it in any of the given equations to find the value of the second variable.

Example: Solve for values of x and y:

$5x - 3y = 12$ and $x + 3y = 6$

Let equations,

$5x - 3y = 12$ ...(i)

$x + 3y = 6$ ...(ii)

From equation (2), we get

$x = 6 - 3y$ ...(iii)

Putting the value of x in equation (i), we get

$5(6 - 3y) - 3y = 12$

$30 - 15y - 3y = 12$

$-18y = 12 - 30 = -18$

$y = \frac{(-18)}{(-18)} = 1$

putting y = 1 in equation (iii), we get

$x = 6 - 3(1) = 3$

Solution is $(x, y) = (3, 1)$

Hence, the value of x and y for a given system of equations is 3 and 1 respectively.

Method 2: Method of Elimination

Following steps are followed:

Step 1: Multiply or divide both the linear equations with a non-zero number so that the coefficient of any one of the variables is the same in both equations.

Step 2: Add or subtract both the equations such that the terms with same coefficient and variables will be eliminated.

Step 3: Solve the equation left in one variable in step II to find out the value of the variable left in the equation.

Step IV: Substitute the value of variable obtained in step III in any of the given equations to find the value of the other variable.

Example: Solve the following linear equations:

$3x - 5y = -30$ and $2x + 7y = 11$

Given equations are,

$3x - 5y = -30$ ...(i)

$2x + 7y = 11$ ...(ii)

Multiply equation (1) by 2 and equation (2) by 3, we get

$6x - 10y = -60$ ...(iii)

$6x + 21y = 33$ ...(iv)

Subtracting equation (iii) from (iv),

$31y = 93 \Rightarrow y = 3$

Putting the value of y = 3 in equation (1) and on solving we get $x = -5$

Hence, the solution of the given equations is (–5, 3).

Note:

Elimination method is faster and better if coefficients of none of the variables are the same whereas substitution method is best if any one of the equations can be expressed either in $x = ay + b$ or $y = ax + b$

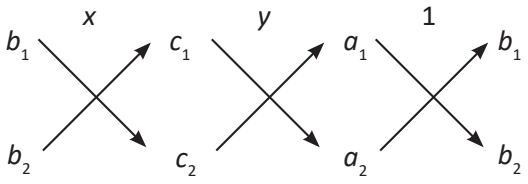
Shortcut Method to Solved 2 linear equations in 2 Variables:

Let two linear equations be:

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

where x and y are variables and $a_1, b_1, c_1, a_2, b_2, c_2$ are constants.



$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1}$$ where

$b_2a_1 - b_1a_2 \neq 0$, $a_1b_2 - a_2b_1 \neq 0$, $a_1b_2 - a_2b_1 \neq 0$

Note:

To find out the value of a variable in the linear equations, the number of variables should be equal to the number of given equations then only a solution can be obtained.

QUADRATIC EQUATIONS

Quadratic equation is defined as an algebraic equation in which the highest power of at least one of the variables is two. In other words, the degree of quadratic equation is two. It is also known as a second-order equation.

For Example: $7x^2 + 10x + 3 = 0$, $x^2 - 8x + 15 = 0$ etc.

General form: $ax^2 + bx + c = 0$ ...(u)

where a, b, c are constants and "x" is a variable.

For the given quadratic equation (i), the value of x which satisfies this equation is known as its root.

Since, it is a quadratic equation therefore it will have two roots.

METHOD TO FIND ROOTS OF QUADRATIC EQUATION

1. Quadratic Formula Method

Consider a quadratic equation, $ax^2 + bx + c = 0$, $a \neq 0$ ...(i)

Two roots by quadratic formula are:

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}, \beta = \frac{-b-\sqrt{b^2-4ac}}{2a} \quad \text{...(ii)}$$

Therefore from equation (2),

Sum of roots = $\alpha + \beta = -\frac{b}{a}$

Product of roots = $\alpha\beta = \frac{c}{a}$

Example: Solve for values of x: $-2x^2 + x + 13 = 0$.

$a = -2$, $b = 1$, $c = 13$

Roots of the equation is:

$$x = \frac{-b+\sqrt{b^2-4ac}}{2a}, \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Putting the values of a, b and c we get

$$x = \frac{-1+\sqrt{1^2-4(-2)(13)}}{2(-2)}, \frac{-1-\sqrt{1^2-4(-2)(13)}}{2(-2)}$$

$$x = \frac{-1+\sqrt{105}}{2(-2)}, \frac{-1-\sqrt{105}}{2(-2)}$$

Hence, the roots are $\frac{1-\sqrt{105}}{4}, \frac{1+\sqrt{105}}{4}$.

Discriminant(D):

Consider a quadratic equation, $ax^2 + bx + c = 0$

then $b^2 - 4ac$ is known as a discriminant of the quadratic equation.

$D = b^2 - 4ac$

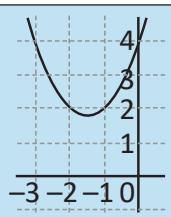
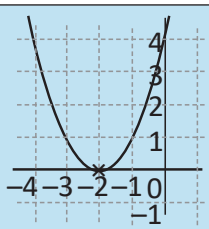
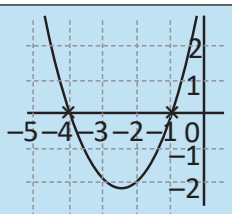
Nature of roots:

Nature of roots means whether the roots are real or non real, equal or unequal, zero or non-zero. It can be interpreted from the nature of discriminant.

Discriminant of a Quadratic Equation

Given the quadratic equation $y = ax^2 + bx + c$

The discriminant $D = b^2 - 4ac$ tells the types of roots the equation has.

Discriminant	D < 0	D = 0	D > 0
Types of Roots	No real roots; Two imaginary roots	One real root	Two distinct real roots
			

The above is true for a>0

Number of roots of the equation is equal to the intersection of the graph with the x-axis.

As it can be clearly seen from the graph that

(i) If $D > 0$, then roots are real and unequal.

(ii) If $D = 0$, then the roots are real and equal. And each root is equal to $\frac{-b}{2a}$

(iii) If $D < 0$ then the roots are non real or imaginary. Imaginary roots are always in conjugate pairs.

It means if one root is $a + ib$ then other root will be $a - ib$ where $b \neq 0$.

2. Factorisation Method

Consider a quadratic equation, $ax^2 + bx + c = 0$ ...(i)

Step 1: Choose two numbers such that their product is equal to ac and sum is equal to b.

Step 2: Let p and q be two numbers such that: $p + q = b$, $pq = ac$.

Step 3: Split the middle term and write it as $px + qx$.

Step 4: Write the equation by grouping the common factors.

For Example:

1. **Find the roots of the equation :** $5x^2 - 31x + 6 = 0$.

   Two numbers such that their sum is equal to –31 and product is equal to 30(i.e. 5×6) are –30 and –1.

   Splitting the middle term, we get

   $5x^2 - 30x - x + 6 = 0$,

   $5x(x - 6) - 1(x - 6) = 0$

   Taking out common factors

   $(5x - 1)(x - 6) = 0$

   $\Rightarrow (5x-1)=0, or\,(x-6)=0$

   $\Rightarrow x = \frac{1}{5}, x = 6$

   Therefore the values of x are $\frac{1}{5}, 6$.

SOME IMPORTANT IDENTITIES

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $a^2 - b^2 = (a + b)(a - b)$
4. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
5. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
8. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
9. $(a + b + c)^3 = a^3 + b^3 + c^3 + 3(a + b)(b + c)(c + a)$

QUESTIONS

Q1. A has some coins. He gives half of the coins and 2 more to B. B gives half of the coins and 2 more to C. C gives half of the coins and 2 more to D. The number of coins D has now, is the smallest two-digit number. How many coins does A have in the beginning? **(UPSC CSAT 2022)**

(a) 76 (b) 68

(c) 60 (d) 52

Ans: (d)

Explanation:

The smallest two-digit number = 10

According to the question,

The number of coins with D = 10

$$D = \left(\frac{C}{2}\right) + 2$$

$$\Rightarrow \frac{C}{2} = 10 - 2 = 8 \text{ or } C = 16$$

$$C = \left(\frac{B}{2}\right) + 2$$

$$\Rightarrow \frac{B}{2} = 16 - 2$$

$$\frac{B}{2} = 14 \text{ or } B = 28$$

$$B = \left(\frac{A}{2}\right) + 2$$

$$\frac{A}{2} = 28 - 2 = 26 \text{ or } A = 52$$

Therefore, A initially had 52 coins.

Q2. The sum of three consecutive integers is equal to their product. How many such possibilities are there? **(UPSC CSAT 2022)**

(a) Only one

(b) Only two

(c) Only three

(d) No such possibility is there

Ans: (c)

Explanation:

Let the three consecutive numbers be

$x - 1$, x, and $x + 1$.

According to the question,

$(x - 1) + x + (x + 1) = (x - 1) \times (x) \times (x + 1)$

$3x = x(x^2 - 1)$

$3x = x^3 - x$

$\Rightarrow x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x = 0, 2$ or -2

Therefore, there are only 3 such possibilities.

And the consecutive numbers would be

1. When $x = 0$ then, –1, 0, 1
2. When $x = 2$ then, 1, 2, 3
3. When $x = -2$ then, –3, –2, –1

Hence, there are three possibilities.

Q3. Let p be a two-digit number and q be the number consisting of the same digits written in reverse order. If p×q=2430, then what is the difference between p and q? **(UPSC CSAT 2022)**

(a) 45 (b) 27

(c) 18 (d) 9

Ans: (d)

Explanation:

$p \times q = 2430$

The last digit of the product is 0, which indicates that one of the two digits must be 5. The other possibility is one of the numbers ending with a 0, however 2-digit number multiplied with a single digit number shall not result in 2430 in any case (Highest possible case is 90×09 = 810

Let the remaining digit be X.

So, p = 'X5' = $10X + 5$

Reverse number, q = '5X' = $50 + X$

According to the question,

$(10X + 5) \times (50 + X) = 2430$

$\Rightarrow 500X + 10X^2 + 250 + 5X = 2430$

$\Rightarrow 10X^2 + 505X - 2180 = 0$

$\Rightarrow 2X^2 + 101X - 436 = 0$ or $2X^2 + 109X - 8X - 436 = 0$

$\Rightarrow X(2X + 109) - 4(2X + 109) = 0$

$\Rightarrow (X - 4)(2X + 109) - 0$

$\Rightarrow X = 4$, or $-\frac{109}{2}$ (neglected)

So, X = 4

Therefore, $p = 10 \times 4 + 5 = 45$

$q = 50+4 = 54$

Difference = 54 – 45 = 9

Q4. A bill for 1840 is paid in the denominations of 50, 20 and 10 notes. 50 notes in all are used. Consider the following statement. **(UPSC CSAT 2022)**

1. **25 notes of 50 are used and the remaining are in the denominations of ₹ 20 and ₹10.**
2. **35 notes of 20 are used and the remaining are in the denominations of ₹50 and ₹10.**
3. **20 notes of 10 are used and the remaining are in the denominations of ₹50 and ₹20.**

Which of the above statements are not correct?

(a) 1 and 2 (b) 2 and 3

(c) 1 and 3 (d) 1, 2 and 3

Ans: (d)

Explanation:

Given, total amount = 1840

Let the number of ₹50, ₹20, and ₹10 notes be a, b and c respectively.

Given, $a + b + c = 50$

Statement 1: $a = 25$

$b + c = 50 - 25 = 25$ ...(i)

$20b + 10c = 1840 - (25 \times 50)$

$\Rightarrow 20b + 10c = 590$ ...(ii)

Solving equations (1) and (2),

$b = 34$, $c = -9$ (not possible)

So, statement (1) is not correct.

Statement 2: b = 35

$a + c = 50 - 35 = 15$ ...(iii)

$50a + 10c = 1840 - (35 \times 20)$

$\Rightarrow 50a + 10c = 1140$ ...(iv)

Solving equations (iii) and (iv)

$a = 99/4$, $c = -\frac{39}{4}$ (not possible)

So, statement (2) is not correct.

Statement 3: $c = 20$

a + b = 50 – 20 = 30 ...(v)

$50a + 20b = 1840 - (10 \times 20)$

$\Rightarrow 50a + 20b = 1640$ ...(vi)

Solving equations (5) and (6),

$a = \frac{104}{3}$, $b = -\frac{14}{3}$ (not possible)

So, statement (3) is not correct.

Hence, statements (1), (2), (3) are not correct.

Q5. Jay and Vijay spent an equal amount of money to buy some pens and special pencils of the same quality from the same store. If Jay bought 3 pens and 5 pencils and Vijay bought 2 pens and 7 pencils, then which one of the following is correct? **(UPSC CSAT 2021)**

(a) A pencil costs more than a pen.

(b) The price of a pencil is equal to that of a pen.

(c) The price of a pen is two times the price of a pencil.

(d) The price of a pen is three times the price of a pencil.

Ans: (c)

Explanation:

Let the price of a pen and pencil be x and y respectively.

Given, $3x + 5y = 2x + 7y$

$3x - 2x = 7y - 5y$

$x = 2y$

Q6. A person P asks one of his three friends X as to how much money he had. X replied, "If Y gives me ₹40, then Y will have half of as much as Z, but if Z gives me ₹40, then three of us will have equal amount". What is the total amount of money that X, Y and Z have? **(UPSC CSAT 2021)**

(a) 420 (b) 360

(c) 300 (d) 270

Ans: (b)

Explanation:

Let the amount of money with *X*, *Y* and Z be ₹*x*, ₹*y* and ₹*z* respectively.

Given, $y - 40 = \frac{z}{2}$

$\Rightarrow z = 2y - 80$ ...(i)

$x + 40 = y = z - 40$

$z = y + 40$ ...(ii)

From equations (i) and (ii), we get

$2y - 80 = y + 40$

$\Rightarrow y = 120$

$x = y - 40 = 120 - 40 = 80$

$z = y + 40 = 120 + 40 = 160$

Therefore,

Total amount of money = 120 + 80 + 160

Total amount of money = ₹360

Q7. A vessel full of water weighs 40 kg. If it is one-third filled, its weight becomes 20 kg. What is the weight of the empty vessel? **(UPSC CSAT 2020)**

(a) 10 kg (b) 15 kg

(c) 20 kg (d) 25 kg

Ans: (a)

Explanation:

Let the weight of an empty vessel = *x* kg

Weight of water when it fills the vessel completely = *y* kg

According to the question,

$x + y = 40$

$\Rightarrow x + \left(\frac{y}{3}\right) = 20$

On solving above equations, we get

$y - \left(\frac{y}{3}\right) = 20$

$\Rightarrow \left(\frac{2}{3}\right) y = 20$

$y = 30$

Hence, the weight of empty vessel (x)

$= 40 - y = 40 - 30 = 10$ kg

Q8. How many pairs of natural numbers are there such that the difference of whose squares is 63? **(UPSC CSAT 2020)**

(a) 3 (b) 4

(c) 5 (d) 2

Ans: (a)

Explanation:

Let the numbers be *x* and *y*.

The difference of the squares of the number is 63.

So, $x^2 - y^2 = 63$

$\Rightarrow (x + y)(x - y) = 63$

Thus, the factors of 63 possibles are 1, 3, 7, 9, 21 and 63

Hence, pairs are (7, 9) (21, 3) and (63, 1)

(i) $x + y = 9$ and $x - y = 7$

On solving above equations,

$x = 8, y = 1$

$\Rightarrow x^2 - y^2 = 8^2 - 1^2 = 63$

(ii) $x + y = 21$ and $x - y = 3$

On solving above equations,

$x = 12, y = 9$

$\Rightarrow x^2 - y^2 = 12^2 - 9^2 = 63$

(iii) $x + y = 63$ and $x - y = 1$

On solving above equations,

$x = 32, y = 31$

$\Rightarrow x^2 - y^2 = 32^2 - 31^2 = 63$

Q9. The ratio of a two-digit natural number to a number formed by reversing its digits is 4: 7. The number of such pairs is **(UPSC CSAT 2019)**

(a) 5 (b) 4

(c) 3 (d) 2

Ans: (b)

Explanation:

Let the digit at unit's place be *x*, and the digit at tenth's place be *y*.

So, original number = $10y + x$.

New number on reversing the digits = $10x + y$.

According to the question,

$\frac{(10y + x)}{(10x + y)} = \frac{4}{7}$

$\Rightarrow 70y + 7x = 40x + 4y$

$\Rightarrow 66y = 33x$

$x = 2y$

When, $x = 2, y = 1$

So, the numbers are 12 and 21.

When, $x = 4, y = 2$

So, the numbers are 24 and 42.

When, $x = 6, y = 3$

So, the numbers are 36 and 63.

When, $x = 8, y = 4$

So, the numbers are 48 and 84.

Now, if we make $y = 5$, then x becomes 10 and the number becomes a three-digit number. So, only 4 pairs of numbers are possible.

Q10. Sunita cuts a sheet of paper into three pieces. Length of the first piece is equal to the average of the three single digit odd prime numbers. Length of the second piece is equal to that of the first piece and one-third the length of the third piece. The third piece is as long as the other two pieces together. The length of the original sheet of paper is **(UPSC CSAT 2019)**

(a) 13 units (b) 15 units

(c) 16 units (d) 30 units

Ans: (d)

Explanation:

Let the length of the first, second and third sheet of paper be x, y and z units, respectively.

Now, we know that the three single digit odd prime numbers are 3, 5 and 7.

$$x = \frac{(3+5+7)}{3=5}$$

According to the question,

$$\text{and } y = 5 + \frac{z}{3} \quad ...(i)$$

$z = x + y = 5 + y$ ...(ii)

Now solving equations (i) and (ii), we get

z = 15 units, y = 10 units

So, total length of paper = $x + y + z$

= 5 + 10 + 15 = 30 units

Q11. There are certain 2-digit numbers. The difference between the number and the one obtained on reversing it is always 27. How many such maximum 2-digit numbers are there? **(UPSC CSAT 2017)**

(a) 3 (b) 4

(c) 5 (d) None of these

Ans: (d)

Explanation:

Let the numbers be of the form $10x + y$, and its reverse is $10y + x$.

Then,

$(10x + y) - (10y + x) = 9x - 9y = 9(x - y)$

Now, difference is 27, so

$9(x - y) = 27$

$x - y = 3$

So, required numbers are 41, 52, 63, 74, 85 and 96.

So, a total of 6 such 2-digit numbers would be there.

Q12. A person has only Rs 1 and Rs 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is Rs 75, then the number of Rs 1 and Rs 2 coins are, respectively **(UPSC CSAT 2011)**

(a) 15 and 35 (b) 35 and 15

(c) 30 and 20 (d) 25 and 25

Ans: (d)

Explanation:

Total number of coins = 50.

Let number of Rs 1 coins = x

and number of Rs 2 coins = y

$x + y = 50$ ...(i)

$x + 2y = 75$ ...(ii)

On solving equation (i) and (ii), we get

x = 25 and y = 25

Hence, there are 25 coins each of Rs 1 and Rs 2 denomination.

Q13. A man takes half the time in rowing a certain distance downstream than upstream. What is the ratio of the speed in still water to the speed of current?

(a) 1 : 2 (b) 2 : 1

(c) 1 : 3 (d) 3 : 1

Ans: (d)

Explanation:

Let the speed of man in still water be x and the speed of current be y.

Let the total distance be D.

So, Downstream speed = $x + y$

And Upstream speed = $x - y$

According to the question,

Time taken in rowing upstream = 2 × Time taken in rowing downstream

$$\Rightarrow \frac{D}{(x-y)} = 2 \times \left[\frac{D}{(x+y)}\right]$$

$\Rightarrow x + y = 2x - 2y$

$\Rightarrow x = 3y$

$\Rightarrow x : y = 3 : 1$

PRACTICE QUESTIONS

1. If $(x-3)$ is a factor of $x^2+3qx-6q=0$ then value of q is

(a) 3 (b) 6

(c) -3 (d) 9

2. Deepesh has only ₹5 and ₹10 coins with him. If the total number of coins is 40, and the amount of money with him is ₹350, then the number of coins of ₹5 and ₹10 respectively, are:

(a) 10 and 30 (b) 30 and 10

(c) 5 and 10 (d) 20 and 10

3. Vidyut gets 4 marks for each correct answer and loses 2 marks for each wrong answer in the mathematics exam. He attempted 60 questions and obtained a total of 120 marks. Find the number of questions he answered correctly.

(a) 30 (b) 10

(c) 40 (d) 25

4. If $\left(3x-\frac{1}{6x}\right)=8$ then what is the value of $\left(27x^3-\frac{1}{216x^3}\right)$?

(a) 512 (b) 524

(c) 216 (d) 144

5. Find the value of 'a' if the division of ax^3+8x^2-6x+1 by $(x+3)$ leaves a remainder 10.

(a) 3 (b) -3

(c) 2 (d) 4

6. The pair of linear equations $9x+4y=6$ & $7x+8y=9$ will have

(a) unique solution (b) Infinite Solution

(c) No Solution (d) Three solution

7. Find the factors of $x^{2/3}-x^{1/3}-42=0$

(a) $(x^{2/3}-7)(x^{2/3}+6)$

(b) $(x-7)(x+6)$

(c) $(x^{1/3}-7)(x^{1/3}+6)$

(d) $(x^{1/3}-14)(x^{1/3}+3)$

8. Determine the value of k for which the equation $2x^2-2kx+32=0$ have equal roots?

(a) 5 (b) ±8

(c) ±4 (d) 2

9. $15x^2+x-6=0$ & $2y^2+15y+25=0$

(a) $x<y$

(b) $x>y$

(c) $x=y$

(d) Relationship cannot be established

10. Let $x, y \in N$ and $4x+15y=270$. The number of solutions is:

(a) 15 (b) 4

(c) 6 (d) infinite

ANSWERS

1. (c) 2. (a) 3. (c) 4. (b) 5. (a) 6. (a) 7. (c) 8. (b) 9. (b) 10. (b)

EXPLANATIONS

1. (c)

Explanation:

Given the equation is $x^2+3qx-6q=0$

$x-3$ is a factor, so put $x=3$ in above equation

$3^2+3q(3)-6q=0$

$9+9q-6q=0$

$3q=-9$

$q=-3$

2. (a)

Explanation:

Let ₹5 coins be x and ₹10 coins be y.,

$x+y=40$ (given) ...(i)

and $5x+10y=350$ (given) ...(ii)

$5y+5(x+y)=350$

$5y+5(40)=350$

$y=30$

$x=10$

Therefore, the number of ₹5 and ₹10 coins respectively are 10 and 30

3. (c)

Explanation:

Let the number of correct answers be 'x' and the number of wrong answers be 'y'

Then, $x+y=60$ (given) ...(i)

Also, he gets 4 marks for each correct answer and loses 2 marks for each wrong answer. So,her total marks can be written as:

$4x - 2y = 120$ ...(ii)

$4(60 - y) - 2y = 120$

$240 - 4y - 2y = 120$

$6y = 120$

$y = 20$

$x = 40$

Therefore, the number of questions he answered correctly is 40.

4. (b)

Explanation:

Given $\left(3x - \frac{1}{6x}\right) = 8$

Formula used: $x^3 - y^3 = (x - y)^3 + 3xy(x - y)$

According to the question

$$\left(27x^3 - \frac{1}{216x^3}\right) = (3x)^3 - \left(\frac{1}{6x}\right)^3$$

Using above formula

$$(3x)^3 - \left(\frac{1}{6x}\right)^3 = \left(3x - \frac{1}{6x}\right)^3 + 3\left(\frac{3x}{6x}\right)\left(3x - \frac{1}{6x}\right)$$

$$\Rightarrow 8^3 + 3\left(\frac{1}{2}\right)(8) = 512 + 12 = 524$$

5. (a)

Explanation:

Given equation $ax^3 + 8x^2 - 6x + 1$

If a polynomial $f(x)$ is divided by $(x - c)$, then the remainder is $f(c)$.

Put $x = -3$ in the above equation

$a(-3)^3 + 8(-3)^2 - 6(-3) + 1 = 10$

$a(-27) + 8(9) + 18 + 1 = 10$

$-27a + 72 + 18 + 1 = 10$

$-27a + 91 = 10$

$-27a = -81$

$a = 3$

6. (a)

Explanation:

Compare with

$ax + by + c = 0$ &

$Px + Qy + R = 0$

$a = 9, b = 4, c = 6$

$P = 7, Q = 8, R = 9$

On comparing

$$\frac{a}{P} \neq \frac{b}{Q}$$

So, the given linear equations have a unique solution.

Also, on solving the equations we get unique values of (x) & (y).

7. (c)

Explanation:

Given $x^{2/3} - x^{1/3} - 42 = 0$

Let's consider $x^{1/3} = p$

Now the equation is $p^2 - p - 42 = 0$

$\Rightarrow p^2 - 7p + 6p - 42 = 0$

$\Rightarrow p(p - 7) + 6(p - 7) = 0$

$\Rightarrow (p - 7)(p + 6) = 0$

Therefore, the factors of $x^{2/3} - x^{1/3} - 42 = 0$ are $(x^{1/3} - 7)(x^{1/3} + 6)$

8. (b)

Explanation:

Given equation $2x^2 - 2kx + 32 = 0$

Now Roots are equal, So $D = 0$

$D = b^2 - 4ac$ (identity)

$(-2k)^2 - 4(2)(32) = 0$

$4k^2 - 256 = 0$

$k^2 = 64$

$k = \pm 8$

Therefore, the value of k for given condition is ±8.

9. (b)

Explanation:

$15x^2 + x - 6 = 0$

$15x^2 + 10x - 9x - 6 = 0$

$5x(3x + 2) - 3(3x + 2) = 0$

$(3x + 2)(5x - 3) = 0$

$x = \frac{-2}{3}, \frac{3}{5}$

$2y^2 + 15y + 25 = 0$

$(2y + 5)(y + 5) = 0$

$y = \frac{-5}{2}, -5$

So, $x > y$

10. (b)

Explanation:

Given that $4x + 15y = 270$

$4x = 270 - 15y$

$x = \frac{270 - 15y}{4}$

$x = \frac{15(18 - y)}{4}$

For 'x' to be a natural number, $18 - y$ has to be divisible by 4, since 15 is not divisible by 4

Now, at $y = 2, 6, 10, 14$

We get $x = 60, 45, 30, 15$

Therefore we have 4 solutions of x, y

Surds and Indices

5

INTRODUCTION

The chapter on surds and indices introduces two important concepts in mathematics. Surds are numbers under a square root that cannot be simplified into a plain number, like √2 or √3. These are exact values and cannot be expressed as simple fractions. The chapter also covers indices, which are about how many times a number is multiplied by itself, known as powers or exponents. This chapter aims to help you understand how to work with these types of numbers, simplifying and solving mathematical problems effectively.

ORDERS OF MAGNITUDE

The factor/quantity which expresses whether a physical quantity is large or small is known as order of magnitude. It is expressed as 10 to the nth power.

Where 'n' represents the order of magnitude.

To determine the order of magnitude of a physical quantity, the quantity is expressed as $x \times 10^{y}$;

where $1 \le x < 10$ and y is a positive or a negative integer.

Example: Calculate the order of magnitude of 4720000.

It can be written as 4.720000×10^{6}.

Hence, $4.720000 \times 10^{6} = 4.72 \times 10^{6}$ and order of magnitude = 6

Powers of Ten			
• Billionths	= 0.000 000 001	$= 10^{9}$	Nano
• Millionths	= 0.000 001	$= 10^{-6}$	Micro
• Thousandths	= 0.001	$= 10^{-3}$	Mili
• Hundredths	= 0.01	$= 10^{-2}$	
• Tenths	= 0.1	$= 10^{-1}$	
• Ones	= 1	$= 10^{0}$	
• Tens	= 10	$= 10^{1}$	
• Hundreds	= 100	$= 10^{2}$	
• Thousands	= 1000	$= 10^{3}$	Kilo
• Millions	= 1,000,000	$= 10^{6}$	Mega
• Billions	= 1,000,000,000	$= 10^{9}$	Giga

SQUARE AND SQUARE ROOT

SQUARE

When a number is multiplied by itself then the product is called the square of that number.

Example: $2 \times 2 = 4$

Methods to Find Square

1. If the number is large then use either of the following technique:
   (i) Express the number as a sum of two numbers and then apply the formula:
   $(a + b)^2 = a^2 + b^2 + 2ab$
   (ii) Express the number as a difference of two numbers and then apply the formula:
   $(a - b)^2 = a^2 + b^2 - 2ab$

Perfect Square

A natural number is called a perfect square if it is the square of any natural number.

Example: 1,4,9,.... are the squares of 1, 2, 3,... respectively.

Square of first 20 natural numbers	
Number	Square
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225
16	256
17	289
18	324
19	361
20	400

How to Find the Number is a Perfect Square

Step 1: Express the number as a product of prime factors.

Step 2: Make pairs of equal factors.

Step 3: If it can be expressed as the product of pairs of equal factors then the number is a perfect square.

Example: Is 90 a perfect square or not ?

Prime factorisation of $90 = 2 \times 3^2 \times 5$

Since 2 and 5 are not in pairs therefore 90 is not a perfect square.

Properties of a Perfect Square Number

- Square of even and odd numbers is even and odd respectively.
- The number of zero at the end of a perfect square ending with zero is always even.
- Perfect square numbers end with either 0, 1, 4, 5, 6 or 9.
- No number can be a perfect square unless its digital sum is 1, 4, 7, or 9.

EXAMPLES

E1. Which is the smallest number by which 800 must be multiplied so that the product is a perfect square?

Explanation:

Given the number is 800, first we resolve it into prime factors.

2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

$800 = \underline{2 \times 2} \times \underline{2 \times 2} \times 2 \times \underline{5 \times 5}$

Clearly 2 has no pair.

Therefore, if we multiply it by 2 then the product will be a perfect square.

Required smallest number is 2.

Shortcut to Find Square of a 2 Digit Number

Case I: When the unit is ≤ 3

Example: find the square of 43

$(43)^2$

$4^2 = 16$ $\quad$ $3^2 = 09$

$4 \times 3 \times 2 = 24$

	16	09
$\therefore$	+ 24	
	1849	

Hence, $(43)^2 = 1849$

Case II: When the unit is ≥ 4

Find the square of 57

$(57)^2$

$5^2 = 25$ $\quad$ $7^2 = 49$

$5 \times 7 \times 2 = 70$

	25	49
+	70	
	3249	

Hence, $(57)^2 = 3249$

SQUARE ROOTS

The square root of a number is a value that, when multiplied by itself, gives the original number. In other words, if you have a number x, the square root of x is a number y such that $y \times y = x$

The symbol used to indicate the square root of a number is "$\sqrt{\ }$".

The square root symbol is called a radical, whereas the number under the square root symbol is called the radicand.

i.e., $\sqrt{625} = 25, \sqrt{484} = 22$ etc.

Note:

We can calculate the square root of positive numbers only.

For Example: $\sqrt{144} = +12$

For any two positive integers x and y,

- $\sqrt{xy} = \sqrt{x} \times \sqrt{y}$
- $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$

HOW TO FIND SQUARE ROOT?

The square root of a perfect square can be calculated easily.

Methods

- Repeated Subtraction Method
- Prime Factorization Method
- Long Division Method

It should be noted that the first two methods can be conveniently used for perfect squares, while the third method, i.e., the long division method can be used for any number whether it is a perfect square or not.

1. Repeated Subtraction Method:

Step 1: Subtract the consecutive odd numbers (Starting with 1) from the number of which square root has to be calculated, till we reach 0.

Step 2: Count the number of times we subtract. The number of times subtraction is done is the square root of the given number.

Let us find the square root of 64 using this method.

- 64 – 1 = 63
- 63 – 3 = 60
- 60 – 5 = 55
- 55 – 7 = 48
- 48 – 9 = 39
- 39 – 11 = 28
- 28 – 13 = 15
- 15 – 15 = 0

Since subtraction is done 8 times to get zero as result.

Therefore, $\sqrt{64} = 8$

2. Prime Factorization Method

Following steps are followed:

Step 1: write the given number into product of its prime factors.

Step 2: Form pairs of factors

Step 3: Take one number from each of the pair.

Step 4: Multiply the numbers and the product is the square root of the given number.

Examples: Find the square root of 1089 by prime factorization method.

3	1089
3	363
11	121
11	11
	1

$1089 = 3 \times 3 \times 11 \times 11 = 3^2 \times 11^2 = (33)^2$

$\sqrt{1089} = \sqrt{33^2} = 33$

3. Long Division Method

This method can be used to find the square root of large numbers where the factors can not be determined easily.

Following steps are followed:

Step 1: Place a bar over every pair of digits of the number starting from right hand side (the units' place) of the number.

Step 2: Start division by dividing the left-most pair by the largest positive number whose square is less than or equal to the number of the left most pair. This number will be divisor as well as quotient and find the remainder.

Step 3: Bring down the next pair to the right of the remainder.

Step 4: For next divisor, multiply quotient by 2.

Step 5: Choose a digit and write it to the extreme right of the number (obtained in step 4). This number is a new divisor for the new dividend which is obtained by carrying down the next pair.

The digit can be found via trial and error method) and the new divisor would be such that its product with the digit gives a result which is less than or equal to the new dividend.

Step 6: Find the remainder and Continue the above steps till all the pairs have been taken up. The required square root of the given number is the quotient obtained at the end of the division.

Example: Find the square root of 527076 by long division method.

	726
7	52 70 76
	49
142	370
	284
1446	8676
	8676
	×

$\therefore \sqrt{527076} = 726$

Note:

Square root of a fraction is calculated by calculating the square root of numerator and denominator separately.

Square Root Table

Number (x)	Square Root ($\sqrt{x}$)
1	1
2	1.414
3	1.732
4	2
5	2.236
6	2.449

CUBE AND CUBE ROOT

Cube

If a number is multiplied by itself thrice then the result is called the Cube of that Number.

Example: 729 = 9 × 9 × 9 i.e., 729 is a cube of 9.

Note: Cube of a number can also be calculated by first expressing the number as either the sum or difference of two numbers and then applying either of the formula as given below:

$(a + b)^3 = a^3 + 3a^2 b + 3ab^2 + b^3$

$(a - b)^3 = a^3 - 3a^2 b + 3ab^2 - b^3$

Perfect Cube Number

If there is an integer m such that $n = m \times m \times m = m^3$ where n is a natural number then "n" is a perfect cube.

Example: 216 is a perfect cube as there is an integer 6 such that $216 = 6 \times 6 \times 6 = 6^3$

Example: What is the smallest number by which 7744 must be multiplied so that the product is a perfect cube.

Resolving 7744 into prime factors, we get

2	7744
2	3872
2	1936
2	968
2	484
2	242
11	121
11	11
	1

$7744 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times 11 \times 11$

2 as a prime factor of 7744 occurs six times but 11 as a prime factor occurs only twice.

Thus, if we multiply 7744 by 11 then 11 will also occur as a prime factor thrice and the product will be

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 11 \times 11 \times 11$, which is a perfect cube.

Cube Root

If $y = x^3$ for some integer x, then y is a perfect cube. Number "x" is called the cube root of "y".

If x is a cube root of y, we write $x = \sqrt[3]{y}$

Example: 4 is a cube root of 64, we write $\sqrt[3]{64} = 4$

Cube Root of Negative Number

for any positive integer 'x'

$(-x)^3 = -x^3$, $\sqrt[3]{(-x)^3} = -x$

Therefore, $\sqrt[3]{-x} = -\sqrt[3]{x}$

Example: Find the cube root of -5832.

$5832 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 2^3 \times (3^2)^3$

$\sqrt[3]{5832} = 2 \times 3^2 = 18$

Therefore, $\sqrt[3]{-5832} = -18$

Note:

For any two real values of x and y,

- $\sqrt[3]{xy} = \sqrt[3]{x} \times \sqrt[3]{y}$
- $\sqrt[3]{\frac{x}{y}} = \frac{\sqrt[3]{x}}{\sqrt[3]{y}}$

METHODS TO FIND CUBE ROOT

Prime Factorization Method

Step 1: write the given number into product of its prime factors.

Step 2: Form triplets of equal factors

Step 3: Take one number from each of the triplet.

Step 4: Multiply the numbers.

Step 5: The product is the cube root of the given number.

EXAMPLES

E2. Find the cube root of 21952 by prime factorisation method.

Explanation:

2	21952
2	10976
2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$21952 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7$

$= (2^2)^3 \times 7^3 = (4)^3 \times 7^3 = (28)^3$

$\sqrt[3]{21952} = \sqrt[3]{28^3} = 28$

Shortcut Method For Finding The Cube Roots of Exact Cubes Having Up To 6 Digits

Step 1: Form the groups of three consecutive digits, starting from the extreme right. The last group may consist of less than three digits.

Step 2: The largest number whose cube is less than or equal to the number under the leftmost bar is the leftmost digit of cube root.

Right most digit depends upon the last digit of the given number and can be found by using the table given below:

Unit digit of cube	Unit digit of cube root
1	1
2	8
3	7
4	4
5	5
6	6
7	3
8	2
9	9
0	0

Example: Find the cube root of 166375

Given number is 166 375,

166 lies between 125(5^3) and 216 (6^3).

Therefore, Left side digit = 5

375 ends in digit 5, Right side digit = 5

It implies the number is 55.

Hence, the cube root of 166375 is 55.

Cube and Cube Root of First 20 Natural Numbers

Number (x)	1	2	3	4	5	6	7	8	9	10	11	12	13
Cube (x^3)	1	8	27	64	125	216	343	512	729	1000	1331	1728	2197

SURDS

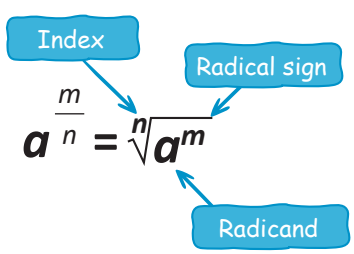


Fig. 5.1

Where a = any positive real number, m = any integer and n = natural numbers.

If the root of a number cannot be exactly obtained, the root is called a surd or an irrational number.

Thus, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$ ..... are all surds or irrational numbers

$\sqrt[3]{2}, \sqrt[3]{3}, \sqrt[3]{5}, \sqrt[3]{7}$..... are all surds or irrational numbers.

$\sqrt[n]{2}, \sqrt[n]{3}, \sqrt[n]{5}, \sqrt[n]{7}$..... are all surds or irrational numbers.

Note:
Every surd is an irrational number but every irrational number is not a surd.

Conjugate Surd

Two surds which differ only in sign (+ or –) between the terms connecting them, are called conjugate surds.

$a + \sqrt{b}$ and $a - \sqrt{b}$ are conjugate surds.

Also $\sqrt{a} + \sqrt{b}$ and $\sqrt{a} - \sqrt{b}$ are conjugate surds.

Laws of Surds

(a) $\sqrt[n]{a} = a^{\frac{1}{n}}$

(b) $\sqrt[m]{ab} = \sqrt[m]{a} \times \sqrt[m]{b} = a^{\frac{1}{m}} \times b^{\frac{1}{m}} = (ab)^{\frac{1}{m}}$

(c) $\sqrt[m]{\frac{a}{b}} = \frac{\sqrt[m]{a}}{\sqrt[m]{b}} = \left(\frac{a}{b}\right)^{\frac{1}{m}}$

(d) $(\sqrt[n]{a})^m = a^{\frac{m}{n}} = \sqrt[n]{a^m}$

(e) $(\sqrt[m]{a})^m = a^{\frac{m}{m}} = a$

(f) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[m]{a^{\frac{1}{n}}} = a^{\frac{1}{mn}}$

Mathematical Operations of Surds

1. **Addition and subtraction of surds:** Addition & subtraction of surds are possible only when order and radicand are same.

   Example: Find the sum of $4\sqrt{12}$ and $3\sqrt{3}$.

   $4\sqrt{12} + 3\sqrt{3} = 4 \times 2\sqrt{3} + 3\sqrt{3} = 11\sqrt{3}$

2. **Multiplication of Surds**:- Surds of the same order can be multiplied according to the following law

   $\sqrt[n]{x}\sqrt[n]{y} = \sqrt[n]{xy}$

   Example:

   Find the product of $4\sqrt{6} \times 3\sqrt{24}$.

   $12\sqrt{6 \times 24} = 12\sqrt{(3 \times 2) \times (2 \times 2 \times 2 \times 3)}$

   $= 12 \times 12 = 144$

3. **Division of Surds:** Surds of the same order can be divided according to the following law: $\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}$

As, $\sqrt[n]{x} = (x)^{\frac{1}{n}}$

Therefore, $\sqrt[n]{\frac{x}{y}} = \left(\frac{x}{y}\right)^{\frac{1}{n}} = \frac{x^{\frac{1}{n}}}{y^{\frac{1}{n}}}$

Example: Find the value of $\frac{5\sqrt[3]{2916}}{9\sqrt[3]{192}}$.

$$\frac{5\sqrt[3]{2916}}{9\sqrt[3]{192}} = \frac{5\sqrt[3]{9\times9\times9\times4}}{9\sqrt[3]{4\times4\times4\times3}}$$

$$= \frac{5\times9\sqrt[3]{4}}{9\times4\sqrt[3]{3}} = \frac{5}{4}\sqrt[3]{\frac{4}{3}}$$

RATIONALIZING FACTOR

Rationalizing factor is a surd which when multiplied with another given surd results in a rational number.

If the given surd is in the form of $\sqrt{a} \pm \sqrt{b}$ then the rationalizing factor is of the form $\sqrt{a} \mp \sqrt{b}$ respectively.

Example: Rationalize $\frac{2+5\sqrt{3}}{2-5\sqrt{3}}$.

Multiplying numerator and denominator by $2+5\sqrt{3}$, we get

$$\frac{2+5\sqrt{3}}{2-5\sqrt{3}} \times \frac{2+5\sqrt{3}}{2+5\sqrt{3}} = \frac{2^2+(5\sqrt{3})^2+2\times2\times5\sqrt{3}}{2^2-(5\sqrt{3})^2}$$

$$= \frac{4+75+20\sqrt{3}}{4-75} = -\frac{79+20\sqrt{3}}{71}$$

INDICES

Positive integral Power

Let 'a' be a real number and 'n' be a positive integer. Then we define a^n as

$a^n = a \times a \times a \times a \times .... \times a$ (n times)

Where a^n is called the n^{th} power of a.

The real number a is called the base and n is called the exponent(indices).

Example:

(i) $3^4 = 3 \times 3 \times 3 \times 3 = 81$

(ii) $\left(\frac{-2}{3}\right)^3 = \frac{(-2)\times(-2)\times(-2)}{3\times3\times3} = \frac{-8}{27}$

Negative integral Power: Let a be any non-zero real number and n be a positive integer.

Then, $a^{-n} = \frac{1}{a^n}$

Example: $5^{-4} = \frac{1}{5^4} = \frac{1}{5\times5\times5\times5} = \frac{1}{625}$

Note:

n^{th} *root* of a negative real number is not defined for any even 'n'

Example: $(-13)^{1/2}$ is not defined.

Let $x = (-13)^{1/2}$

$x^2 = -13$, which is not possible as x^2 should always be positive for a real number x.

LAW OF INDICES

Let a, b, c be positive real numbers, and let m, n and p be rational numbers.

Law of Indices

(a) $a \times a \times a \times a \times a ... n$ times $= a^n$

(b) $a^m \times a^n \times a^p = a^{m+n+p}$, $(a \neq 0)$

(c) $(a^m)^n = a^{m\times n} = a^{n\times m} = (a^n)^m$

(d) $\frac{a^m}{a^n} = a^{m-n}\ (m > n) = \frac{1}{a^{n-m}}\ (m < n),\ (a \neq 0)$

(e) $(abc)^n = a^n \times b^n \times c^n$

(f) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\ (b \neq 0)$

(g) $\left(\frac{a}{b}\right)^m = \left(\frac{b}{a}\right)^{-m}$

Examples:

1. $(a^m)\,(a^n) = a^{m+n}$

   Examples: $9^5 \times 81^3 \times 243^2 = (3^2)^5 \times (3^4)^3 \times (3^5)^2$

   $= (3)^{10} \times (3)^{12} \times (3)^{10}$

   $= (3)^{10+12+10} = (3)^{32}$

2. $\frac{a^m}{a^n} = a^{(m-n)}$

   Example: $\frac{49^5}{7^5} = \frac{7^{10}}{7^5}$

   $= 7^{10-5} = 7^5$

3. $(a^m)^n = a^{mn}$

   Example: The value of $(3^2)^3$ is equal to

   $(3^2)^3 = (3)^6 = 729$

4. $(ab)^m = a^m b^m$

Example: The value of $(5 \times 3)^3$ is equal to

$(5 \times 3)^3 = (5)^3 \times (3)^3$

$= 125 \times 27 = 3375$

5. $a^0 = 1$

Any number or polynomial raise to power zero is equal to one.

Note:

- $(a)^m + (a)^n \neq (a)^{m+n}$
- $(a)^m \times (b)^n \neq (ab)^{m+n}$

QUESTIONS

Q1. Which of the following is pure form of surd $\frac{4}{5}\sqrt[3]{\frac{625}{256}}$.

(a) $\sqrt[3]{\frac{5}{4}}$ (b) $\sqrt[3]{\frac{3}{4}}$

(c) $\sqrt[3]{\frac{4}{5}}$ (d) $\sqrt[3]{\frac{5}{3}}$

Ans: (a)

Explanation:

$$\frac{4}{5}\sqrt[3]{\frac{625}{256}} = \left(\frac{4}{5}\right)\left(\frac{625}{256}\right)^{\frac{1}{3}}$$

$$= \left(\frac{5}{4}\right)^{-1}\left(\frac{625}{256}\right)^{\frac{1}{3}} = \left(\frac{5}{4}\right)^{-1}\left(\frac{5}{4}\right)^{\frac{4}{3}}$$

$$= \left(\frac{5}{4}\right)^{\frac{1}{3}}$$

Q2. What is the value of x if $16^x = \frac{64}{4^{2x}}$.

(a) $\frac{4}{3}$ (b) $\frac{5}{4}$

(c) $\frac{3}{4}$ (d) $\frac{3}{5}$

Ans: (c)

Explanation:

$$16^x = \frac{64}{4^{2x}}$$

$$= (4)^{2x} = \frac{4^3}{4^{2x}} \Rightarrow (4)^{2x} = (4)^{3-2x}$$

Comparing, we get

$2x = 3 - 2x$ or $4x = 3$ or $x = \frac{3}{4}$

Q3. Find the value of $\sqrt[3]{\frac{(0.0025)^3 + (0.343)^3 + (0.0927)^3}{(0.025)^3 + (3.43)^3 + (0.927)^3}}$.

(a) $\frac{7}{10}$ (b) $\frac{1}{10}$

(c) $\frac{9}{10}$ (d) $\frac{3}{10}$

Ans: (b)

Explanation:

$$\sqrt[3]{\frac{(0.0025)^3 + (0.343)^3 + (0.0927)^3}{(0.025)^3 + (3.43)^3 + (0.927)^3}}$$

$$= \sqrt[3]{\frac{\left(\frac{0.025}{10}\right)^3 + \left(\frac{3.43}{10}\right)^3 + \left(\frac{0.927}{10}\right)^3}{(0.025)^3 + (3.43)^3 + (0.927)^3}}$$

$$= \sqrt[3]{\frac{1}{1000} \times \frac{(0.025)^3 + (3.43)^3 + (0.927)^3}{(0.025)^3 + (3.43)^3 + (0.927)^3}}$$

$$= \sqrt[3]{\frac{1}{1000}} = \sqrt[3]{\left(\frac{1}{10}\right)^{\frac{1}{3}}} = \frac{1}{10}$$

Q4. Calculate the value of z (x, y, z is not equal to zero) if $5^x = 7^y = 35^z$.

(a) $\frac{1}{(x+y)}$ (b) $\frac{y}{(x+y)}$

(c) $\frac{x}{(x+y)}$ (d) $\frac{xy}{(x+y)}$

Ans: (d)

Explanation:

$5^x = 7^y = 35^z$

From the above equation,

$5 = 35^{\frac{z}{x}}$ and $7 = 35^{\frac{z}{y}}$

Now, $5 \times 7 = 35^{\frac{z}{x}} \times 35^{\frac{z}{y}} = 35^{\frac{z}{x}+\frac{z}{y}}$

Thus, $\frac{z}{x} + \frac{z}{y} = 1$

$$z = \frac{xy}{(x+y)}$$

Q5. What is the value of "x" if $49^{x-5} = 117649$?

(a) 5 (b) 3

(c) 8 (d) None of these

Ans: (c)

Explanation:

$7^{2(x-5)} = 7^6$

$\Rightarrow 2(x-5) = 6$

$\Rightarrow x - 5 = 3$

$\Rightarrow x = 5 + 3 = 8$

Q6. Calculate the cube root of 74.088.

(a) 2.4 (b) 4.2

(c) 5.3 (d) 4.5

Ans: (b)

Explanation:

$$\sqrt[3]{74.088} = \sqrt[3]{\frac{74088}{1000}}$$

Now, $\sqrt[3]{74088} = \sqrt[3]{2^3 \times 3^3 \times 7^3}$

$= 2 \times 3 \times 7 = 42$

Therefore, $\sqrt[3]{\frac{74088}{1000}} = \frac{42}{10} = 4.2$

Q7. The square root of 649.2304 is equal to

(a) 25.48 (b) 24.58

(c) 28.45 (d) 28.54

Ans: (a)

Explanation:

$$\sqrt{649.2304} = \sqrt{\frac{6492304}{10000}}$$

Now, $\sqrt{6492304} = \sqrt{2^4 \times 7^4 \times 13^2}$

$= 2^2 \times 7^2 \times 13 = 2548$

Therefore, $\sqrt{\frac{6492304}{10000}} = \frac{2548}{100} = 25.48$

Q8. Find the value of $36 \times (216)^{-\frac{2}{3}}$.

(a) 0 (b) 1

(c) 6 (d) 36

Ans: (b)

Explanation:

$36 \times (216)^{-\frac{2}{3}} = 6^2 \times (6^3)^{-\frac{2}{3}}$

$= 6^2 \times (6)^{3 \times \left(-\frac{2}{3}\right)} = 6^2 \times 6^{-2}$

$= 6^{2+(-2)} = 6^0 = 1$

Anything raised to power zero is 1.

Q9. If $x = \sqrt{3 - 2\sqrt{2}}$ then the value of x is equal to:

(a) $\sqrt{2} + 2$ (b) $\sqrt{2} - 2$

(c) $\sqrt{2} + 1$ (d) $\sqrt{2} - 1$

Ans: (d)

Explanation:

$x = \sqrt{3 - 2\sqrt{2}} = \sqrt{2 + 1 - 2\sqrt{2}}$

$= \sqrt{(\sqrt{2})^2 + 1^2 - 2 \times 1 \times \sqrt{2}}$

$= \sqrt{(\sqrt{2} - 1)^2}$

$x = (\sqrt{2} - 1)$

Q10. Find the value of $\sqrt{7 + 4\sqrt{3}} + \sqrt{7 - 4\sqrt{3}}$.

(a) $2\sqrt{3}$ (b) 4

(c) $\sqrt{3}$ (c) 2

Ans: (b)

Explanation:

$\sqrt{7 + 4\sqrt{3}} + \sqrt{7 - 4\sqrt{3}}$

$= \sqrt{2^2 + (\sqrt{3})^2 + 2(2)(\sqrt{3})}$

$\sqrt{2^2 + (\sqrt{3})^2 - 2(2)(\sqrt{3})}$

$= \sqrt{(2 + \sqrt{3})^2} + \sqrt{(2 - \sqrt{3})^2}$

$= (2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$

Q11. Find the value of $\frac{\sqrt{5 + 2\sqrt{6}}}{\sqrt{5 - 2\sqrt{6}}}$.

(a) $5 - 2\sqrt{6}$ (b) $5 + \sqrt{6}$

(c) $5 + 2\sqrt{6}$ (d) $5 - \sqrt{6}$

Ans: (c)

Explanation:

$$\sqrt{5+2\sqrt{6}} = \sqrt{3+2+2\sqrt{6}}$$

$$= \sqrt{(\sqrt{3})^2 + (\sqrt{2})^2 + 2\times\sqrt{3}\times\sqrt{2}}$$

$$= \sqrt{(\sqrt{3}+\sqrt{2})^2} = (\sqrt{3}+\sqrt{2})$$

Similarly,

$$\sqrt{5-2\sqrt{6}} = (\sqrt{3}-\sqrt{2})$$

$$\Rightarrow \frac{\sqrt{5+2\sqrt{6}}}{\sqrt{5-2\sqrt{6}}} = \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})}$$

Rationalizing the denominator we get,

$$\Rightarrow \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})} \times \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})} = \frac{(\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$$

$$= \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2 - (\sqrt{2})^2}$$

$$= \frac{(\sqrt{3}+\sqrt{2})^2}{3-2}$$

$$= \frac{3+2+2\sqrt{6}}{1}$$

Therefore, $\frac{\sqrt{5+2\sqrt{6}}}{\sqrt{5-2\sqrt{6}}} = 5+2\sqrt{6}$.

Q12. What is the largest number among the following? **(UPSC CSAT 2020)**

(a) $\left(\frac{1}{2}\right)^{-6}$ (b) $\left(\frac{1}{4}\right)^{-3}$

(c) $\left(\frac{1}{3}\right)^{-4}$ (d) $\left(\frac{1}{6}\right)^{-2}$

Ans: (c)

Explanation:

Given numbers can be simplified as follows

Option (a): $\left(\frac{1}{2}\right)^{-6} = (2)^6 = 64$

Option (b): $\left(\frac{1}{4}\right)^{-3} = (4)^3 = 64$

Option (c): $\left(\frac{1}{3}\right)^{-4} = (3^4) = 81$

Option (d): $\left(\frac{1}{6}\right)^{-2} = (6^2) = 36$

We can see that 81 is the largest number among the given four numbers.

PRACTICE QUESTIONS

1. If $z = \sqrt{3} + \sqrt{2}$, then the value of $\frac{(z+1)}{(z-1)} - \frac{(z-1)}{(z+1)}$ is?

   (a) $\sqrt{7}$ (b) $\sqrt{5}$

   (c) $\sqrt{3}$ (d) $\sqrt{2}$

2. Which among the following numbers $\sqrt[3]{216}, \sqrt[3]{1728}, \sqrt[4]{256}$ and $\sqrt[5]{243}$ is greatest?

   (a) $\sqrt[3]{216}$ (b) $\sqrt[3]{1728}$

   (c) $\sqrt[4]{256}$ (d) $\sqrt[5]{243}$

3. If $(16)^3$ is added to the square of a number, the answer obtained is 6905. What is the magnitude of the number?

   (a) 53 (b) 47

   (c) 67 (d) 73

4. Find the least number multiplied by 9746100 which will make it a perfect square.

   (a) 85 (b) 221

   (c) 145 (d) 235

5. A college professor draws up his 22252 students in the form of a solid square and finds that he has 51 students left over. Find the number of students in the front.

   (a) 179 (b) 149

   (c) 127 (d) 97

6. What is the largest number among the following?

   (a) $\left(\frac{1}{8}\right)^{-3}$ (b) $\left(\frac{1}{2}\right)^{-6}$

   (c) $\left(\frac{1}{7}\right)^{-4}$ (d) $\left(\frac{1}{6}\right)^{-5}$

7. Find the total number of subscribers when a certain group of people agree to contribute the same amount in rupees as there are subscribers, and the total contribution amounts to Rs. 3214849.

   (a) 17833 (b) 1433

   (c) 16643 (d) 1793

8. Which one of the following will have a minimum change in its value if 3 is added to both the numerator and the denominator of the fractions $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}$ and $\frac{6}{7}$?

   (a) $\frac{1}{3}$ (b) $\frac{2}{5}$

   (c) $\frac{3}{7}$ (d) $\frac{6}{7}$

9. To accommodate 8504 chairs in an auditorium, the event organizer arranged them in a grid with an equal number of rows and columns. Consequently, the organizer had to remove a minimum number of chairs, denoted as '*P*', from the total of 8504 chairs. What is the minimum value of '*P*'?

   (a) 69 (b) 40

   (c) 43 (d) 29

10. In Ranthambore National Park, one-fourth of a streak of Tiger was seen in the forest. Twice the square root of the streak had gone to mountains and the remaining 15 Tiger were seen on the bank of a river. Find the total number of Tigers.

    (a) 56 (b) 46

    (c) 27 (d) 36

ANSWERS

1. (*d*) 2. (*b*) 3. (*a*) 4. (*b*) 5. (*b*) 6. (*d*) 7. (*d*) 8. (*d*) 9. (*b*) 10. (*d*)

EXPLANATIONS

1. (d)

Explanation:

$$\frac{(z+1)}{(z-1)} - \frac{(z-1)}{(z+1)} = \frac{(z+1)^2 - (z-1)^2}{(z-1)(z+1)}$$

$$= \frac{(z^2+1+2z) - (z^2+1-2z)}{(z^2-1)} = \frac{4z}{(z^2-1)}$$

$$= \frac{4(\sqrt{3}+\sqrt{2})}{\{(\sqrt{3}+\sqrt{2})^2 - 1\}} = \frac{4(\sqrt{3}+\sqrt{2})}{(3+2+2\sqrt{6}-1)}$$

$$= \frac{4(\sqrt{3}+\sqrt{2})}{(4+2\sqrt{6})}$$

$$= \frac{4(\sqrt{3}+\sqrt{2})}{2\sqrt{2}(\sqrt{3}+\sqrt{2})} = \sqrt{2}$$

2. (b)

Explanation:

$\sqrt[3]{216}, = (216)^{\frac{1}{3}} = 6$

$\sqrt[3]{1728}, = (1728)^{\frac{1}{3}} = 12$

$\sqrt[4]{256}, = (256)^{\frac{1}{4}} = 4$

$\sqrt[5]{243}, = (243)^{\frac{1}{5}} = 3$

3. (a)

Explanation:

Let the number be x.

According to the question,

$x^2 + (16)^3 = 6905$

$x^2 = 6905 - (16)^3 = 6905 - 4096 = 2809$

$x = \sqrt{2809} = \pm 53$

4. (b)

Explanation:

9746100 is the multiple of:

$9746100 = 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 \times 7 \times 13 \times 17$

Least number which should be multiplied to 9746100 to make it a perfect square = $13 \times 17 = 221$

5. (b)

Explanation:

Let the total number of students in front be x.

Then, the total number of students in the form of a solid square = x^2

According to the question, $x^2 + 51 = 22252$

$x^2 = 22201$

$x = \sqrt{22201}$

$x = +149$ (–149 can't be the value as the number of students can't be negative)

6. (d)

Explanation:

For every number "a" with negative exponents "$-n$" (i.e.) a^{-n}, take the reciprocal of the base number and multiply the value according to the value of the exponent number. a^{-n} can be written as $\left(\frac{1}{a}\right)^n$

Option (a) – $\left(\frac{1}{8}\right)^{-3} = 8^3 = 512$

Option (b) – $\left(\frac{1}{2}\right)^{-6} = 2^6 = 64$

Option (c) – $\left(\frac{1}{7}\right)^{-4} = 7^4 = 2401$

Option (d) – $\left(\frac{1}{6}\right)^{-5} = 6^5 = 7776$

7. (d)

Explanation:

Let the total number of subscribers be x.

Then, x^2 **= 3214849**

$x = \sqrt{3214849} = 1793$

8. (d)

Explanation:

Original fractions and their decimal values are given below:

$\frac{1}{3} = 0.333$

$\frac{2}{5} = 0.40$

$\frac{3}{7} = 0.428$

$\frac{6}{7} = 0.857$

The new fractions (after adding 3 to numerator and denominator) and their decimal values are given below:

$\frac{4}{6} = 0.666$

$\frac{5}{8} = 0.625$

$\frac{6}{10} = 0.60$

$\frac{9}{10} = 0.90$

Now, change in values of fractions are:

$\left(\frac{4}{6}\right) - \left(\frac{1}{3}\right) = 0.666 - 0.333 = 0.333$

$\left(\frac{5}{8}\right) - \left(\frac{2}{5}\right) = 0.625 - 0.400 = 0.225$

$\left(\frac{6}{10}\right) - \left(\frac{3}{7}\right) = 0.60 - 0.428 = 0.172$

$\left(\frac{9}{10}\right) - \left(\frac{6}{7}\right) = 0.90 - 0.857 = 0.043$

Hence, we can see that the minimum change in value is in the fraction $\frac{6}{7}$.

9. (b)

Explanation:

According to question

8504 – P must be a perfect square. ...(i)

Now, using the option, we will find the value of P

We will put the value of P in the above equation (i)

Option (a)

8504 – 69 = 8435 (Not a Perfect Square)

Option (b)

8504 – 40 = 8464 is a perfect square of 92.

Option (c)

8504 – 43 = 8461 (Not a Perfect Square)

Option (d)

8504 –29 = 8475 (Not a Perfect Square)

10. (d)

Explanation: Let the total number of tigers be T

So,

- Tigers seen in forest = $\frac{T}{4}$
- Tigers on mountain = $2\sqrt{T}$
- Remaining Tigers = 15

According to the question,

$$\frac{T}{4}+2\sqrt{T}+15=T$$

We can put the value of T from options:

Right–hand Side should be equal to Left Hand Side

Option (a)

$$\frac{56}{4}+2\sqrt{56}+15=56$$

14 + 2 × 7.48 + 15 = 56

14 + 15 + 15 = 56

44 = 56 (Option (a) is not correct.)

Option (b)

$$\frac{T}{4}+\sqrt{T}+15=T$$

$$\frac{46}{4}+2\sqrt{46}+15=46$$

11.5 + 2 × 6.78 + 15 = 46

40.06 = 46 (Option (b) is not correct.)

Option (c)

T/4 + 2√T + 15 = T

27/4 + 2√27 + 15 = 27

6.75 + 2 × 5.19 + 15 = 27

32.13 = 27 (Option (c) is not correct.)

Option (d)

$$\frac{T}{4}+2\sqrt{T}+15=T$$

$$\frac{36}{4}+2\sqrt{46}+15=36$$

9 + 2 × 6 + 15 = 36

36 = 36 (Option (d) is correct)

Average

6

INTRODUCTION

The average is a measure of the central tendency of a set of numbers. An average or an arithmetic mean of a given set of observations is the sum of all the observations divided by the total number of observations. It is a very effective way of representing an entire group of data by a single value.

$$\text{Average or Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

Example:

Find the average of 9, 22, 8, 11, 15, 14, 1.

Sum of given data = 9+22+8+11+15+14+1 = 80

Number of data = 7

$$\text{Average} = \frac{\text{Sum of given quantities}}{\text{Number of quantities}} = \frac{80}{7} = 11.43$$

IMPORTANT FACTS ABOUT AVERAGE

1. If all the given quantities have the same value, then the average is equal to the number itself.

   Example: Let 10, 10, 10, 10 and 10 be the set of observations, then the average is 10.

2. If all the given quantities are not the same, then the average of any given number of quantities lies between the lowest and highest values of the given data.

   Example: Find the average of 5, 7, 13, 12, 8, 15, 1, 9, 6, 4.

   $$Average = \frac{Sum\ of\ all\ observations}{Number\ of\ observations}$$

   $$Average = \frac{5+7+13+12+8+15+1+9+6+4}{10}$$

   $$= \frac{80}{10} = 8$$

   Average of given data is 8 which is greater than least number i.e., 5 and less than greatest number which is 15.

3. If each quantity is increased/decreased by a value 'p' then the new average is increased/decreased by p.

   Example: Increasing each observation in the last Example by 2: 5+2, 7+2, 13+2, 12+2, 8+2, 15+2, 1+2, 9+2, 6+2, 4+2.

   $$\text{Average} = \frac{(7+9+15+14+10+17+3+11+8+6)}{10}$$

   $$= \frac{100}{10} = 10$$

   Hence the average also increased by 2 when each observation increased by 2.

4. If each quantity is multiplied/divided by a certain value p, then the new average is obtained by multiplying/dividing the old average by p.

   Example: Monthly expenditure of Ram on food, travel, rent and shopping is Rs. 2000, Rs. 800, Rs. 1200 and Rs. 5000 respectively. If the monthly expenditure on each item is increased by three times then the average monthly expenditure of Ram is equal to:

   Monthly expenditure of Ram on food, travel, rent and shopping is Rs. 2000, Rs. 800, Rs. 1200 and Rs. 5000.

   Average monthly expenditure

   $$= \frac{2000+800+1200+5000}{4} = \frac{9000}{4} = Rs.2250$$

   Since expenditure on each is increased by three times therefore,

   New average monthly expenditure = Old Average × 3

   Old Average × 3

   = 2250 × 3

   = Rs. 6750

Note:
If the same value is added to half of the quantities and the same value is subtracted from other half quantities then there will not be any change in the final value of the average.

AVERAGE AGE PROBLEMS

1. When a person leaves a group and another person joins the same group causing a change in the average age of the group;

Case-I: When this causes an increase in average age,

Age of new person can be determined as = Age of person who left the group + (Number of persons in the group including new person × Increase in average age of the group)

Case-II: Decrease in average age,

Age of new person can be determined as = Age of person who left the group – (Number of persons in the group including new person × Decrease in average age of the group)

EXAMPLES

E1. The average age of 8 persons is increased by 2 years, when one of them, whose age is 24 years, is replaced by a new person. The age of the new person is :

(a) 42 years (b) 40 years

(c) 38 years (d) 45 years

Ans: (b)

Explanation:

Age of new person can be determined as = Age of person who left the group + (Number of persons in the group including new person × Increase in average age of the group)

Age of new person = 24 + (8 × 2)

= 24 + 16 = 40

Age of new person = 40 years.

E2. Out of 10 teachers of a school, one teacher retires and in his place a new teacher of age 25 years joins. As a result, the average age of the teachers is reduced by 3 years. The age of the retired teacher is

(a) 60 years (b) 58 years

(c) 56 years (d) 55 years

Ans: (d)

Explanation:

Here, Number of teachers, n = 10,

Reduction in average age,p = 3

Thus, Age of new teacher = Age of retired teacher - n × p

$\Rightarrow$ 25 = Age of retired teacher - 10 × 3

Age of retired teacher = 25 + 30 = 55 years

2. When a person joins a group without replacing any previous person from that group, then

Case-I: Increase in average age,

Age of new member = Previous average of the age + Number of all persons including new member × Increase in average age

Case-II: Decrease in average age,

Age of new member = Previous average of the age – Number of all persons including new member × Increase in average age

EXAMPLES

E3. 3 years ago, the average age of a family of 5 members was 17 years. A baby having been born, the average age of the family is the same today. The present age of the baby is

(a) 1 year (b) 1.5 year

(c) 2 years (d) 3 years

Ans: (c)

Explanation:

Present age of 5 members = 17 × 5 + 3 × 5

= 100 years

Present age of 5 members and a baby

= 17 × 6= 102 years

Present age of child

= 102 – 100

= 2 years

3. When a person leaves the group but nobody joins that group, then

Case-I: Increase in average age,

Age of person left = Previous average of the age - (Number of persons present x Increase in average age)

Case-II: Decrease in average age,

Age of person left = Previous average of the age +(Number of persons present x Decrease in average age)

EXAMPLES

E4. The average age of a family with 5 members is 28 years. If one of the members of age 20 years is excluded, the average age of the family becomes

(a) 25 years (b) 20 years

(c) 30 years (d) 24 years

Ans: (c)

Explanation:

Since the age of person who left is less than the average, therefore there will be increase in average age

Given, Age of person left = 20

Previous average of the age = 28

Number of persons present = 4

Age of person left = Previous average of the age - (Number of persons present × Increase in average age)

$\Rightarrow$ 20 = 28 – (4x Increase in average age)

$\Rightarrow$ Increase in average age= $\frac{28-20}{4} = \frac{8}{4} = 2$

Hence the new average age= 28+2= 30 Years.

AVERAGE OF NATURAL NUMBERS

(i) Average of first 'n' consecutive natural numbers = $\frac{(n+1)}{2}$

(ii) Average of squares of first 'n' consecutive natural numbers = $\frac{(n+1)(2n+1)}{6}$

(iii) Average of cubes of first 'n' consecutive natural numbers = $\frac{n(n+1)^2}{4}$

EXAMPLES

E5. The average of the squares of first ten natural numbers is

(a) 35.5 (b) 36

(c) 37.5 (d) 38.5

Ans: (d)

Explanation:

Average of sum of squares of first n natural numbers

$= \frac{1^2+2^2+3^2+4^2+....+n^2}{n} = \frac{n(n+1)(2n+1)}{6n}$

$= \frac{(n+1)(2n+1)}{6}$

Given, n = 10

$\frac{1^2+2^2+3^2+4^2+....+n^2}{n} = \frac{(10+1)(2(10)+1)}{6} = 38.5$

E6. Calculate the average of cubes of first 28 consecutive natural numbers.

(a) 4732 (b) 2347

(c) 7423 (d) 3247

Ans: (a)

Explanation: Average of cubes of first 'n' consecutive natural numbers = $\frac{\left[\frac{n(n+1)}{2}\right]^2}{n} = \frac{n(n+1)^2}{4}$

Here, n = 28

Hence,

Average = $\frac{28(28+1)^2}{4}$

$= 7(29)^2 \quad = 5887$

AVERAGE OF EVEN NUMBERS

(i) Average of first n consecutive even numbers = (n+1)

(ii) Average of squares of first n consecutive even numbers = $\frac{2(n+1)(2n+1)}{3}$

(iii) Average of cubes of 1st n even numbers = $2n(n+1)^2$

EXAMPLES

E7. The average of squares of first 50 consecutive even numbers is equal to:

(a) 3000 (b) 3223

(c) 4334 (d) 3434

Ans: (d)

Explanation:

First 50 even consecutive numbers are:

2, 4, 6, 8.....100

Average of squares of 1st n consecutive even number

$= \frac{2(n+1)(2n+1)}{3}$

Given, n = 50

Average of squares of 1st 50 consecutive even number

$= \frac{2(50+1)(2*50+1)}{3} = \frac{2(51)(101)}{3} = 3434$

E8. The sum of three consecutive even numbers is 28 more than the average of these three numbers. Then the smallest of these three numbers is

(a) 6 (b) 12

(c) 14 (d) 16

Ans: (b)

Explanation:

Let the consecutive even numbers be x, $x + 2$ and $x + 4$.

then, $x + x + 2 + x + 4 = 28 + \frac{(x+x+2+x+4)}{3}$

$\Rightarrow (3x+6) - \frac{3x+6}{3} = 28$

$\Rightarrow 3x + 6 - x - 2 = 28$

$\Rightarrow 2x = 24 \Rightarrow x = 12$

AVERAGE OF ODD NUMBERS

(i) Average of first n consecutive odd numbers [1, 3, 5……., (2n-1)] = n

(ii) Average of sum of squares of first "n" odd natural numbers = $\frac{(2n+1)(2n-1)}{3}$

(iii) Average of sum of cubes of first "n" odd natural numbers = $n(2n^2 - 1)$

EXAMPLES

E9. The average of first 50 odd numbers

(a) 50.5 (b) 50

(c) 49.5 (d 49

Ans: (b)

Explanation:

Average of first n natural odd numbers = n

Given, $n = 50$

Hence, Average of first 50 odd numbers = 50

E10. The average of three consecutive odd numbers is 12 more than one third of the first number of these numbers. What is the largest number of the three numbers?

(a) 15 (b) 17

(c) 19 (d) Data inadequate

Ans: (c)

Explanation:

Let the three consecutive odd numbers are

$= x$, $x + 2$ and $x + 4$

According to the question,

$\frac{x+x+2+x+4}{3} = \frac{x}{3} + 12$

$\Rightarrow 3x + 6 = x + 36$

$\Rightarrow 2x = 30$

$\Rightarrow x = 15$

Hence the largest number = $x + 4 = 15 + 4 = 19$

E11. If a, b, c, d, e are five consecutive odd numbers, their average is

(a) $5(a + 4)$ (b) $\frac{abcde}{5}$

(c) $5(a + b + c + d + e)$ (d) $a + 4$

Ans: (d)

Explanation:

a, b, c, d, e are five consecutive odd numbers, then

$b = a + 2$

$c = b + 2 = a + 4$

$d = c + 2 = b + 4 = a + 6$

$e = d + 2 = a + 8$

Required average = $\frac{a+a+2+a+4+a+6+a+8}{5}$

$= \frac{5a+20}{5}$

$= a + 4$

AVERAGE SPEED PROBLEMS

$$\text{Average Speed} = \frac{\text{Total distance traveled}}{\text{Total time taken}}$$

1. If a person covers Distance A km at a km/hr and B km at b km/hr

   Average speed of total journey = $\frac{A+B}{\left(\frac{A}{a}+\frac{B}{b}\right)} km/hr$

2. If a person travels from point P to point Q at a speed of a km/hr and then returns back from point Q to point P at a speed of b km/hr

   Average speed of total journey $= \frac{2ab}{a+b} km/hr$

3. If a person covers Distance A km at a km/hr and B km at b km/hr and C km at c km/hr,

   Average speed of total journey = $\frac{A+B+C}{\left(\frac{A}{a}+\frac{B}{b}+\frac{C}{c}\right)} km/hr$

4. If same distance is traveled with three different speeds a km/hr, b km/hr and c km/hr, then

   Average speed of total journey

   $= \frac{3abc}{ab+bc+ca} km/hr$

5. If a person covers x^{th} part of the distance at a km/h, y^{th} part of the distance at b km/h and remaining z^{th} part at c km/h, then the average speed during entire journey

$$=\left(\frac{1}{\frac{x}{a}+\frac{y}{b}+\frac{z}{c}}\right)$$

EXAMPLES

E12. Aakash travels to a place 453 km away at an average speed of 37 km/hr and returns at 43 km/hr. His average speed for the whole journey in km/hr is :

(a) 38.97 km/h (b) 39.87 km/h

(c) 37.98 km/h (d) 38.87 km/h

Ans: (b)

Explanation:

Average speed during the whole journey $=\frac{2ab}{a+b}$ km/hr

Let a = 37 km/hr and b = 43 km/hr

Therefore, Average speed $=\frac{2(37)(43)}{37+43}$

$=\frac{2\times(37)\times(43)}{80}$

$=39.78\,km/hr$

E13. A train traveling between two stations P and Q covers one fifth of the total distance at 10 km/h, one third at 20 km/hr and the rest distance at 30 km/hr. Find the average speed of the train for the journey.

(a) 25.71 km/h (b) 27.51 km/h

(c) 21.75 km/h (d) Cannot be determined

Ans: (a)

Explanation:

Distance covered at 10 km/h is $\frac{1}{5}th$

Distance covered at 20 km/h is $\frac{1}{3}rd$

Distance covered at 30 km/h is $\frac{7}{15}$ th

Therefore, $x=\frac{1}{5}$, $y=\frac{1}{3}$, $z=\frac{7}{15}$

a = 10 km/h, b= 20 km/h, c =30 km/h

Average speed during entire journey

$$\left(\frac{1}{\frac{x}{a}+\frac{y}{b}+\frac{z}{c}}\right)=\left(\frac{1}{\frac{1/5}{10}+\frac{1/3}{20}+\frac{7/15}{30}}\right)$$

$$=\left(\frac{10}{\frac{1}{5}+\frac{1}{6}+\frac{7}{45}}\right)=\left(\frac{10}{\frac{18+15+2}{90}}\right)$$

$=\frac{900}{35}=25.71$ km/h

TYPES OF QUESTIONS

There are two types of questions on Average

TYPE 1: QUESTIONS BASED ON AGE/ WEIGHT

QUESTIONS

Q1. The average weight of A, B, C is 40 kg, the average weight of B, D, E is 42 kg and the weight of F is equal to that of B. What is the average weight of A, B, C, D, E and F? **(UPSC CSAT 2022)**

(a) 40.5 kg

(b) 40.8 kg

(c) 41 kg

(d) Cannot be determined as data is inadequate

Ans: (c)

Explanation:

According to the question,

$\frac{(A+B+C)}{3}=40$

$(A + B + C) = 120$ ...(i)

$\frac{(B+D+E)}{3}=42$

$\Rightarrow$ (B + D + E) = 126 ...(ii)

F = B ...(iii)

From equations (i) and (ii), we get

A + B + C + B + D + E = 120 + 126

$\Rightarrow$ A + B + C + D + E + B = 246

$\Rightarrow$ A + B + C + D + E + F = 246 ($\because$ B=F)

So, average weight of A + B + C + D + E + F

$=\frac{246}{6}=41$kg

Q2. There are two classes A and B having 25 and 30 students respectively. In class A the highest score is 21 and lowest score is 17. In class B the highest score is 30 and lowest score is 22. Four students are shifted from class A to class B. Consider the following statements

1. The average score of class B will definitely decrease.
2. The average score of class A will definitely increase.

Which of the above statements is/are correct?

(UPSC CSAT 2021)

(a) Only 1 (b) Only 2

(c) Both 1 and 2 (d) Neither 1 nor 2

Ans: (a)

Explanation:

Statement 1. Since, 4 students are moved from A to B, the average of class B will definitely decrease as the range of marks for class A(17 to 21) is less than the range of marks for class B(22 to 30). Therefore, Statement 1 is correct.

Statement 2. According to the question, we are not sure regarding the marks of the 4 students that have been moved from class A. Therefore, We can not determine whether the average score of class A will increase or decrease.

Q3. Consider the following data. **(UPSC CSAT 2020)**

	Average marks in English	Average marks in Hindi
Girls	9	8
Boys	8	7
Overall average marks	8.8	*x*

What is the value of x in the above table?

(a) 7.8 (b) 7.6

(c) 7.4 (d) 7.2

Ans: (a)

Explanation:

Let the number of girls = a

Number of boys = b

According to the question.

$9a + 8b = 8.8(a + b)$ $9a\text{-}8.8a = 8.8b\text{-}8b$

$0.2a = 0.8b$

$a = 4b$

Again, $8a + 7b = x(a + b)$

$\Rightarrow 8\times 4b + 7b = x\times 5b$

$\Rightarrow x = \frac{39}{5} = 7.8$

Q4. In a class, there are three groups A, B and C. If one student from group A and two students from group B are shifted to group C, then what happens to the average weight of the students of the class?

(UPSC CSAT 2020)

(a) It increases

(b) It decreases

(c) It remains the same

(d) No conclusion can be drawn due to insufficient data

Ans: (c)

Explanation:

Since, there is only shifting of students from one group to another. Hence, there is no change in the total weight and the total number of students present in the three groups. Therefore, the average weight of the students in the class will remain the same.

Q5. The average score of a batsman after his 50th innings was 46.4. After 60th innings, the average score increases by 2.6. What was his average score in the last ten innings? **(UPSC CSAT 2020)**

(a) 122 (b) 91

(c) 62 (d) 49

Ans: (c)

Explanation:

Given, average score of batsman after 50th innings was 46.4, so total score after 50th innings

$= 46.4 \times 50 = 2320$

And, average score of batsman after 60th innings

$= 46.4 + 2.6 = 49$

Hence, total score after 60th innings

$= 49 \times 60 = 2940$

So, total score in last ten innings

$= 2940 - 2320 = 620$

Hence, average score in the last ten innings

$= \frac{620}{10} = 62.$

Q6. The average age of a teacher and three students is 20 years. If all the three students are of same age and the difference between the age of the teacher and each student is 20 years, then what is the age of the teacher? **(UPSC CSAT 2020)**

(a) 25 years (b) 30 years

(c) 35 years (d) 45 years

Ans: (c)

Explanation:

Let the age of the teacher be x, and the age of each student be y.

Now according to question,

$$\frac{x+3y}{4}=20$$

$$\Rightarrow x+3y=80 \quad ...(i)$$

Now according to given condition,

$x-y=20$ ...(ii)

From equations (i) and (ii),

x = 35 and y = 15

Hence, the age of the teacher is 35 years.

Q7. The average marks of 100 students are given to be 40. It was found later that marks of one student were 53 which were misread as 83. The corrected mean marks are **(UPSC CSAT 2019)**

(a) 39 (b) 39.7

(c) 40 (d) 40.3

Ans: (b)

Explanation:

Total marks of 100 students = 100 × 40 = 4000.

New corrected total marks

= 4000 – 83 + 53 = 3970

So, corrected mean = $\frac{3970}{100}=39.7$

Q8. A family has two children along with their parents. The average of the weights of the children and their mother is 50kg. The average of the weights of the children and their father is 52 kg. If the weight of the father is 60 kg, then what is the weight of the mother? **(UPSC CSAT 2019)**

(a) 48 kg (b) 50 kg

(c) 52 kg (d) 54 kg

Ans: (d)

Explanation:

Let the weight of the mother be x kg.

Let the total weight of two children by y kg.

According to the question,

$$\frac{y+60}{3}=52$$

$$\Rightarrow y+60=156$$

$$\Rightarrow y=156-60$$

$$\Rightarrow y=96 \quad ...(i)$$

also, $\frac{y+x}{3}=50$

Using equation (i), we get

$x=150-96=54$

Hence, the weight of the mother is 54 kg.

Q9. The average rainfall in a city for the first four days was recorded to be 0.40 inch. The rainfall on the last two days was in the ratio of 4 : 3. The average of six days was 0.50 inch. What was the rainfall on the fifth day? **(UPSC CSAT 2017)**

(a) 0.60 inch (b) 0.70 inch

(c) 0.80 inch (d) 0.90 inch

Ans: (c)

Explanation:

Let the rainfall on the last two days be 4x and 3x, respectively. Now as per question,

$$0.50=\frac{4\times0.40+4x+3x}{6}$$

$$\Rightarrow 0.50=\frac{1.6+7x}{6}$$

$$\Rightarrow 3=1.6+7x$$

$$\Rightarrow 7x=3-1.6$$

$$\Rightarrow 7x=1.4$$

x = 0.2 inch

Hence, rainfall on the fifth day

= 4x = 4 × 0.2= 0.80 inch

Q10. Suppose the average weight of 9 persons is 50 kg. The average weight of the first 5 persons is 45 kg, whereas the average weight of the last 5 persons is 55kg. then, the weight of the 5th person will be **(UPSC CSAT 2017)**

(a) 45 kg (b) 47.5 kg

(c) 50 kg (d) 52.5 kg

Ans: (c)

Explanation:

Total weight of 9 persons

= 50 × 9 = 450 kg

Total weight of first 5 persons

= 45 × 5 = 225 kg

Therefore the total weight of last 4 persons

= 450 – 225 = 225 kg

Total weight of last 5 persons

= 55 × 5 = 275 kg

Hence, weight of the 5th person

= 275 – 225 = 50 kg

Q11. The monthly average salary paid to all the employees of a company was Rs. 5000. The monthly average salary paid to male and female employees was Rs. 5200 and Rs. 4200, respectively. Then, the percentage of males employed in the company is **(UPSC CSAT 2016)**

(a) 75% (b) 80%

(c) 85% (d) 90%

Ans: (b)

Explanation:

Let the number of male and female employees be x and y, respectively.

Given,

Monthly average salary of employees

= Rs. 5000

Monthly average salary of male employees

= Rs. 5200

Monthly average salary of female employees

= Rs. 4200.

According to the questions,

$5000\ (x + y) = 5200 + 4200\ y$

$5000\ x + 5000\ y = 5200\ x + 4200\ y$

$200x = 800y$

$x = 4y$ ...(i)

So, required percentage

$$= \frac{x}{x+y} \times 100$$

$$= \frac{x}{x+\frac{x}{4}} \times 100 \quad \text{[From (i)]}$$

$$= \frac{4x}{5x} \times 100 = 80\%$$

Q12. For five children with ages a < b < c < d < e; any two successive ages differ by 2 years.

Question: What is the age of the youngest child?

Statement-1: The age of the eldest is 3 times the youngest.

Statement-2: The average age of the children is 8 years

Which one of the following is correct in respect of the above Question and the Statements?

(a) The Question can be answered by using one of the Statements alone.

(b) The Question can be answered by using either Statement alone.

(c) The Question can be answered by using both the Statement together, but cannot be answered using either Statement alone.

(d) The Question cannot be answered even by using both the Statements together.

Ans: (b)

Explanation:

Given: $a < b < c < d < e$

Since, any two successive ages differ by 2 years, the ages can also be written as: a, $a + 2$, $a + 4$, $a + 6$, $a + 8$

Statement 1: The age of the eldest is 3 times the youngest.

So, $a + 8 = 3a$

Or $2a = 8$ Or $a = 4$

So, Statement 1 alone is sufficient.

Statement 2: The average age of the children is 8 years.

$$\frac{(a + a + 2 + a + 4 + a + 6 + a + 8)}{5} = 8$$

Or $\frac{(5a + 20)}{5} = 8$

Or $5a + 20 = 40 \Rightarrow a = 4$

So, Statement 2 alone is sufficient.

TYPE 2: QUESTIONS BASED ON AVERAGE SPEED

QUESTIONS

Q13. A car travels from a place *X* to place *Y* at an average speed of v km/hr, from *Y* to *X* at an average speed of 2v km/hr, again from *X* to *Y* at an average speed of 3v km/hr and again from *Y* to *X* at an average speed of 4v km/hr. Then, the average speed of the car for the entire journey **(UPSC CSAT 2020)**

(a) is less than v km/hr

(b) lies between v and 2v km/hr

(c) lies between 2v and 3v km/hr

(d) lies between 3v and 4v km/hr

Ans: (b)

Explanation:

Let the distance between *X* and *Y* be *D*.

Now, average speed

$$=\frac{\text{distance covered}}{\text{time taken}}$$

$$=\frac{D+D+D+D}{D\times\frac{1}{v}+D\times\frac{1}{2v}+D\times\frac{1}{3v}+D\times\frac{1}{4v}}$$

$$=\frac{4D}{\frac{D}{v}\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)}$$

$$=\frac{4v}{50/24}$$

$$=\left(\frac{4\times24}{50}\right)v$$

= 1.92 v

Therefore, the average speed of the car for the entire journey lies between v and 2v km/hr.

Q14. Two cities A and B are 360 km apart. A car goes from A to B with a speed of 40 km/h and returns to A with a speed of 60 km/hr. What is the average speed of the car? **(UPSC CSAT 2015)**

(a) 45 km/h (b) 48 km/h

(c) 50 km/h (d) 55 km/h

Ans: (b)

Explanation:

Given distance between the cities A and B = 360 km.

So, average speed

$$=\frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$=\frac{360+360}{\frac{360}{40}+\frac{360}{60}}$$

$$=\frac{720}{9+6}$$

$$=\frac{720}{15}=48\text{km/h}$$

TYPE 3: MISCELLANEOUS QUESTIONS

QUESTIONS

Q15. The average monthly savings of Prakash was Rs 200 during the first 4 months; Rs 275 during the next 3 months and Rs 350 during the remaining months of a year. If the total expenditure during the year were Rs 25503, then Calculate the average monthly income of Prakash.

(a) Rs. 2604.50 (b) Rs.2406.50

(c) Rs. 2460.05 (d) Rs. 2640.50

Ans: (b)

Explanation:

Total annual Income = Expenditure + Savings

Total annual Income

= 25503 + $(200\times4+275\times3+350\times5)$

= 25503 + (800 + 825 + 1750)

= 25503 + 3375 = 28878

Average monthly Salary = $\frac{28878}{3+4+5}=\frac{28878}{12}$

$=Rs.2406.50$

Q16. If the average weight of 6 students is 50 kg; that of 2 students is 51 kg; and that of other 2 students is 55 kg; then the average weight of all students is

(a) 61 kg (b) 51.5 kg

(c) 52 kg (d) 51.2 kg

Ans: (d)

Explanation:

Average Weight

$$=\frac{(50\times6+51\times2+55\times2)}{6+2+2}$$

$$=\frac{300+102+110}{10}=\frac{512}{10}=51.2\,kg$$

E17. The average weight of five people sitting in a boat is 38 kg. The average weight of the boat and the people sitting in the boat is 52kg. What is the weight of the boat?

(a) 228 kg (b) 122 kg

(c) 232 kg (d) 242 kg

Ans: (b)

Explanation:

Average weight of 5 people sitting in the boat = 38 kg

Total weight = 5 × 38 = 190 kg

Average weight of the boat and the people sitting in the boat = 52 kg

Total weight = 6 × 52 = 312 kg

Weight of Boat = (Total weight of boat and five people) - (Total weight of 5 people)

$= 6\times52-5\times38$

$=312-190$

$=122\,kg$

E18. The average of marks of 17 students in an examination was calculated as 71. But it was later found that the mark of one student had been wrongly entered as 65 instead of 56 and another as 24 instead of 50. The correct average is

(a) 70 (b) 71

(c) 72 (d) 73

Ans: (c)

Explanation:

Total correct marks of 17 students

$=17\times71-65-24+56+50$

$=1207-89+106=1224$

Required correct average = $\frac{1224}{17}=72$

E19. When the average age of a husband and wife and their son was 42 years, the son got married and a child was born just one year after the marriage. When the child turned five years old, then the average age of the family became 36 years. What was the age of the daughter-in-law at the time of marriage ?

(a) 26 years (b) 25 years

(c) 24 years (d) 23 years

Ans: (b)

Explanation:

Let H, W, S, D, C denote the initial age of husband, wife, son, daughter-in-law and child respectively.

$\frac{H+W+S}{3}$ = 42

or, H + W + S = 42 × 3 = 126 years

After 6 Years, The total age of H+W+S+D+C= 36 × 5 = 180 years

[After 6 Years, Age of Husband= H+6, Age of Wife= W+6, Age of Son= S+6, Age of wife= D+6

Total age of Husband, Wife and Son

= 126+ 6+6+6= 144]

Therefore,

144 + (D+6) + 5 = 180 (Where age of child is 5 years because he is born after one year of marriage)

D= 180-144- 6- 5= 25

Hence, the age of the daughter-in-law at the time of marriage = 25 years

Note:

1. If the average of 'm' students in a class is a, where average of passed students is x and average of failed students is y, then

   Number of students passed= $\frac{m(a-y)}{x-y}$

2. Bowling Average = $\frac{\text{Total runs given}}{\text{Total wickets taken}}$

3. Batting Average = $\frac{\text{Total runs scored}}{\text{Total number of times batsman got out}}$

PRACTICE QUESTIONS

1. **In a set of 21 numbers, the average is 15.5. The first eleven numbers have an average of 14.8, and the last eleven numbers have an average of 16.3. What is the value of the number in the middle of the set?**

   (a) 13.4 (b) –13.4

   (c) 16.6 (d) –16.6

2. **The average age of a coach and four athletes is 25 years. If all four athletes are of the same age, and the difference between the age of the coach and each athlete is 15 years, then what is the age of the coach?**

   (a) 35 years (b) 37 years

   (c) 25 years (d) 27 years

3. **The average of three numbers *p*, *q* and *r* is 15. The average of *r*, *s* and *t* is 21. The average of *t* and *u* is 26, and the average of *t* and *r* is 19. Find the average of all six numbers: *p*, *q*, *r*, *s*, *t*, and *u*.**

   (a) 19.33 (b) 21.33

   (c) 20.33 (d) 23.33

4. **The average speed of a car for the first half of a journey is 60 km/h. If the average speed for the entire journey is 70 km/h, what was the speed of the car during the second half of the journey?**

   (a) 65 km/h (b) 75 km/h

   (c) 80 km/h (d) 84 km/h

5. **The average height of 50 students is recorded as 160 cm. However, it was later discovered that the height of one student was misread as 130 cm when it should have been 170 cm. What is the corrected average height of the 50 students?**

   (a) 160.8 cm (b) 168 cm

   (c) 162.8 cm (d) 165 cm

6. **The average temperature in a city over 30 days was 25°C. After the next 10 days, the average temperature increased by 3.5°C. What was the average temperature in the last 10 days?**

   (a) 22.5°C (b) 25°C

   (c) 28.5°C (d) 39°C

7. **Out of three subjects, English, science and maths, each with maximum marks of 300, Anita secured 45% and 70% marks in the English and science tests. How many marks should Anita score in the Maths test to make her overall average equal to 85%?**

   (a) 280 (b) 270

   (c) 290 (d) Not possible

8. **In a school, the average age of teachers and students is 25 years. The average age of all 30 students is 20 years, and the average age of teachers is 30 years. If 15 teachers are married, how many unmarried teachers are there in the school?**

   (a) 15 (b) 25

   (c) 30 (d) 32

9. **The average score of Aakash and Vikas in a math quiz is 90 points. The average score of Vikas and Deepti in the same quiz is 85 points, and the average score of Aakash and Deepti is 88 points. What is Vikas's score in the quiz?**

   (a) 86 (b) 87

   (c) 88 (d) 89

10. **In a college, there are 120 employees, and their average salary is ₹ 50,000 per year. If the average salary of male employees is ₹ 60,000, and the average salary of female employees is ₹ 45,000, find out the number of male employees in the college.**

    (a) 80 (b) 60

    (c) 40 (d) 20

11. **What will be the average age of Akansha and Ekta two years from now if their current age ratio is in ratio 5:2, and three years ago, it was 7:1?**

    (a) 12 years (b) 10 years

    (c) 6 years (d) 9 years

12. **The average of four numbers is 50. The first number is three times the second number, the second number is four times the third number, and the third number is five times the fourth number. What is the difference between the largest and the smallest number?**

(a) 137.21 (b) 132.71

(c) 133.21 (d) 173.21

13. The average salary of an employee for the first three weeks of February 2023 is ₹ 2,000, and for the last two weeks, it is ₹ 2,500. If the employee's salary for the third week is ₹ 2,400, what is the employee's average weekly salary for the entire month?

(a) ₹ 2150 (b) ₹ 2350

(c) ₹ 2400 (d) ₹ 2300

14. The average weight of all the passengers on a bus is 70 kg. If the average weight of 5 passengers sitting in the front row is 80 kg, and the average weight of the rest of the passengers is 68 kg, how many passengers are there on the bus?

(a) 25 (b) 35

(c) 30 (d) 40

15. The average number of pages read by five students in a week is 50 pages. If the number of pages read by the teacher is also included, the average number of pages read per week increases by 25%. What is the number of pages read by the teacher?

(a) 125 (b) 150

(c) 175 (d) 200

16. The average speed of the first 7 cars among 8 cars is 80 km/h. The speed of the 8th car is 40 km/h more than the average speed of all 8 cars. What is the speed of the 8th car?

(a) 127.51 km/h (b) 125.71 km/h

(c) 152.71 km/h (d) 175.71 km/h

17. Two statements are given in the question below as Assertion (A) and Reason (R). Read the statements, and choose the appropriate option.

Assertion (A): If one value in a set of data is changed, the average will always change.

Reason (R): The average is sensitive to changes in individual values in the given data.

(a) Both A and R are true, and R is the correct explanation of A.

(b) Both A and R are true, but R is not the correct explanation of A.

(c) A is true, but R is false.

(d) A is false, but R is true.

18. Two statements are given in the question below as Assertion (A) and Reason (R). Read the statements, and choose the appropriate option.

Assertion (A): In a set of 10 numbers, if the average is 75, the sum of all the numbers must be 750.

Reason (R): The average is calculated by dividing the sum of all numbers by the total number of values.

(a) Both A and R are true, and R is the correct explanation of A.

(b) Both A and R are true, but R is not the correct explanation of A.

(c) A is true, but R is false.

(d) A is false, but R is true.

19. The average marks obtained by students in a class of 50 is 80. The average mark of the top 10 students in the class is 95. If the average mark of the bottom 10 students is 75, what are the average marks of the students who are neither in the top 10 or bottom 10 students in the class?

(a) 66.67 (b) 67.67

(c) 7.67 (d) 76.67

20. Two statements are given in the question below as Assertion (A) and Reason (R). Read the statements, and choose the appropriate option.

Assertion (A): If the average of three consecutive odd numbers is 21, the middle number must be 21.

Reason (R): The average of three consecutive odd numbers is a even number.

(a) Both A and R are true, and R is the correct explanation of A.

(b) Both A and R are true, but R is not the correct explanation of A.

(c) A is false, but R is true.

(d) A is true, but R is false

ANSWERS

1. (c)	2. (b)	3. (c)	4. (d)	5. (a)	6. (d)	7. (d)	8. (a)	9. (b)	10. (c)
11. (d)	12. (a)	13. (a)	14. (c)	15. (a)	16. (b)	17. (a)	18. (a)	19. (d)	20. (d)

EXPLANATIONS

1. (c)

Explanation:

The average of 21 numbers is 15.5.

Total Sum of 21 numbers = $15.5 \times 21 = 325.5$

The average of the first eleven numbers is 14.8,

Sum of First eleven numbers = $14.8 \times 11 = 162.8$

The average of the last eleven numbers is 16.3,

Sum of Last eleven numbers = $16.3 \times 11 = 179.3$

Value of Middle Number(11th) = (Sum of first eleven numbers + sum of last eleven numbers) – sum of 21 numbers

Value of Middle Number = (162.8+179.3) –325.5 = 342.1 – 325.5 = 16.6

2. (b)

Explanation:

Let the age of the coach be "x" years and the age of each athlete be "y" years.

The average age of the coach and four athletes is 25 years.

$$\frac{x+4y}{5}=25$$

$$x+4y=125$$

The difference between the age of the coach and each athlete is 15 years.

$x - y = 15$

From $x + 4y = 125$ and $x - y = 15$

We get, $x = 37$

So, the age of coach is 37 years.

3. (c)

Explanation:

Given,

$p + q + r = 15 \times 3 = 45$ ...(i)

$r + s + t = 21 \times 3 = 63$ ...(ii)

$t + u = 26 \times 2 = 52$ ...(iii)

$t + r = 19 \times 2 = 38$ ...(iv)

From Eqs. (ii) and (iv), we get

$38 + s = 63$

$s = 25$ ...(v)

Now adding Eqs. (i),(iii), (v), we get

$p + q + r + s + t + u = 45 + 25 + 52 = 122$

Average of p, q, r, s, t and $u = \frac{122}{6} = 20.33$

4. (d)

Explanation:

Let the distance for the first half of the journey as "x" and the speed during the first half be "a" = 60 km/h distance for the second half as "x" and speed during the second half be "b".

Average speed = Total distance / Total time

$70 \text{ km/h} = 2x / [(x / a) + (x / b)]$

$70 \text{ km/h} = 2ab / (a + b)$

$70 \times (60 + b) = 2(60)b$

$7 \times (60 + b) = 12b$

$5b = 420$

$b = 84$

The speed of the car during the second half of the journey is 84 km/h.

5. (a)

Explanation:

Average height of 50 students was recorded as 160 cm.

The sum of heights = $50 \times 160 = 8000\,\text{cm}$

One student's height was misread as 130 cm when it should have been 170 cm.

Difference in height = Correct height – Misread height

= 170 cm – 130 cm = 40 cm

Now, we can correct the sum of heights:

Corrected sum of heights

= Sum of heights + Difference in height

= 8000 cm + 40 cm = 8040 cm

Corrected average height

$$= \frac{\text{Corrected sum of heights}}{\text{Number of students}} = \frac{804}{50} = 160.8 \text{ cm}$$

6. (d)

Explanation:

Total temperature for 30 days = $25°C \times 30 = 750°C$

Total temperature for 40 days = $28.5°C \times 40 = 1,140°C$

Total temperature for last 10 days

$= 1,140°C - 750°C = 390°C$

Average temperature in the last 10 days

$$= \frac{390°C}{10} = 39°C$$

7. (d)

Explanation:

Total required marks in the three examinations to make an overall average of 85%

to make an overall average of 85%

$$= \frac{(3 \times 300 \times 85)}{100} = 765$$

Marks secured in English = 45% of 300

$$= \frac{(45 \times 300)}{100} = 135$$

Marks secured in Science= 70% of 300

$$= \frac{(70 \times 300)}{100} = 210$$

Thus, the required marks in Maths

= 765 – (135 + 210) = 765 – 345 = 420

420 is greater than the maximum mark (300) of the subject which is not possible.

8. (a)

Explanation:

Let the number of teachers be ''x''.

Then, $30x + 30 \times 20 = 25(30 + x)$

$\Rightarrow 30x + 600 = 750 + 25x$

$\Rightarrow 30x - 25x = 750 - 600$

$5x = 150 \quad x = 30$

Number of married teachers = 15

Number of Unmarried teachers = 30 – 15 = 15

9. (b)

Explanation:

Let scores of Aakash, Vikas, and Deepti be x, y and z respectively.

Then,

$x + y = 2 \times 90 = 180$ ...(i)

$y + z = 2 \times 85 = 170$ ...(ii)

$x + z = 2 \times 88 = 176$ ...(iii)

On adding Eqs. (i), (ii) and (iii), we get

$2(x + y + z) = 526$

$\Rightarrow x + y + z = 263$ ...(iv)

On subtracting Eq. (iii) from Eq. (iv), we get

$y = 263 - 176$

$y = 87$

10. (c)

Explanation:

Let the number of male employees = a

then, number of female employees = $120 - a$

Sum of salaries of all employees = sum of salaries of female employees + sum of salaries of male employees

$120 \times 50,000 = (120 - a) \times 45,000 + a \times 60,000$

$\Rightarrow 60,00,000 = 54,00,000 - 45,000a + 60,000a$

$\Rightarrow 15,000a = 6,00,000$

$a = 40$

∴ Number of male employees = 40

11. (d)

Explanation:

Let the present ages of Akansha be 5x and Ekta be 2x.

3 year ago, Akansha's age and Ekta's age was (5x – 3) and (2x – 3) respectively

$$\Rightarrow \frac{(5x-3)}{(2x-3)} = \frac{7}{1}$$

$(5x - 3) = 7(2x - 3)$

$5x - 3 = 14x - 21$

$9x = 18$

$x = 2$

Present ages of Akansha = 5(2) = 10 years and Ekta = 2(2) = 4 years.

Two years from now,

Akansha's age = 10 + 2 = 12 years

Ekta's age = 4 + 2 = 6 years

$\therefore$ Average age $= \frac{(12+6)}{2} = 9$ years

12. (a)

Explanation:

Let Fourth number = x

Then, third number = $5x$

Second number = 4 ($5x$) = $20x$

and first number = 3(20x) = $60x$

Average of four numbers = 50

$\Rightarrow \frac{(x+5x+20x+60x)}{4} = 50$

$\Rightarrow 86x = 50 \times 4$

$\Rightarrow x \approx 2.3256$

Difference between the largest and the smallest number = $60x - x = 59x$

$= 59(2.3256) \approx 137.2$

13. (a)

Explanation:

Total salary for the month

= Sum of salary of first three weeks + Sum of salary of last two weeks – Salary of third week

$= 3 \times 2000 + 2 \times 2500 - 2400$

= 6000 + 5000 – 2400 = 11000 – 2400 = ₹ 8600

$\therefore$ Average weekly salary for the entire month

= ₹ 8600 / 4 = ₹ 2150

14. (c)

Explanation:

Let the total number of passengers be ''p''.

The sum of weights of all passengers = sum of the weight of 5 passengers sitting in the front row + sum of weight of (p – 5) passengers sitting in the bus.

$70 \times p = 5 \times 80 + 68 \times (p - 5)$

$\Rightarrow 70p = 400 + 68p - 340$

$\Rightarrow 2p = 60$

$p = 30$

So the total number of passengers in the bus = 30

15. (a)

Explanation:

New average = 125% of old average = 125% of 50

$= \frac{(125 \times 50)}{100} = 62.5$

Number of pages read by the teacher = Number of pages read by the teacher and students – Number of pages read by the students

$= 6 \times 62.5 - 5 \times 50 = 375 - 250$

= 125 pages

16. (b)

Explanation:

Let the speed of the 8th car be ''x'' km/h.

The average speed of 8 cars $= \frac{7 \times 80 + x}{8} = \frac{560 + x}{8}$ km/h

The speed of the 8th car is 40 more than the average:

$x = \frac{560 + x}{8} + 40 = \frac{560 + x + 320}{8}$

$x = \frac{x + 880}{8}$

$8x = x + 880$

$7x = 880$

$x = 125.71 km/h$

17. (a)

Explanation:

Both the assertion and reason are true, and R is the correct explanation of A.

If one value in a data set is changed, the average will always change because the average is sensitive to changes in individual data points. The reason correctly explains why the assertion is true.

18. (a)

Explanation:

If the average of 10 numbers is 75,

the sum of all the numbers = 10×75 = 750

Thus, the Assertion(A) is true.

In a set of numbers, the average is calculated by dividing the sum of all numbers by the total number of values.

Thus, Reason (R) is true and Reason (R) is the correct explanation of Assertion (A).

19. (d)

Explanation:

The average marks of the entire class of 50 students are 80.

Total marks for 50 students = 80×50 = 4000

Total marks for top 10 students = 95×10 = 950

Total marks for bottom 10 students = 75×10 = 750

Total marks of top 10 and bottom 10 students = 950 + 750 = 1700

Total marks for remaining 30 students = 4000 – 1700 = 2300

Average marks of the remaining 30 students are = 2300 / 30 = 76.6666 ≈ 76.67

20. (d)

Explanation:

The average of three consecutive odd numbers is 21.

Sum of these three numbers = 3 × 21 = 63.

Let the three consecutive odd numbers be

$(x – 2)$, x and $(x + 2)$, where x is the middle number.

$(x – 2) + x + (x + 2) = 63$

$x = 21$

Hence, Assertion is true.

The reason provided is irrelevant and incorrect.

As, the average of three consecutive odd numbers will be an odd number. So the reason is wrong.

Percentage

7

INTRODUCTION

Percentage is a number or ratio expressed as a fraction. "Per cent" is derived from the latin words "per centum", which means "per hundred".

A percentage is a fraction with denominator hundred. Symbol % is used to denote percentage and the numerator of the fraction is called the rate per cent.

For example: 43 percentage is written as 43 % and it means $\frac{43}{100}$.

- x as a percent of y can be expressed as: $\frac{x}{y} \times 100$.
- y % of $x = \frac{y}{100} \times x$.

Example:

Calculate 35% of 350.

y % of $x = \frac{y}{100} \times x$

$x = 350$, $y = 35\%$

$35 \text{ \% of } 350 = \frac{35}{100} \times 350$

$= \frac{35}{10} \times 35$

$= \frac{1225}{10} = 122.5$

CONVERSION OF FRACTION INTO EQUIVALENT PERCENTAGE

To Convert a fraction/Decimal into a percentage multiply it by 100.

Example:

1. Express $\frac{5}{7}$ in percentage form.

   $\frac{5}{7} \times 100 = 5 \times \frac{1}{7} \times 100 = 5 \times 14.28\%$

   $= 71.43\%$

2. Write 0.725 in percentage form.

   0.725 in percentage = $0.725 \times 100 = 72.5\%$

CONVERSION OF PERCENTAGE INTO FRACTION

To convert a percent into a fraction divide it by 100.

Example:

1. Write 7.5% in fraction form.

   $7.5\% = \frac{7.5}{100} = \frac{3}{40}$

2. Convert 26.66% into fraction.

   26.66% can be written as:

   $20\ \% + 6.66\% = \frac{1}{5} + \frac{1}{15}$

   $= \frac{3+1}{15}$

   Hence, $20\ \% + 6.66\% = \frac{4}{15}$

FRACTION PERCENTAGE CHART

Fraction	Percentage
1	100%
$\frac{1}{2}$	50%
$\frac{1}{3}$	33.33%
$\frac{1}{4}$	25%
$\frac{1}{5}$	20%
$\frac{1}{6}$	16.66%
$\frac{1}{7}$	14.28%
$\frac{1}{8}$	12.5%
$\frac{1}{9}$	11.11%
$\frac{1}{10}$	10%

$\frac{1}{11}$	9.09%
$\frac{1}{12}$	8.33%
$\frac{1}{13}$	7.69%
$\frac{1}{14}$	7.14%
$\frac{1}{15}$	6.66%
$\frac{1}{16}$	6.25%
$\frac{1}{17}$	5.88%
$\frac{1}{18}$	5.55%
$\frac{1}{19}$	5.26%
$\frac{1}{20}$	5%

APPLICATION OF PERCENTAGE

TYPE 1: BASED ON INCREASE/ DECREASE OF A QUANTITY

1. If amount/quantity is increased, then

$$\%\ \text{Increase} = \frac{\text{Increase in value}}{\text{Original Value}} \times 100$$

Let p and q be the initial and final amount such that q is greater than p.

$$\%\text{Increase} = \frac{q-p}{p} \times 100$$

Example: If the fees for a mathematics course is increased from Rs. 5000 to Rs. 6500. Calculate the percentage increase in the fees.

Initial fees = Rs.5000

New fees = Rs. 6500

Increase in fees = Rs. 6500 – Rs. 5000 = Rs. 1500

$$\%\text{Increase} = \frac{1500}{5000} \times 100 = 30\ \%.$$

2. If amount/quantity is decreased, then

$$\%\text{Decrease} = \frac{\text{Decrease in value}}{\text{Original Value}} \times 100$$

3. If an amount is increased by *x*% and then it is decreased by *x*% again, then there is always an overall percentage decrease which is equal to:

$$\%\,Decrease = \frac{x^2}{100}\ \%$$

4. If an amount is increased by *x*% and then it is decreased by *y*% again, then the resultant change in percentage is,

$$\%\,change = \left(x - y - \frac{xy}{100}\right)\%$$

5. If an amount is decreased by x% and then it is decreased by y% again, then the resultant change in percentage is,

$$\%\,change = \left(-x - y + \frac{xy}{100}\right)\%$$

6. If an amount is increased by x% and then it is increased by y% again, then the resultant change in percentage is,

$$\%\,change = \left(x + y + \frac{xy}{100}\right)\%$$

7. If an amount is increased by x% and then again increased by y% and finally increased by z%, So that resultant quantity is 'A', then,

$$\text{Initial quantity} = \frac{100 \times 100 \times 100 \times A}{(100+x)(100+y)(100+z)}$$

QUESTIONS

Q1. Price of an article is increased by 25% and then it is decreased by 20% , then the resultant change in the price of the article in percentage is equal to

(a) +5% (b) –25%

(c) +20% (d) None of these

Ans: (d)

Explanation:

$$\%\text{change in the price} = \left(x - y - \frac{xy}{100}\right)\%$$

Given, x = 25 %, y = 15 %

$$\%\text{change} = \left(25 - 20 - \frac{(25)(20)}{100}\right)\%$$

$$= (25 - 20 - 5)\ \% = (25 - 25)\% = 0$$

Hence the price of the article will remain the same.

Q2. A number *x* is increased by 20%, then by 30% and finally increased by 50%. If the resultant number is 234 then find the value of *x*.

(a) 100 (b) 184

(c) 143 (d) 134

Ans: (a)

Explanation:

Using Formula:

Initial quantity = $\frac{100\times100\times100\times A}{(100+x)(100+y)(100+z)}$

Initial number = x, A = 234

Number (x)= $\frac{100\times100\times100\times234}{(100+20)(100+30)(100+50)}$

$= \frac{100\times100\times100\times234}{120\times130\times150}$

$= \frac{1000\times234}{12\times13\times15}$

= 100.

Q3. A and B are two heavy steel blocks. If B is placed on the top of A, then weight increases by 60%. How much weight will reduce with respect to the total weights of A and B, if B is removed from the top of A? **(UPSC CSAT 2019)**

(a) 60% (b) 45.5%

(c) 40% (d) 37.5%

Ans: (d)

Explanation:

Let the weight of block A is 100 kg.

Then, after placing B on A, the total weight becomes

= 100 + 60 = 160 kg

Hence the weight of Block B

= 160 – 100 = 60 kg

Now if B is removed from top, then reduction in weight (percent) with respect to total weight of A and B

$= \frac{160-100}{160}\times100$

$= \frac{60}{160}\times100 = 37.5\%$

Note:

For such questions one can also assume a convenient known value for an unknown variable in the question to arrive at the answer.

TYPE 2: BASED ON INCOME/ EXPENDITURE

1. If the cost of an article is increased by *x*%, then how much to decrease the consumption of article, so that expenditure remains same is given by,

   % decrease in consumption $= \left(\frac{x}{100+x}\times100\right)\%$

2. If the cost of an article is decreased by x%, then how much to increase the consumption of article, so that expenditure remains same is given by,

   % increase in consumption $= \left(\frac{x}{100-x}\times100\right)\%$

3. If the income of a person *A* is *x*% more than person *B*, then income of *B* is less in comparison to the *A* by, $\left(\frac{x}{100+x}\times100\right)\%$.

4. If the income of a person *A* is *x*% less than person *B*, then income of *B* is more in comparison to the *A* by, $\left(\frac{x}{100-x}\times100\right)\%$.

QUESTIONS

Q4. If the price of an article is increased by 25%, then by what percent the consumption is to be decreased so that there is no change in expenditure?

(a) 20% (b) 25 %

(c) 12.5% (d) 10 %

Ans: (a)

Explanation:

% decrease in consumption

$= \left(\frac{x}{100+x}\times100\right)\%$

x = 25%

Hence, % decrease in consumption

$= \left(\frac{25}{100+25}\times100\right)\%$

$= \left(\frac{25}{125}\times100\right)\% = 20\%$

Q5. The full tank petrol in Arun's motorcycle lasts for 10 days. If he starts using 25% more everyday, how many days will the tank full petrol last?

(UPSC CSAT 2013)

(a) 5 (b) 6

(c) 7 (d) 8

Ans: (d)

Explanation:

Let us assume that Arun uses x units of petrol everyday. So, the amount of petrol in the tank when it is full will be 10x. If he starts using 25% more petrol everyday then the units of petrol he now used everyday will be

$$x\left(1+\frac{25}{100}\right)=1.25x$$

So, the number of days his petrol will now last

$$=\frac{\text{Amount of Petrol in Tank}}{\text{Number of units used everyday}}$$

$$=\frac{10x}{1.25x}=8$$

Q6. In a city, 12% of households earn less than Rs. 30000 per year, 6% households earn more than Rs. 200000 per year, 22% households earn more than Rs. 100000 per year and 990 households earn between Rs. 30000 and Rs. 100000 per year. How many households earn between Rs. 100000 and Rs. 200000 per year? **(UPSC CSAT 2017)**

(a) 250 (b) 240

(c) 230 (d) 225

Ans: (b)

Explanation:

According to question,

6% of households earn> Rs. 200000.

22% of households earn> Rs. 100000.

Therefore, (22 – 6)

= 16% of households income is between Rs. 100000 to Rs. 2000000.

So, 12% + 990 + 16% + 6% = 100%

$\Rightarrow$ 990 = (100 – 34)%

$\Rightarrow$ 990 = 66%

$\Rightarrow$ 1%= 15

Hence, 16% = $\frac{990}{66}\times 16$ = 240

So, 240 households earn between Rs. 100000 and Rs. 20000.

TYPE 3: BASED ON POPULATION/ PRICE/ QUANTITY

1. If the "population/cost" of a "town/article", is P and annual increment is r%, then

   - After 't' years population/cost $=P\left(1+\frac{r}{100}\right)^t$
   - Before 't' years population/cost $=\frac{P}{\left(1+\frac{r}{100}\right)^t}$

2. If the "population/cost" of a "town/article" is P and it decreases at the rate of r% annually, then,

   - After 't' years population/cost $=P\left(1-\frac{r}{100}\right)^t$
   - Before 't' years population/cost = $\frac{P}{\left(1-\frac{r}{100}\right)^t}$

3. If the present population of a town is P and the population increases or decreases at rate of x%, y% and z% in the first, second and third year respectively.

   - Then the population of town after 3 years $=P\left(1\pm\frac{x}{100}\right)\left(1\pm\frac{y}{100}\right)\left(1\pm\frac{z}{100}\right)$

   + sign indicates increase in population.

   – Sign indicates decrease in population.

4. If x% of a quantity is taken by 1st person and y% of remaining sum is taken by 2nd person and z% of remaining sum is taken by 3rd person, then if the remaining quantity is 'A' then,

   Initial quantity = $\frac{100\times 100\times 100\times A}{(100-x)(100-y)(100-z)}$

QUESTIONS

Q7. The population of a town decreases at the rate of 10% for the first year, it increases at the rate of 22% for the second year and for the third year it is increased at the rate of 15%. Then what will be the population after 3 years if the present population of the town is 3250000?

(a) 4107375 (b) 4103775

(c) 4105773 (d) 4107735

Ans: (b)

Explanation:

Using formula:

Population after 3 years

$$= P\left(1-\frac{x}{100}\right)\left(1+\frac{y}{100}\right)\left(1+\frac{z}{100}\right)$$

Given, $P = 3250000$,

$x = 10\%$, $y = 22\%$, $z = 15\%$

Population of town after 3 years

$$= P\left(1-\frac{10}{100}\right)\left(1+\frac{22}{100}\right)\left(1+\frac{15}{100}\right)$$

$$= 3250000\left(\frac{90}{100}\right)\left(\frac{122}{100}\right)\left(\frac{115}{100}\right)$$

= 3250000(0.9) (1.22) (1.15)

= 4103775

Q8. In the adult population of a city, 40% men and 30% women are married. What is the percentage of the married adult population if no man marries more than one woman and no woman marries more than one man; and there are no windows and widowers? **(UPSC CSAT 2020)**

(a) $31\frac{1}{7}\%$ (b) 34%

(c) $34\frac{2}{7}\%$ (d) 35%

Ans: (c)

Explanation:

Let the total adult population of the city be z, with the number of men are x and the number of women are y.

According to question,

0.4 x = 0.3 y

$$\therefore \frac{x}{y} = \frac{3}{4} \quad \text{...(i)}$$

and $x + y = z$ ...(ii)

$$\Rightarrow x = \frac{3z}{7} \text{ and } y = \frac{4z}{7}$$

Hence, number of married men

$$= 0.4 * x = 0.4 \times \frac{3z}{7} = 1.2\frac{z}{7}$$

and number of married women

$$= 0.3 * x = 0.4 \times \frac{4z}{7} = 1.2\frac{z}{7}$$

Therefore, population of married adults

$$= \frac{2.4z/7}{z} \times 100$$

$$\frac{240}{7}\% = 34\ \frac{2}{7}\%$$

Q9. A person ordered 5 pairs of black socks and some pairs of brown socks. The price of a black pair was thrice that of a brown pair. While preparing the bill, the bill clerk interchanged the number of black and brown pairs by mistake which increased the bill by 100%. What was the number of pairs of brown socks in the original order? **(UPSC CSAT 2015)**

(a) 10 (b) 15

(c) 20 (d) 25

Ans: (d)

Explanation:

Assume that the person bought x pairs of brown socks and the price of each brown pair is Rs y.

Given, the price of each black pair be 3y

So, total cost = $5 \times 3y + x \times y$

and changed cost = $5 \times y + x \times 3y$

According to the question,

$$100 = \frac{5y + 3xy - (15y + xy)}{15y + xy} \times 100$$

$\Rightarrow 5y + 3xy = 15y + xy + 15y + xy$

$\Rightarrow 5y + 3xy = 30y + 2xy$

$\Rightarrow xy = 25y$

Hence $x = 25$

Therefore, the number of pairs of brown socks was 25.

TYPE 4: BASED ON EXAMINATION MARKS

1. The percentage of passing marks in an examination is a %. If a candidate who scores b marks fails by c marks, then the maximum marks(M) are given by,

$$M = \frac{100(b+c)}{a}$$

2. An examinee scored x% marks in an exam, and failed by 'a' mark. In the same examination another examinee obtained y% marks and passed with 'b' more marks than minimum, then,

$$\text{Maximum marks} = \frac{100}{(y-x)} \times (a+b)$$

3. In a certain examination, '*x*' boys and '*y*' girls participated. *a*% of boys and *b*% of girls passed the examination, then,

Percentage of passed students of the total students

$$= \left(\frac{x.a + y.b}{x+y}\right)\%$$

QUESTIONS

Q10. In an examination, A has scored 20 marks more than B. If B has scored 5% less marks than A, how much has B scored? (UPSC CSAT 2019)

(a) 360 (b) 380

(c) 400 (d) 420

Ans: (b)

Explanation:

Assume that *B* scored *x* marks.

Then, the score of *A* is = (*x* + 20) marks.

According to the question, $\frac{x+20-x}{x+20} = \frac{5}{100}$

$\Rightarrow \frac{20}{x+20} = \frac{5}{100}$

$\Rightarrow x = 380$

So, B scored 380 marks.

Q11. A student has to get 40% marks to pass in an examination. Suppose he gets 30 marks and fails by 30 marks, then what are the maximum marks in the examination? (UPSC CSAT 2018)

(a) 100 (b) 120

(c) 150 (d) 300

Ans: (c)

Explanation:

Method I: Let x be the total marks. So, 40% of x = passing marks. Given, the student got 30 marks and failed by 30 marks. So,

Therefore passing mark = 60 marks

$\therefore 60 = \frac{40}{100} \times x$

$\therefore x = 150$ marks

So, maximum marks in the examination would be 150.

Method II: 40% marks is required to pass an examination.

The percentage of passing marks in an examination is a %. If a candidate who scores b marks fails by c marks, then the maximum marks(M)

$$M = \frac{100(b+c)}{a}$$

In this case, $a = 40\%$, $b = 30$, $c = 30$

Hence, Maximum marks(M)

$$= \frac{100(30+30)}{40} = \frac{100 \times 60}{40} = 150$$

Q12. Anita's mathematics test had 70 problems carrying equal marks, i.e., 10 arithmetic, 30 algebra and 30 geometry. Although she answered 70% of the arithmetic, 40% of the algebra and 60% of the geometry problems correctly, she did not pass the test because she got less than 60% marks. The number of more questions she would have to answer correctly to earn a 60% passing marks, is (UPSC CSAT 2016)

(a) 1 (b) 5

(c) 7 (d) 9

Ans: (b)

Explanation:

Let's assume that each question carries 1 mark.

Now, the number of questions attempted by Anita = 70% of 10 + 40% of 30 + 60% of 30

= 7 + 12 + 18 = 37.

Given also, passing marks = 60% of 70 = 42.

So, required number of questions to be attempted

= 42 – 37 = 5

Q13. In a test, a candidate attempted only 8 questions and secured 50% marks in each of the questions. If he obtained a total of 40% in the test and all questions in the test carried equal marks, how many questions were there in the test? (UPSC CSAT 2015)

(a) 8 (b) 10

(c) 15 (d) 16

Ans: (b)

Explanation:

Let's assume that each question carried x marks

Then, student's total score

= 8 × 0.5*x* = 4*x* marks

Let total number of questions in test were n, so total marks of test was = *nx* marks

Hence as per the question,

4x = 40% of *nx*

$\Rightarrow 4x = (40/100) \times nx$

$\Rightarrow 4x = 0.4 \times nx$

Thus, *n* = 10; implying a total of 10 questions in the test.

Q14. Candidates in a competitive examination consisted of 60% men and 40% women, 70% men and 75% women cleared the qualifying test and entered the final test where 80% men and 70% women were successful. **(UPSC CSAT 2015)**

Which of the following statements is correct?

(a) Success rate is higher for women

(b) Overall success rate is below 50%

(c) More men cleared the examination than women

(d) Both (A) and (B) above are correct.

Ans: (c)

Explanation:

Let there are 100 persons in which 60 are men and 40 are women

So, number of men who cleared the qualifying test

$$=60\times\frac{70}{100}=42$$

Number of women who cleared the qualifying test

$$=40\times\frac{75}{100}=30$$

Number of men who got success in final test

$$=42\times\frac{80}{100}=33.6$$

Number of women who got success in final test

$$=30\times\frac{70}{100}=21$$

Hence, more men cleared the examination than women.

Q15. In an examination, every candidate took Physics or Mathematics or both. 65.8% took Physics and 59.2% took Mathematics. The total number of candidates was 2000. How many candidates took both Physics and Mathematics? **(UPSC CSAT 2001)**

(a) 750 (b) 500

(c) 250 (d) 125

Ans: (b)

Explanation:

Let z% candidates take both the subjects.

Percentage of candidates who opted for Physics = 65.8% (This includes students who opted for only Physics(x) and those who opted for Physics and Maths(z))

Percentage of candidates who opted for Mathematics = 59.2% (This includes students who opted for only Mathematics(y) and those who opted for Physics and Maths(z))

Therefore, $(x + z) + (y + z) = 65.8 + 59.2 = 125\%$

However, $x + y + z = 100\%$

So, $z = 125 - 100 = 25\%$

Now, total number of candidates = 2000.

So, number of candidates who opted both the subjects = 25% of 2000

$$=\frac{25\times2000}{100}=500$$

Q16. In an examination, 70% of the students passed in the paper I, and 60% of the students passed in the paper II. 15% of the students failed in both the papers while 270 students passed in both the papers. What is the total number of students? **(UPSC CSAT 2008)**

(a) 600 (b) 580

(c) 560 (d) 540

Ans: (a)

Explanation:

Percentage of students failed in paper I

= 100 – 70 = 30%

Percentage of students failed in paper II

= 100 – 60 = 40%.

Total percentage of students who failed

= 30 + 40 – 15 = 55%

Total percentage of students who passed in both papers 100 – 55 = 45%.

Let, total number of students be x. So,

45% of x = 270

$$\Rightarrow\frac{45\times x}{100}=270$$

$$\Rightarrow x = 600$$

TYPE 5: BASED ON GEOMETRICAL FIGURES

1. If the sides of an equilateral triangle, square, rhombus or radius of a circle are increased by a%, then its area is increased by $\frac{a(a+200)}{100}\%$

2. If the sides of an equilateral triangle, square, rhombus or radius of a circle decreased by a % then its area is decreased by $\frac{a(200-a)}{100}\%$

3. If the length of a rectangle is increased by x% and breadth is increased by y%, then the area of rectangle will increase by,

$$\%\text{change}=\left(x+y+\frac{xy}{100}\right)\%$$

$$\text{Percentage error}=\frac{\text{Error}}{\text{True Value}}\times100\ \%$$

SUCCESSIVE PERCENTAGE CHANGE

If a quantity X is increased/decreased successively by a% followed by b% and then c % then the final value of X is

$$X\left(1\pm\frac{a}{100}\right)\left(1\pm\frac{b}{100}\right)\left(1\pm\frac{c}{100}\right)$$

QUESTIONS

Q17. If the radius of a circle is increased by 25% then the percentage increase in the area is equal to

(a) 50.5% (b) 52.25%

(c) 56.25% (d) 40%

Ans: (c)

Explanation:

If radius of a circle are increased by a%, then

Area is increased by $\frac{a(a+200)}{100}\%$

a = 25 %

Therefore % increase in area = $\frac{25(25+200)}{100}\%$

$=\frac{(225)}{4}\%=56.25\%$

Q18. Length, breadth and height of a cuboid are increased by 10%, 20%, 25% respectively then percentage increase in the volume of cuboid is equal to

(a) 55% (b) 45%

(c) 50% (d) 65%

Ans: (d)

Explanation:

Using formula:

If the length, breadth and height of a cuboid are increased by x %, y %, z% respectively, then,

% Increase in volume

$$=\left[x+y+z+\frac{xy+yz+xz}{100}+\frac{xyz}{(100)^2}\right]\%$$

x = 10%, y = 20%, z = 25%

% Increase in volume

$$=\left[10+20+25+\frac{(10)(20)+(20)(25)+(10)(25)}{100}+\frac{(10)(20)(25)}{(100)^2}\right]\%$$

$$=\left[55+\frac{200+500+250}{100}+\frac{5000}{(100)^2}\right]\%$$

$$=\left[55+\frac{950}{100}+\frac{5000}{10000}\right]\%$$

$=[55+9.5+0.5]\%$

= 65%

TYPE 6: MISCELLANEOUS QUESTIONS

QUESTIONS

Q19. When a runner was crossing the 12km mark, she was informed that she had completed only 80% of the race. How many kilometers was the runner supposed to run in this event? (UPSC CSAT 2019)

(a) 14 (b) 15

(c) 16 (d) 16.5

Ans: (b)

Explanation:

Let the total length of the race be x km.

$\therefore$ 80% of x = 12

$$\Rightarrow\frac{80}{100}\times=12$$

$$\Rightarrow x=\frac{12\times100}{80}=15$$

Hence, the runner was supposed to run 15 km in this event.

Q20. A and B decide to travel from place X to place Y by bus. A has Rs. 10 with him and he finds that it is 80% of the bus fare for two persons. B finds that he has Rs. 3 with him and hands it over to A. In this context, which one of the following statements is correct? (UPSC CSAT 2014)

(a) Now, the money A has is just enough to buy two tickets

(b) A still needs Rs. 2 for buying the tickets

(c) After buying the two tickets A will be left with 50 paise

(d) The money A now has is still not sufficient to buy two tickets

Ans: (c)

Explanation:

Let the fair of the bus for two passengers be Rs. x.

Now, A has Rs. 10, i.e., 80% of total fair, so

80% of x = 10

$$\Rightarrow\frac{80}{100}\times=10$$

$$\therefore\ x=\frac{10\times100}{80}=12.5$$

Now, B has given Rs. 3.

So, total amount A has = 10 + 3 = Rs. 13.

Money left after purchasing two tickets

= 13 – 12.5

= Rs. 0.5 = 50 paise

So, after buying the two tickets, A will be left with 50 paise.

Q21. There are 100 students in a particular class. 60% students play cricket, 30% students play football and 10% students play both the games. What is the number of students who play neither cricket nor football? **(UPSC CSAT 2011)**

(a) 25 (b) 20

(c) 18 (d) 15

Ans: (b)

Explanation:

Out of 100 students, 60 students play cricket, 30 students play football and 10 students play both So, number of students who play neither cricket nor football

= [100 – (60 + 30 – 10)]

= 100 – 80

= 20

Q22. In a group of persons, 70% of the persons are male and 30% of the persons are married. If two sevenths of the males are married, what fraction of the females is single? **(UPSC CSAT 2011)**

(a) $\frac{2}{7}$ (b) $\frac{1}{3}$

(c) $\frac{3}{7}$ (d) $\frac{2}{3}$

Ans: (d)

Explanation:

Let us assume there are 100 persons in the group, out of which 70 will be male and 30 will be female. Out of male, two sevenths are married, that is 20 males are married. Since, total married persons are 30, this implies 10 females are married. So, fraction of unmarried females $=\frac{20}{30}=\frac{2}{3}$

Q23. Two numbers X and Y are respectively 20% and 28% less than a third number Z. By what percentage is the number Y less than the number X? **(UPSC CSAT 2010)**

(a) 8% (b) 9%

(c) 10% (d) 12%

Ans: (c)

Explanation:

Let value of Z be 100

Then, X = Z – 20 = 100 20 = 80

Y = Z – 28 = 100 – 28 = 72

So, required percentage

$=\frac{Y-X}{X}\times 100$

$=\frac{72-80}{80}\times 100=-10\%$

Hence, Y is 10% less than the number X.

Method II: If two numbers are respectively x% and y% less than the third number,

Second number as a percentage of first is

$\left(\frac{100-y}{100-x}\times 100\right)\%$

x = 20 % and y = 28 %

$=\left(\frac{100-28}{100-20}\times 100\right)\%$

$=\left(\frac{72}{80}\times 100\right)\%$

= 90%

Second number as a percentage of the first is 90 %.

Therefore it is 10 % less than the first number.

Q24. Two candidates X and Y contested an election. 80% of voters cast their vote and there were no invalid votes. There was no NOTA (None of the above) option. X got 56% of the votes cast and won by 1440 votes. What is the total number of voters in the voters list?

(a) 15000 (b) 12000

(c) 9600 (d) 5000

Ans: (a)

Explanation:

Let the total number of voters in the voter list be a.

Number of casted votes = 80% of a = 0.8a

X got 56% of the casted votes.

Thus percentage of votes gained by Y = 44% (as there were no invalid or NOTA)

Winning percentage of X = (56 – 44)%, i.e. 12% of the casted votes.

Now as per given statement

12% of 0.8a = 1440

Or $a=\left(\frac{144\times 100}{0.8\times 12}\right)=15000$

Accordingly, the total number of voters in the voter list =15000 voters.

PRACTICE QUESTIONS

1. **How much lower are marks of Aakash compared to marks of Aarti if Aarti's marks are 20% higher than Aakash?**
   (a) 16.67% (b) 15.66%
   (c) 20.56% (d) 25.07%

2. **A student multiplied a number by $\frac{2}{5}$ instead of $\frac{5}{2}$ then the percentage error in the calculation is**
   (a) 82.5% (b) 80%
   (c) 72.5% (d) 84%

3. **A number exceeds 60% of itself by 64 then the number is?**
   (a) 640 (b) 400
   (c) 124 (d) 160

4. **What number when decreased by 45% and then added to 225 equals the original number?**
   (a) 280 (b) 350
   (c) 500 (d) 550

5. **If 50 % of *P* = 25% of *Q*, then *P* = *x* % of Q. Find *x*.**
   (a) 125 (b) 25
   (c) 150 (d) 50

6. **The cost of a smartphone is ₹80000. It appreciates in value by 15% during the first year, then depreciates by 8% during the second year, and finally appreciates by 12% during the third year. What will be the approximate value of the smartphone after 3 years?**
   (a) ₹ 94900 (b) ₹ 94700
   (c) ₹ 94800 (d) ₹ 94600

7. **Ramesh plans to purchase a laptop that costs ₹ 40,000. However, he realizes that he only has 60% of the required amount. To cover the remaining cost, he borrows ₹ 10,000 from his brother. What percentage of the laptop's price does Ramesh have after borrowing the money?**
   (a) 85% (b) 80%
   (c) 75% (d) 70%

8. **In a school, 85% of the students passed their final examination, and 45 students failed. What was the total number of students who took the examination?**
   (a) 130 (b) 300
   (c) 260 (d) 330

9. **In a mathematics test, a student needs to score at least 50% to pass. If a student scores 60 marks and fails by 20 marks, what are the maximum marks possible for the test?**
   (a) 80 (b) 100
   (c) 140 (d) 160

10. **The population of a town decreases at the rate of 8% per annum. Two years ago if its population was 25,000 then what is the present population of the town?**
    (a) 14600 (b) 18500
    (c) 20160 (d) 21160

11. **If a number is reduced by 35% and then the resulting value is increased by 35%, what is the overall percentage change?**
    (a) 25% (b) 17.5%
    (c) 12.25% (d) No change

12. **B's salary is 40% higher than A's, and C's salary is 25% lower than B's. By what percentage is C's salary higher than A's?**
    (a) 5% (b) 7.5%
    (c) 15% (d) 10%

13. **The population of a city increases by 20% in the first year, then decreases by 20% in the second year, and the same pattern repeats for the third and fourth years. If the current population is 15000, what will be the population after four years?**
    (a) 15884 (b) 13284
    (c) 14584 (d) 13824

14. **A news network broadcasts news bulletins from 1:00 p.m. to 10:00 p.m. In that period, they show 10 live bulletins, each lasting 30 minutes, and 20 highlight reels, each lasting 10 minutes. What percentage (in approximate terms) of the total airtime in a day is dedicated to live bulletins and highlight reels?**
    (a) 90% (b) 87%
    (c) 97% (d) 93%

15. **260% of a number is 52. What is 52% of the same number?**
    (a) 10.4 (b) 13.2
    (c) 12. 6 (d) Cannot be determined

16. **The tuition fee for a college course accounts for 70% of the books for a student. If the cost of books decreases by 15%, but the tuition fee increases by 25%, what will be the percentage change (approximately) in the total expenses for a student if the total expenses equals the combined cost of tuition fee and the cost of book?**

(a) 2.3% (b) 1.5%

(c) 5.6% (d) 4.4%

17. **In a book comprising 40 pages, each page contains 32 lines with 21 characters on each line. If this content is transcribed into a different notebook with 40 lines on each page and 28 characters per line, what percentage change will there be in the number of pages in the new notebook compared to the original one?**

(a) 40% (b) 25%

(c) 50% (d) No change

18. **The difference between the value of a number increased by 30% and the value of the original number decreased by 45% is 69. What is the original number?**

(a) 86 (b) 80

(c) 92 (d) 95

19. **Student A received 28% of the maximum marks in the exam and failed by 23 marks. On the other hand, student B scored 45% of the total marks and exceeded the passing marks by 28. What were the passing marks for this examination?**

(a) 84 (b) 99

(c) 107 (d) 112

20. **The current population of tigers in a country is 4400. If the number of males increases by 10% and the number of females increases by 22%, the population will become 5068. What is the current population of female tigers in the country?**

(a) 1900 (b) 2500

(c) 1700 (d) 2100

ANSWERS

1. (*a*)	2. (*d*)	3. (*d*)	4. (*c*)	5. (*d*)	6. (*c*)	7. (*a*)	8. (*b*)	9. (*d*)	10. (*d*)
11. (*c*)	12. (*a*)	13. (*d*)	14. (*d*)	15. (*a*)	16. (*b*)	17. (*a*)	18. (*c*)	19. (*c*)	20. (*a*)

EXPLANATIONS

1. (a)

Explanation:

Let Aakash's marks as "*x*" and Aarti's marks as "*y*".

Aarti's marks are 20% higher than Aakash. Thus,

$y = x + 0.20x = 1.20x$

Aakash's marks are compared to Aarti's and expressed in terms of it $y - x = 1.20x - x = 0.20x$

In terms of percentage,

Aakash's marks as compared to Aarti

$$=\left(\frac{0.20x}{1.20x}\right)*100=16.67\%$$

So, Aakash's marks are 16.67% lower than Aarti's marks.

2. (d)

Explanation:

Let the original number be '*a*'

$$\text{Error}=\frac{5}{2}a-\frac{2}{5}a$$

$$=\frac{25-4}{10}a=\frac{21}{10}a$$

$$\text{Hence the percentage error is } \frac{\frac{21a}{10}}{\frac{5a}{2}}\times 100=\frac{21}{10}\times\frac{2}{5}\times 100$$

$= 0.84 \times 100 = 84\%$

3. (d)

Explanation:

Let the required number be N.

Then, N – 60% of N = 64

$$\Rightarrow N-\frac{60}{100}\times N=64$$

$$\Rightarrow \frac{40}{100}\times N=64$$

$$\Rightarrow N=\frac{64\times 100}{40}=160$$

4. (c)

Explanation:

Let 'x' be the required number

$\Rightarrow 0.55x + 225 = x$

$\therefore 0.45x = 225$

$x = \frac{225}{0.45} = 500$

5. (d)

Explanation:

Given, 50% of P is equal to 25% of Q

$\frac{50}{100}P = \frac{25}{100}Q$

$0.50P = 0.25Q$

P as a percentage of Q (P = x% of Q)

$P = \frac{0.25}{0.50}Q$

$P = \frac{1}{2}Q$

$P = \frac{1}{2}Q \times 100 = 50\%Q$

So, P is 50% of Q.

Therefore, $x = 50$.

6. (c)

Explanation:

Cost of a smartphone is ₹ 80000.

Value of the smartphone after 3 years

$= 80000\left(1+\frac{15}{100}\right)\left(1-\frac{8}{100}\right)\left(1+\frac{12}{100}\right)$

$= 80000(1.15)(0.92)(1.12)$

= ₹ 94796.8 ≅ ₹ 94800

7. (a)

Explanation:

The amount Ramesh initially has = 60% of the laptop's price = 60% of ₹ 40,000 = ₹ 24,000

Total amount Ramesh has after borrowing = Initial amount + Borrowed money = ₹ 24,000 + ₹ 10,000 = ₹ 34,000

Percentage of laptop's price that Ramesh has after borrowing

$= \frac{\text{Total amount}}{\text{Laptop's price}} \times 100 = \frac{34,000}{40,000} \times 100 = 85\%$

Therefore, Ramesh has 85% of the laptop's price after borrowing the money.

8. (b)

Explanation:

If the total number of students be 'x', then

The number of students who failed the exam is–

15% of x = 45

$x = \frac{45}{15} \times 100 = 300$

9. (d)

Explanation:

Let ''x'' be the maximum marks in the examination.

The student must get 50% marks to pass.

Passing marks (50% of maximum marks) = $0.50x$

The student gets 60 marks and fails by 20 marks.

Marks obtained by the student = Passing marks – 20

$60 = 0.50x - 20$

$0.50x = 60 + 20 = 80$

$x = 160$

10. (d)

Explanation:

The population of a town decreases at the rate of 8% per annum.

Population (2 years ago) = 25000

Present Population = $25000\left(1-\frac{8}{100}\right)^2$

$= 25000 \times (0.92)^2 = 21160$

11. (c)

Explanation:

Let the original value be 100.

A decrease of 35% will make it 65.

Now, an increase of 35% in 65 = 1.35 × 65 = 87.75.

Change in the value = 100 – 87.75 = 12.25

Therefore, Overall percentage change = 12.25%

12. (a)

Explanation:

Let A's salary be Rs100

B's salary is 40% higher than A's

⇒ B's salary = Rs140

C's salary is 25% lower than B's

⇒ C's salary = 0.75×140 = Rs105

C's salary higher than A's (Percentage)

= [(105 – 100)/100]×100 = 5%

13. (d)

Explanation:

Population after 1st year = 120% of 15000

Population after 2nd year = 80% of Population after 1st year

Population after 3rd year = 120% of Population after 2nd year

Population after 4th year = 80% of Population after 3rd year

i.e.,1.2 × 0.8 × 1.2 × 0.8 × 15000 = 13824

14. (d)

Explanation:

Total time from 1:00 p.m. to 10:00 p.m. = 9 hours = 540 minutes

Number of live bulletins = 10; Duration of each live bulletin = 30 minutes

Total airtime for live bulletins = Number of bulletins × Duration per bulletins = 10 × 30 = 300 minutes

Number of highlight reels = 20; Duration of each highlight reel = 10 minutes

Total airtime for highlight reels = Number of reels × Duration per reel = 20 × 10 = 200 minutes

Total airtime = Total airtime for live bulletins + Total airtime for highlight reels = 300 + 200 = 500 minutes

Percentage of the total airtime dedicated to live bulletins and highlight reels

$$=\frac{500}{540}\times 100\approx 93\%$$

15. (a)

Explanation:

Let the given number be ×

According to the question, 260% of × = 52

i.e., $\frac{260}{100}\times x=52 \Rightarrow$ × = 20

Thus, 52% of x = $\frac{52}{100}\times 20=$ 10.4

16. (b)

Explanation:

Let the cost of books be 1 ⇒ Cost of tuition fee = 0.7

Total expenses = 1.7

New cost of books = 0.85 (15% decrease);

New cost of tuition fee = 125% of 0.7 = 0.875

New expenses = 1.725

Percentage increase in expenses

$$=\frac{1.725-1.7}{1.7}\times 100\approx 1.5\%$$

17. (a)

Explanation:

Let the number of pages of the new notebook be x

Then, 40 × 32 × 21 = x × 40 × 28 ⇒ x = 24

Percentage change in number of pages

$$=\frac{40-24}{40}\times 100=40\%$$

18. (c)

Explanation:

Let the original number be N

According to the question,

130% of N – 55% of N = 69 ⇒ 75% of N = 69

$$\Rightarrow \frac{75}{100}\times N=69\Rightarrow N=92$$

19. (c)

Explanation:

Let the maximum marks be x. Then, according to the conditions given in the question.

Marks obtained by student A = 28% of x = 0.28x

Marks obtained by student B = 45% of x = 0.45x

Passing marks = (28% of x + 23) or (45% of x – 28)

28% of x + 23 = 45% of x – 28 ⇒ 17% of x = 51 ⇒ .17x = 51

x = 300

Passing marks = 28% of 300 + 23 = 107

20. (a)

Explanation:

Let the number of males be x.

Then, the number of females = 4400 – x

According to the question,

110% of x + 122% of (4400 – x) = 5068

$$\Rightarrow \frac{110}{100}x+\frac{122}{100}(4400-x)=5068$$

1.1x + 5368 – 1.22x = 5068

300 = 0.12x

⇒ x = 2500

Thus, the number of female tigers

= 4400 – 2500 = 1900

Ratio, Proportion and Mixture

8

INTRODUCTION

This chapter deals with comparing two or more quantities and changes in their magnitudes. For example, you can compare ages, weights, income, savings, heights, volume, density, temperature, etc. This chapter is useful for solving problems related to data interpretation. The concepts of ratio, proportion, and variations are very useful in solving most of the arithmetic problems.

RATIO

The ratio is a way of comparing two quantities in terms of magnitude. It tells us how many times one quantity is greater or lesser than the other.

For example: A has 3 books and B has 5 books. It implies that the ratio of number of books between A and B is 3 to 5. Which is expressed as 3:5.

Keep in mind that in ratio, the order of terms is very important as in the above example the required ratio is 3:5 and not 5:3.

So the ratio of any two quantities is expressed as $\frac{a}{b}$ or a:b.

Note:
If you want to compare two quantities, they must be of the same kind or in the same units. You cannot compare 5 apples and 3 oranges directly because they are not of the same kind. To compare them, you need to express them in the same units. For example, you can compare the number of apples and oranges by weight, such as 5 apples weighing 1 kg and 3 oranges weighing 0.5 kg. Then, you can compare the weight of the apples and oranges by expressing them in the same units.

PROPERTIES AND IMPORTANT RESULTS OF RATIO

1. Value of the ratio does not change when the numerator and denominator both are multiplied or divided by the same number or quantity.

$$\frac{a}{b}=\frac{na}{nb}=\frac{ma}{mb}=\frac{a/c}{b/c}=\frac{a/d}{b/d}$$

Example: $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}$

etc. have same ratios

Following example will help us in better understanding:

EXAMPLES

E1. Consider the following ratios:

I. 5 : 7 II. 25 : 35

III. 45 : 62 IV. 105 : 147

Which of the above ratios are equal?

(a) I and II only (b) I, II and III only

(c) II and IV only (d) I, II and IV only

Ans: (d)

Explanation:

Consider the ratio I : 5 : 7

Now I, II and IV are equal because

$$\frac{5}{7}=\frac{5\times5}{7\times5}=\frac{25}{35}=\frac{21\times5}{21\times7}=\frac{105}{147}$$

2. The ratio of two fractions can be expressed in the ratio of integers.

$$e.g.\ \frac{a}{b}:\frac{c}{d}=a\times d:b\times c, i.e \frac{2}{3}:\frac{4}{5}=2\times5:4\times3=10:12=5:6$$

Following example will help us in better understanding:

EXAMPLES

E2. Which of the following is simplification of the ratio $\frac{5}{6}:\frac{7}{8}$?

(a) 20 : 21 (b) 35 : 48

(c) 30 : 56 (d) 21 : 48

Ans: (a)

Explanation:

By formula $\frac{a}{b}:\frac{c}{d}=a\times d:b\times c$

Hence, $\frac{5}{6}:\frac{7}{8}=5\times8:6\times7=40:42=20:21$

3. **Compounded ratio:** when two or more than two ratios are multiplied with each other, then it is called as compounded ratio e.g. compounded ratio of 2 : 3 and 3 : 4 is 6 : 12 = 1 : 2
4. If a:b and b:c are given then a:b : c = (a.b) : (b.b) : (b.c)
   a,b and c should be taken as : $\begin{matrix} a & & b \\ \downarrow & \swarrow & \downarrow \\ b & & c \end{matrix}$ for better understanding look at the following example:

EXAMPLES

E3. If a : b = 3 : 2 and b : c = 6 : 5 then a : b : c is equal to

(a) 9 : 6 : 5 (b) 9 : 6 : 10
(c) 3 : 3 : 5 (d) 3 : 6 : 5

Ans: (a)

Explanation:

a : b = 3 : 2 b : c = 6 : 5

a : b : c = (a.b) : (b.b) : (b.c)

implies, a : b : c = (3.6) : (2.6) : (2.5)

a : b : c = 18 : 12 : 10 = 9 : 6 : 5.

Alternatively,

a : b = 3 : 2 and b : c = 6 : 5 are given

We can see that "b" is common in both the ratios but with different values so to find a : b : c we can equate the "b" in both the ratio

In 1st ratio b = 2 and in 2nd ratio b = 6, to make them equal we can multiply 1st ratio by 3

So a : b = 9 : 6 (by 1st property: ratio does not change if we multiply both the terms by same number)

Now a : b = 9 : 6 and b : c = 6 : 5 now we can assume

a = 9, b = 6 and c = 5

Hence a : b : c = 9 : 6 : 5.

5. If, a : b, b : c, c : d and d : e are given then
   a : b : c : d : e = (a.b.c.d) : (b.b.c.d) : (b.c.c.d) : (b.c.d.e)
   Following example will help us in better understanding of this concept:

EXAMPLES

E4. If a : b = 1 : 2, b : c = 3 : 4, c : d = 5 : 6 and d : e = 7 : 8 then a : d equals to:

(a) 1 : 2 (b) 35 : 128
(c) 35 : 112 (d) 3 : 4

Ans: (c)

Explanation:

Given that:

a : b = 1 : 2

b : c = 3 : 4

c : d = 5 : 6

d : e = 7 : 8

Now, a : b : c : d : e = (a.b.c.d) : (b.b.c.d) : (b.c.c.d) : (b.c.d.e)

a : b : c : d : e = (1.3.5.7) : (2.3.5.7) : (2.4.5.7) : (2.4.6.7) : (2.4.6.8)

a : b : c : d : e = 105 : 210 : 280 : 336 : 384

So a : d = 105 : 336 = 35 : 112

Now, remember the trick:

a : b = 1 : 2

b : c = 3 : 4

c : d = 5 : 6

d : e = 7 : 8

Observe the above ratios

In a : b : c : d : e to find "a" we take left side of all ratios i.e 1.3.5.7

To find "b" we take upper one from right side and lower 3 from left side i.e 2.3.5.7

To find "c" we take upper two from right side and lower two from left i.e 2.4.5.7

To find "d" we take upper three from right side and lower one from left i.e 2.4.6.7

to find "e" we take right side of all ratios i.e 2.4.6.8

Similarly we can find a : b : c : d if

a : b, b : c and c : d are given.

6. If the two numbers are in the ratio a : b and the sum of these numbers is A, then these number will be $\frac{aA}{a+b}$ and $\frac{bA}{a+b}$ following example will help us in better understanding:

EXAMPLES

E5. If ratio of two numbers is 10:19 and their sum is 290, then these numbers are

(a) 100, 190 (b) 150, 140
(c) 130, 160 (d) 125, 165

Ans: (a)

Explanation:

According to the question a = 10, b = 19, A = 290

So the first number = $\frac{10\times290}{10+19}=100$

Second number = $\frac{19\times290}{10+19}=190$

7. If three numbers are in the ratio of a : b : c and the sum of these numbers is A, then these numbers will be, $\frac{aA}{a+b+c}$, $\frac{bA}{a+b+c}$ and $\frac{cA}{a+b+c}$ respectively.

QUESTIONS

Q1. The sum of two natural numbers is 72. Which of the following cannot be the ratio of these two numbers?

(a) 3:5 (b) 1:3
(c) 7:17 (d) 3:4

Ans: (d)

Explanation:

Let's check the option one by one

$3x + 5x = 8x$

$8x = 72$

$x = 9$, which is possible

The numbers are 27,45

Checking option *b*

$x + 3x = 4x$

$4x = 72$

$x = 18$ which is possible

The numbers are 18,54

Checking option c

$7x + 17x = 24x$ $24x = 72$

$x = 3$, which is possible

The numbers are 21,51

Checking option d

$3x + 4x = 7x$

$7x = 72$

$x = \frac{72}{7}$, Which is not possible.

Q2. *A*, *B* and *C* have 40, *x* and *y* balls with them respectively. If *B* gives 20 balls to *A*, he is left with half as many balls as *C*. If together they had 60 more balls, each of them would have had 200 balls on average. What is the value of *x* : *y*?

(a) 3: 2 (b) 9:16
(c) 2:16 (d) 3:4

Ans: (b)

Explanation:

From the above statement

$\frac{(40 + x + y + 60)}{3} = 200$

$\Rightarrow x + y = 500$ ...(i)

Again from first statement

$(x - 20) = \frac{y}{2}$

$2x - y = 40$ ...(ii)

Solving Equation (i) and (ii),

We get

$x = 180$ $y = 320$

$x : y \Rightarrow 9:16$

Q3. The incomes of A B, and C are in the ratio of 12:8:9 and their spendings are in the ratio 15:9:8. If A saves 25% of his income. What is the ratio of the savings of A, B and C?

(a) 15:13:21 (b) 9:16:13
(c) 12:13:14 (d) 3:4:17

Ans: (a)

Explanation:

Income = Expenditure + Saving

$A \rightarrow 12x = 15y + 3x$ ($3x$ = 25% of $12x$)

$B \rightarrow 8x = 9y + (8x - 9y)$

$C \rightarrow 9x = 8y + (9x - 8y)$

Therefore, $12x - 3x = 15y$

$\Rightarrow \frac{x}{y} = \frac{5}{3}$

$\Rightarrow y = \frac{3x}{5}$

Savings = Income – Expenditure

$A = 12x - 9x = 3x$

$B = \frac{8x - 27}{5} = \frac{13x}{5}$

$C = \frac{9x - 24}{5} = \frac{21x}{5}$

The ratio of savings of A:B:C

$A : B : C = 3x : \frac{13x}{5} : \frac{21x}{5}$

= 15:13:21

Q4. There are coins consisting of 20 paise, 50 paise and Rs.1 in the ratio of 7:8:5. What is the number of coins of 20 paise if the difference between the amount yielded by 50 paise and Rs.1 coin is 5?

(a) 32 (b) 35
(c) 26 (d) 56

Ans: (b)

Explanation:

Let the 20 paise, 50 paise and Rs.1 coins be $7x$, $8x$ and $5x$ respectively.

Therefore,

$(1 \times 5x) - (0.5 \times 8x) = 5$

$5x - 4x = 5$

$x = 5$

Total no. of 20 paise coin = $7 \times 5 = 35$

Q5. The ratio of a two-digit natural number to a number formed by reversing its digits is 4 : 7. The number of such pairs is **(UPSC CSAT 2019)**

(a) 5 (b) 4
(c) 3 (d) 2

Ans: (b)

Explanation:

Let the number, be ab

Which can be written as $10a + b$

Reverse of "ab" = "ba" = 10b + a

According to question,

$\frac{10a+b}{10b+a}=\frac{4}{7}$ which gives $b = 2a$

Therefore, total, 4 pairs are possible

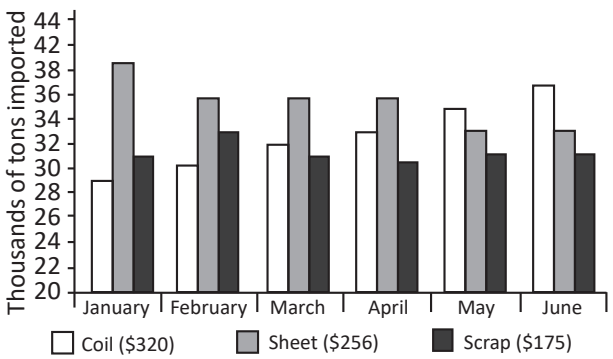
(12, 21)

(24, 42)

(36, 63)

(48, 84)

Q6. The following three items are based on the graph given below which shows imports of three different types of steel over a period of six months of a year. Study the graph and answer items that follow.



The figures in the brackets indicate the average cost per ton over a six month period. What was the approximate ratio of sheet steel and scrap steel imports in the first three months of the year ?

(UPSC CSAT 2018)

(a) 1 : 1 (b) 1.2 : 1

(c) 1.4 : 1 (d) 1.6 : 1

Ans: (b)

Explanation:

Sheet steel = 40+ 37+ 36 = 113

Scrap steel = 32 + 34 + 32 = 98

113 : 98 ≅ 1.2 :1

Q7. The monthly incomes of X and Y are the ratio of 4 : 3 and their monthly expenses are in the ratio of 3 : 2. However, each saves Rs. 6000 per month. What is their total monthly income? **(UPSC CSAT 2017)**

(a) ₹ 28000 (b) ₹ 42000

(c) ₹ 56000 (d) ₹ 84000

Ans: (b)

Explanation:

Let the monthly income of X and Y is $4x$ and $3x$ respectively

And their expenses are $3y$ and $2y$ respectively

Then according to question,

$4x - 3y = 6000$ and $3x - 2y = 6000$

By solving linear equation we get $x = y = 6000$

So, income of $X = 4x = 24000$ and

Income of $Y = 3x = 18000$

Their total income = 24000 + 18000 = 42000

Q8. The monthly income of Peter and Paul are in the ratio of 4 : 3. Their expenses are in the ratio of 3 : 2. If each saves Rs. 6000 at the end of the month, their monthly incomes respectively are (in ₹)

(UPSC CSAT 2015)

(a) 24,000 and 18,000

(b) 28,000 and 21,000

(c) 32,000 and 24,000

(d) 34,000 and 26,000

Ans: (a)

Explanation:

Let the monthly income of Peter and Paul is 4x and 3x respectively

And their expenses are 3y and 2y respectively

Then according to question,

$4x - 3y = 6000$ and $3x - 2y = 6000$

By solving linear equation we get $x = y = 6000$

So, income of Peter = $4x = 24000$ and

Income of paul = $3x = 18000$

PROPORTION

An equality of two ratios is called a proportion and we say that four numbers are in proportion.

i.e if $a : b = c : d$ or $\frac{a}{b}=\frac{c}{d}$ then we can say that a, b, c and d are in proportion which can be written as : $a : b :: c : d$, where "::" is symbol for proportion as can be read as "***a* is to *b* as *c* is to *d*.**''

Here a and d are called "**extremes**" and b and c and are called **"means".**

Thus, four numbers are said to be in proportion if the ratio of the first two numbers is equal to the ratio of the last two numbers. For example, 1 : 2 :: 2 : 4.

TEST OF PROPORTIONALITY

If you have four numbers in proportion, it means that if you multiply the first and last numbers (extremes), it will be the same as multiplying the middle two numbers (means). But if the numbers are not in proportion, then multiplying the extremes won't be equal to multiplying the means.

Hence if $a : b :: c : d$ then $a \times d = b \times c$

So, if you know three out of the four terms in a proportion, you can figure out the fourth term by applying this test of proportionality.

CONTINUED PROPORTION

Continued proportion, also known as continued proportionality, refers to a sequence of numbers in which the ratio of any two successive numbers is constant.

- a, b and c are in continued proportion if $a : b :: b : c$ then, $\mathbf{b^2 = ac}$

Here ***b*** is said to be the mean proportional to ***a***, ***c*** and, and ***c*** is said to be the third proportional to ***a, b***.

For example: 4 : 8 :: 8 : 16 here 8 x 8 = 4 x 16 That means 4, 8 and 16 are in continued proportion.

Theorem Related To Proportion and Continued Proportion

1. **Invertendo:** if $a : b :: c : d$ then, $b : a :: d : c$
2. **Alternendo:** if $a : b :: c : d$ then, $a : c :: b : d$
3. **Componendo:** if $a : b :: c : d$ then, $a + b : b :: c + d : d$
4. **Dividendo:** if $a : b :: c : d$ then, $a - b : b :: c - d : d$
5. **Componendo and Dividendo:** if $a : b :: c : d$ then, $a + b : a - b :: c + d : c - d$

   For example: let 1,2, 4 and 8 are values of a, b, c and d respectively then a, b, c and d are proportional such that 1 : 2 :: 4 : 8

   Now 2 : 1 :: 8 : 4 or 2,1, 8 and 4 are also proportional (Invertendo)

   1 : 4 and 2 : 8 are also in same ratio (Alternendo)

   1 + 2 : 2 = 3 : 2 and 4 + 8 : 8 = 12 : 8 are also same ratio (Componendo)

   Similarly we can understand other results as well using this example.
6. If a, b and c are three numbers such that $a : b :: b : c$ then
   - $b^2 = ac$, For example 4 : 8 :: 8 : 16 here $8^2 = 16 * 4$
   - **b : a :: c : b** (Invertendo)
   - **a : c ::** $a^2 : b^2$,

   For example:

   4 : 8 :: 8 : 16 here 4 : 16 :: 4^2: 8^2 = 4 : 16 :: 16 : 64

Unitary Method

In this approach, we start by determining the value of one unit, and then we calculate the value of the required quantity by multiplying the value of one unit with the required number of units.

For Example:

let's say the cost of 3 notebooks is $15. To find the cost of 8 notebooks using the unitary method, we first establish the cost of one notebook, which is $5. Therefore, we can find the cost of 8 notebooks by multiplying the cost of one notebook by the number of notebooks we want, resulting in $40.

Direct Proportion

Two quantities are said to be directly proportional if the increase (or decrease) in one quantity causes the increase (or decrease) in the other quantity by the same proportion. e.g.,

(i) The cost of things varies directly with the number of things. More things cost more whereas less things cost less.

(ii) The work done varies directly with the number of men (work force) at work. More men at work, more work done at the same time. Less men, less work done in the same time

Inverse proportion

Two quantities are said to vary inversely if the increase (or decrease) in one quantity causes the decrease (or increase) in the other quantity by the same proportion. For example, the time taken to finish a work varies inversely to the number of men at work. More men at work, less time taken to finish the same work. Less men at work, more time taken to finish the same work.

QUESTIONS

Q9. The first, second and fourth terms of a proportion are 15,5 and 90 respectively.

The third term will be

(a) 270 (b) 350

(c) 260 (d) 370

Ans: (a)

Explanation:

15:5:: x : 90

15 × 90 = 5 × x

x = 270

Q10. The mean proportional between 15 and 60 is :

(a) 15 (b) 50

(c) 30 (d) 112

Ans: (c)

Explanation:

15: x :: x :60

$x^2 = 15 \times 60$

$x = 30$

Q11. The students in three classes are in the ratio of 4:5:6. If 40 students are added in each class, the ratio becomes 6:7:8. Find the total number of students in all the three classes

(a) 270 (b) 180

(c) 126 (d) 300

Ans: (d)

Explanation:

$4x + 40 = 6y$ ...(i)

$5x + 40 = 7y$ ...(ii)

$6x + 40 = 8y$ ...(iii)

Solving above eq we get $x = y$

$2x = 40$

$x = 20$

Total number of students $= 4x + 5x + 6x = 15x$

$\Rightarrow 15 \times 20 = 300$

Q12. The dimensions of a photograph are 10 and 4.5 cm. If the breadth of the reduced photo is 1.8 cm and it was reduced proportionally then what is the new length of the new photograph?

(a) 6 (b) 4

(c) 10 (d) 9

Ans: (b)

Explanation:

10 : 4.5:: x :1.8

$4.5 \times x = 10 \times 1.8$

$x = 4$

Q13. A student appeared in 6 papers. The maximum marks are the same for each paper. His marks in these papers are in the proportion of 5:6:7:8:9:10. Overall he scored 60%. In how many papers did he score less than 60% of the maximum marks?

(UPSC CSAT 2021)

(a) 2 (b) 3

(c) 4 (d) 5

Ans: (b)

Explanation:

Let the maximum marks in each subject = 100

Total maximum marks in 6 subjects = 100 + 100 + 100 + 100 + 100 + 100 = 600

Marks scored = 60% of 600 = 360

His marks in these papers are in the proportion of 5:6:7:8:9:10.

Let the marks in each paper be $5x$, $6x$, $7x$, $8x$, $9x$ and $10x$

$\therefore 5x + 6x + 7x + 8x + 9x + 10x = 360$

$45x = 360$

$x = 8$

Marks in each of the 6 subjects are

$5 \times 8, 6 \times 8, 7 \times 8, 8 \times 8, 9 \times 8, 10 \times 8 = 40, 48, 56, 64, 72, 80$

So, in 3 subjects he scored less than 60% marks (40, 48 and 56)

VARIATION

When things are connected, and if one thing changes, the other thing changes too, we call it a kind of relationship. There are two main types:

1. **Direct Variation:** This is like when you earn more money, you tend to save or spend more. A quantity A is said to vary directly if the increase (or decrease) in B yields increase (or decrease) in A but not in proportion. It is expressed as $A \propto B \Rightarrow A = KB$ where K is proportionality constant, $K = \frac{A}{B}$

   Example: If your salary increases, your savings or spending also increases.

2. **Inverse Variation:** A quantity A is said to vary inversely if the increase (or decrease) in B yields decrease (or increase) in A but not in the same proportion. It is expressed as: $A \propto \frac{1}{B} \Rightarrow A = \frac{K}{B}$, Where K is proportionality constant and K = AB

   Example: If the number of guests in a hotel increases, the expenses per guest may decrease.

   In summary, it's all about how things change together:- either going up together (direct variation) or going in opposite directions (inverse variation).

QUESTIONS

Q14. M varies directly as N and inversely as O. M is 15 when N is 9 and O is 3. What is the value of M when N is 16 and O is 4?

(a) 6 (b) 24

(c) 20 (d) 18

Ans: (c)

Explanation:

$\Rightarrow M \propto N$

$\Rightarrow M \propto 1/O$

$\Rightarrow M \propto \frac{N}{O}$

$\Rightarrow M = K\frac{N}{O}$

First Condition

$\Rightarrow M = K \times \frac{N}{O}$

$15 = K \times \frac{9}{3} \quad \Rightarrow \quad K = 5$

Second Condition where N = 16 and O = 4

$M = 5 \times \frac{16}{4} = 5 \times 4 = 20$

M = 20

Q15. The value of a coin varies directly to the square of its radius, when its thickness is constant. The radius of a coin is 1.5 cm and its value is Rs. 1. What will be the radius of a coin if its value is Rs. 2?

(a) 2.6 (b) 2.4

(c) 2.5 (d) 2.1

Ans: (d)

Explanation:

From question $v \propto r^2$

[v = value r = radius]

$\Rightarrow v = Kr^2$

$1 = K(1.5)^2$

$\Rightarrow K = \frac{4}{9}$

Now finding the radius

$2 = \frac{4}{9}r^2$

$r = \sqrt{\frac{2 \times 9}{4}}$

$r = \frac{3}{2}\sqrt{2}$

r =2.1 cm

PARTNERSHIP

When a group of people comes together to start a business by putting in their money and resources, we call it a joint venture or a partnership. These people are called partners because they've invested something to make the business work. It's like a team effort where everyone shares in the business and works together to make it successful.

PARTNERSHIP RULES

(i) If the partners invest different amounts for the same period of time, then the profits of all the partners are shared in the ratio of their investments.

(ii) If the partners invest the same amount for the different time periods, then the profits of all the partners are shared in the ratio of time periods for which their amounts were invested.

(iii) If the partners invest different amounts for different time periods, then their profits are shared in the ratio of products of respective investments with the time period for each partner, individually. Thus gain or loss is divided in the ratio of 'money-time' capitals.

Basic Formula for Partnership

Ratio of profit of two partners $P_1 : P_2$ is equal to ratio of product of their investment (say, I_1 and I_2) and time period of investment (say, T_1 and T_2)

i.e $P_1 : P_2 = (I_1 \times T_1) : (I_2 \times T_2)$

e.g. If A and B invested in ratio of 5 : 4 for 1 yr and 2 yr respectively then ratio of their profit

P1 : P2 = 5 × 1 : 4 × 2 = 5 : 8

QUESTIONS

Q16. Amit and Rahul started a business by investing Rs.30000 and Rs. 60000. Find the share of each, out of an annual profit of Rs. 5700.

(a) 1600,4100 (b) 1400,4300

(c) 1000,4700 (d) 1900,3800

Ans: (d)

Explanation:

Ratio of shares of Aman and Rahul

= 30000:60 000 = 1:2

$\therefore$ Share of Aman = $5700 \times \frac{1}{3}$ = Rs.1900

Share of Rahul = $5700 \times \frac{2}{3}$ = Rs. 3800

Q17. Vinay started a business by investing Rs. 36000. After 6 months Karan joined him with some investment. At the end of the year, the total profit was divided between them in the ratio of 3:2. How much capital was invested by Karan in the business?

(a) 35000 (b) 48000

(c) 39000 (d) 40000

Ans: (b)

Explanation:

Let the amount invested by Karan by x

$$\frac{36000 \times 12}{x \times 6} = \frac{3}{2}$$

$$\Rightarrow \frac{72000}{x} = \frac{3}{2}$$

$\Rightarrow x = 48000$

Q18. Arun starts some business with Rs. 70000. After 4 months Akash joins him with Rs. 80000. At the end of the year, in what ratio should they share the profits?

(a) 21:16 (b) 21:23

(c) 17:27 (d) 13:24

Ans: (a)

Explanation:

Ratio of their investment

= 70000 : 80000

Ratio of time period

= 12 : 8

$\therefore$ Ratio of their shares in total profit

$$= 70000 \times 12 : 80000 \times 8$$

$\Rightarrow$ 21:16

Q19. P works thrice as fast as Q, whereas P and Q together can work four times as fast as R. If P, Q and R together work on a job, in what ratio should they share the earnings? **(UPSC CSAT 2017)**

(a) 3 : 1 : 1 (b) 3 : 2 : 4

(c) 4 : 3 : 4 (d) 3 : 1 : 4

Ans: (a)

Explanation:

Here, $P = 3Q$ and

$P + Q = 4R$ which implies, $4Q = 4R$ or $Q = R$

Hence the ratio of their earning = $3Q : Q : Q$

Or 3 : 1 : 1

Q20. The total emoluments of two persons are the same, but one gets allowances to the extent of 65% of his basic pay and the other gets allowances to the extent of 80% of his basic pay. The ratio of the basic pay of the former to the basic pay of the latter is : **(UPSC CSAT 2016)**

(a) 16 : 13 (b) 5 : 4

(c) 7 : 5 (d) 12 : 11

Ans: (d)

Explanation:

Let the basic pay of 1st person and 2nd person be x and y respectively.

Emoluments of 1st person

$= x + 65\%$ of $x = \frac{165x}{100}$

Emoluments of 2nd person

$= y + 80\%$ of $y = \frac{180y}{100}$

Since the total emoluments of both the persons are the same.

$\Rightarrow \frac{165x}{100} = \frac{180y}{100}$ or $165x = 180y$

Implies $\frac{x}{y} = \frac{12}{11}$

So, the ratio of basic pay of the former to the basic pay of latter is 12: 11

MIXTURE

When you combine two or more things in a specific amount, the result is a new product. We call this new product a mixture of the ingredients you used. It's like making a blend by putting different things together in just the right proportions.

MIXTURE RULES

1. To find the average value of a mix when you know the prices and how much of each item you're mixing, follow these steps. The mix's value will always be more than the cheapest item and less than the most expensive one. It's like figuring out the middle value when you're blending different things together.
2. If two quantities are in a ratio then

   Quantity of cheaper: quantity of dearer = (Cost price of dearer - mean price) : (mean price - Cost price of cheaper)

   Following example will help us in better understanding

EXAMPLES

E6. In what ratio must the grocer mix two types of wheat costing Rs. 6.50 per kg and Rs. 10 per kg respectively, so as to get a mixture worth Rs. 7.25 per kg?

(a) 11 : 3 (b) 3 : 11

(c) 4 : 5 (d) 5 : 4

Ans: (a)

Explanation:

Here the price of cheaper wheat

= Rs. 6. 50 per kg

The price of dearer wheat = Rs. 10 per kg

And the mean price = Rs. 7.25 per kg

So according to the formula the ratio is

: 10 – 7.25 : 7.25 - 6.50 = 2.75 : 0.75 = 11 : 3

3. If there's a container with x units of liquid and you take out y units, replacing it with water. After doing this n times, the amount of pure liquid left in the container can be found using this given formula:

   Quantity of pure liquid left = $\left[x\left(1-\frac{y}{x}\right)^{n}\right]$

   Following example will help us in better understanding:

EXAMPLES

E7. From a tank of petrol, which initially contains 200 litres of petrol, the seller replaces it each time with kerosene when he sells 40 litres of petrol (or its mixture). Everytime he sells out only 40 litres of petrol (pure or impure). After replacing the petrol with kerosene 4th time, the total amount of kerosene in the mixture is :

(a) 81. 92. litres (b) 96 litres

(c) 118. 08 litres (d) none of these

Ans: (c)

Explanation:

In this question $x = 200$

$Y = 40$ and $n = 4$

So the quantity of petrol in the mixture

$$=\left[200\left(1-\frac{40}{200}\right)^{4}\right]=200\left(\frac{4}{5}\right)^{4}=81.92 \text{ litres}$$

So the quantity of kerosine

= 200 – 81.92 = 118.08 litres.

QUESTIONS

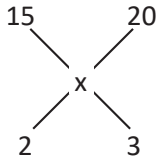
Q21. Two varieties of soda water with different prices are mixed in the ratio of 2: 3. The price of first soda water is Rs. 15 per litre while the price of second soda water is Rs. 20 per litre, respectively. The average price of the mixture (per litre) is :

(a) Rs. 12 (b) Rs. 13

(c) Rs. 14 (d) Rs. 18

Ans: (d)

Explanation:



$$\frac{x-15}{20-x}=\frac{3}{2}$$

$\Rightarrow 2(x-15) = 3(20-x)$

$\Rightarrow 2x - 30 = 60 - 3x$

$\Rightarrow x = 18$

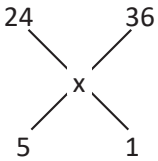
Q22. 1 kg of superior quality of sugar is mixed with 5 kg of inferior quality sugar. The price of superior quality and inferior quality sugar is Rs. 36 and Rs. 24 respectively. The average price per kg of the mixture is:

(a) Rs. 13 (b) Rs. 15

(c) Rs. 26 (d) Rs. 21

Ans: (c)

Explanation:



$\frac{x-24}{36-x}=\frac{1}{5}$

$\Rightarrow 5(x-24)=1(36-x)$

$\Rightarrow 5x-120=36-x$

$6x=156 \Rightarrow x=26$

Q23. The average weight of girls is 30 and the average weight of boys is 60. If the number of boys are 6, then the number of girls is 12 .The average weight of boys and girls both is :

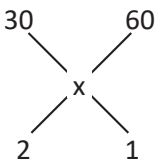
(a) 34 (b) 40

(c) 23 (d) 27

Ans: (b)

Explanation:

Ratio of boys and girls is 1:2



$\frac{x-30}{60-x}=\frac{1}{2}$

$\Rightarrow 2(x-30)=1(60-x)$

$\Rightarrow 2x-60=60-x$

$\Rightarrow x=40$

Q24. A bottle contains 20 litres of liquid A. 4 litres of liquid A is taken out of its and replace by same quantity of liquid B. Again 4 litres of the mixture is taken out and replaced by same quantity of liquid B. What is the ratio of quantity of liquid A to that of liquid B in the final mixture? (UPSC CSAT 2020)

(a) 4 : 1 (b) 5 : 1

(c) 16 : 9 (d) 17 : 8

Ans: (c)

Explanation:

Quantity of pure liquid left $= \left[x\left(1-\frac{y}{x}\right)^n\right]$

Here $x = 20$ and $y = 4$ and $n = 2$

Hence quantity of liquid A

$=\left[20\left(1-\frac{4}{20}\right)^2=20\times\frac{4}{5}\times\frac{4}{5}=\frac{64}{5}l\right]$

Quantity of liquid $B = 20-\frac{64}{5}=\frac{36}{5}$ ratio of quantity of liquid A to that of liquid B in the final mixture

$=\frac{64}{5}:\frac{36}{5}=16:9$

Alternate solution:

Initial quantity in bottle is: 20 litres of liquid A and 0 litre of liquid B

1st Mixing:

Now we take 4 litres of liquid A from the bottle and replace it with liquid B.

Quantities after 1st mix: 16 litres of liquid A and 4 litres of liquid B

2nd Mixing:

Again 4 litres of the mixture is taken out and replaced by the same quantity of liquid B.

Now 4 litres of mixture contain:

20 litres of mixture contain 16 litres of liquid A and 4 litres of liquid B

1 litre of mixture contain: $\frac{16}{20}$ litres of liquid A and $\frac{4}{20}$ litres of liquid B

4 litres of mixture contain: $\frac{16}{5}$ litres of liquid A and $\frac{4}{5}$ litres of liquid B.

Quantities after 2nd mix:

Liquid A: $16-\frac{16}{5}=\frac{64}{5}$ litres

Liquid B: $8-\frac{4}{5}=\frac{36}{5}$ litres

Now ratio of liquids in mixture:

Liquid A: Liquid B $=\frac{64}{5}:\frac{36}{5}=16:9$

PRACTICE QUESTIONS

1. **If the ratio of two numbers is 13:17 and their sum is 90, then what is the product of these numbers?**
   (a) 1800 (b) 2210
   (c) 1669 (d) 1989
2. **The sum of squares of three positive numbers is 2100 and the ratio of the first number to the second number is 2:1 and the ratio of second number to the third number is 2:1. What is the second number?**
   (a) 40 (b) 20
   (c) 10 (d) 50
3. **P and Q together have ₹1710. If one-fourth of P's share is equal to three-seventh of Q's share then how much money does Q have?**
   (a) 710 (b) 610
   (c) 630 (d) 690
4. **80 kg of alloy A is mixed with 50kg of alloy B. If alloy A has Silver and Copper in the ratio 5:3 and alloy B has copper and zinc in the ratio of 1:4, then the amount (in kg) of Copper in the new alloy is?**
   (a) 40 (b) 30
   (c) 35 (d) 25
5. **A sum of money is to be distributed among A, B, C, and D in the proportion of 6:5:4:3. If C gets Rs.1000 more than D, what is the share of B?**
   (a) 3000 (b) 5000
   (c) 3500 (d) 2000
6. **In a mixture of 90 litres, the ratio of milk and water is 2:1. If this ratio is to become 1:2 then the quantity of water that must be added (in litres) is?**
   (a) 30 (b) 45
   (c) 60 (d) 90
7. **If 4, 8 and X are in continued proportion then what is the value of X?**
   (a) 16 (b) 12
   (c) 10 (d) 14
8. **Find the mean proportion between 3 and 12.**
   (a) 9 (b) 6
   (c) 10 (d) 36
9. **30 m of a uniform wire weighs 40 kg. What will be the weight of the same wire of 132 m length?**
   (a) 148 (b) 160
   (c) 120 (d) 176
10. **The monthly ratio of expenditure to savings of Jack is 23:6. If his monthly income is Rs.7192, then what is his monthly savings?**
    (a) 7200 (b) 1450
    (c) 1445 (d) 1488
11. **A 70-litre mixture of milk and water contains 10% water. How many litres of water must be added to increase the water content to 30%?**
    (a) 45 (b) 15
    (c) 20 (d) 30
12. **In what ratio must tea at ₹90 per kg be mixed with tea at ₹120 per kg so that the mixture must be worth ₹100 per kg?**
    (a) 4:5 (b) 1:5
    (c) 2:1 (d) 3:2
13. **In what ratio should water be mixed in grape juice which costs ₹50 per litre such that the price of the resultant mixture is ₹30 per litre?**
    (a) 4:5 (b) 2:7
    (c) 2:3 (d) 3:5
14. **The ratio of weights of A, B, C and D is 1:2:3:4. If the difference between the weights of D and B is 48 Kg, what is the cumulative weight of A, B, C and D in kg?**
    (a) 240 (b) 500
    (c) 480 (d) 300
15. **In a board examination, the ratio of passed to failed students was 3 : 1. Had 15 students less appeared and 10 students less passed, the ratio of passes to failures would have been 4 : 1. Find the total number of students who appeared in the examination.**
    (a) 45 (b) 36
    (c) 72 (d) 40

ANSWERS

1. (*d*) 2. (*b*) 3. (*c*) 4. (*a*) 5. (*b*) 6. (*d*) 7. (*a*) 8. (*b*) 9. (*d*) 10. (*d*)
11. (*c*) 12. (*c*) 13. (*c*) 14. (*a*) 15. (*d*)

EXPLANATIONS

1. (d)

Explanation:

Let the two numbers be $13x$ and $17x$

So, $13x + 17x = 90$

$\Rightarrow 30x = 90 \Rightarrow x = 3$

Therefore, the numbers are 39 and 51

So, the product of these two number is 1989

2. (b)

Explanation:

Let the first three numbers be a, b, c respectively

Now $a : b = 2 : 1$

And $b : c = 2 : 1$

$A : b : c = 2 \times 2 : 2 \times 1 : 1 \times 1 = 4 : 2 : 1$

Let $a = 4x$, $b = 2x$, $c = x$

So, $(4x)^2 + (2x)^2 + (x)^2 = 2100$

$\Rightarrow 16x^2 + 4x^2 + x^2 = 2100$

$\Rightarrow 21x^2 = 2100 \Rightarrow x^2 = 100 \Rightarrow x = 10$

Therefore, the second number is $b = 2x = 2 \times 10 = 20$

3. (c)

Explanation:

$$\frac{1}{4}\,P = \frac{3}{7}\,Q$$

$$\Rightarrow P = \frac{3}{7} \times \frac{4}{1}\,Q$$

$$\Rightarrow \frac{P}{Q} = \frac{12}{7}$$

Now Q's share $= \frac{7}{19} \times 1710 = 630$

4. (a)

Explanation:

Quantity of Copper in 80kg of A $= \frac{3}{8} \times 80 = 30\text{kg}$

Quantity of Copper in 50kg of B $= \frac{1}{5} \times 50 = 10\text{kg}$

Quantity of Copper in new alloy = (30 + 10) = 40kg

5. (b)

Explanation:

Let the shares of A, B, C and D are ₹$6x$, ₹$5x$, ₹$4x$ and ₹$3x$ respectively

According to the question

$4x - 3x = 1000 \qquad x = 1000$

Now, B's share is $5x = 5 \times 1000 = 5000$

6. (d)

Explanation

Quantity of milk in the mixture $= \frac{2}{3} \times 90 = 60\text{L}$

Quantity of water = 90 – 60 = 30L

Let the quantity of water to be added further be x litres

Then, Milk : Water is $\frac{60}{30+x}$

New (milk : water) ratio should be 1:2

Now, $\frac{60}{30+x} = \frac{1}{2}$

$\Rightarrow 120 = 30 + x \Rightarrow x = 90$

Quantity of water to be added is 90L.

7. (a)

Explanation:

When a, b and c are in continued proportion, they can be written as

$a : b :: b : c$

Therefore, $4 : 8 :: 8 : X$

Now, $\frac{4}{8} = \frac{8}{X}$

X = 16

8. (b)

Explanation:

(Mean proportion: If three quantities a, b and x are such that $a:x :: x:b$, then $x^2 = ab$ and x is called the mean proportion of a and b.)

Mean proportion between 3 and 12

$= \sqrt{3 * 12} = \sqrt{36} = 6$

9. (d)

Explanation

According to the question

$$\frac{30}{40} = \frac{132}{X}$$

$$\Rightarrow x = \frac{132 * 40}{30}$$

$\Rightarrow x = 44 \times 4$

$\Rightarrow x = 176$

Therefore, the weight of 132 m long wire will be 176 kg.

10. (d)

Explanation:

Let the expenditures be ₹23x and savings be ₹6x.

$\therefore 23x + 6x = 7192$

$\Rightarrow 29x = 7192$

$\Rightarrow x = \frac{7192}{29} = 248$

$\therefore$ Savings = $6x$ = ₹1488

11. (c)

Explanation:

Amount of water in the mixture = $\frac{10}{100} \times 70 = 7L$

Amount of milk in the mixture = 70 – 7 = 63L

Let x litre of water is added

Now according to the question

$\frac{7+x}{70+x} = \frac{30}{100}$

$70 + 10x = 210 + 3x$

$7x = 140 \Rightarrow x = 20$

Therefore, 20L of water must be added

12. (c)

Explanation:

Let's denote tea bags with A & B

Now Let tea bag A that is of ₹ 90/kg be x Kg

Let tea bag B that is of ₹120/kg be y Kg

Price of x kg tea = ₹ 90x

Price of y kg tea = ₹ 120y

The mixture costs = ₹ 100/kg

$\therefore$ Total price of mixture = 100(x + y)

Now, 90x + 120y = 100 (x + y)

$\Rightarrow$ 20y = 10x

$\Rightarrow \frac{20}{10} = \frac{x}{y} \Rightarrow \frac{x}{y} = \frac{2}{1}$

Hence the required ratio is 2:1

13. (c)

Explanation:

The cost price of grape juice = ₹50 per litre

The cost price of water = ₹0 per litre

The cost price of mixture = ₹30 per litre

Let the quantity of grape juice and water in the final mixture be x and y respectively.

According to the question:

$50x + 0y = (x + y) \times 30$

$\Rightarrow 50x = 30x + 30y \Rightarrow 20x = 30y$

$\Rightarrow x : y = 3 : 2$

Therefore the ratio in which water and grape juice should be mixed is 2 : 3

14. (a)

Explanation

Let the weight of A, B, C, D is x, 2x, 3x, 4x

Now, According to the question,

The difference between the weights of D and B = 48 Kg

$\Rightarrow (4x – 2x) = 48 \Rightarrow 2x = 48 \Rightarrow x = 24$

Now, the weight of A, x = 24

The weight of B = 2x = (2 x 24) = 48

The weight of C = 3x = (3 x 24) = 72

The weight of D = 4x = (4 x 24) = 96

Now, the cumulative weight of A, B, C and D = (24 + 48 + 72 + 96) = 240 kg

15. (d)

Explanation:

Suppose x students passed and y students failed

$\frac{x}{y} = \frac{3}{1}$

$x = 3y$ ...(i)

In the second case;

number of students appeared = $x + y - 15$

and number of students who passed = $x - 10$

Number of failed Student

$= x + y - 15 - (x - 10) = y - 5$

Now according to the question

$\frac{x-10}{y-5} = \frac{4}{1}$

$x – 10 = 4y – 20$

Put the value of x from (i)

$3y – 10 = 4y – 20 \Rightarrow y = 10$

So, $x = 30$

Total number of students is $x + y = 40$

Profit, Loss and Discount

9

INTRODUCTION

This chapter holds significant importance within the Quantitative Aptitude domain. Given that the computation of profit, loss, and discounts is a routine necessity, particularly for those engaged in business activities. The assessment of profit and loss remains crucial for individuals involved in any transactions related to the purchase or sale of goods and services. This importance arises from the desire to maximize earnings, minimize expenditures, and optimize profitability while minimizing losses. In situations where multiple options for buying and selling exist, individuals typically engage in price comparisons to make informed decisions. This explains why virtually every competitive examination incorporates problems based on this chapter, and it is noteworthy that a substantial number of such problems can be found in the Data Interpretation section.

TERMINOLOGY

Cost Price (CP): The amount paid by the shopkeeper to the manufacturer or wholesaler for the purchase of goods is referred to as the cost price (CP) of the acquired goods.

Selling Price (SP): The selling price (SP) is the amount at which the shopkeeper sells the goods to customers.

Profit: Profit occurs when the selling price of an article is higher than its cost price. In terms of calculation, Profit = SP – CP, where SP is greater than CP.

Loss: Loss is incurred when the selling price of an article is less than its cost price. The loss is calculated as Loss = CP – SP, where CP is greater than SP.

IMPORTANT FORMULAE

(i) Profit = SP – CP

(ii) Loss = CP – SP

(iii) Profit percentage

$$= \frac{\text{profit}}{\text{cost price}} \times 100 = \left(\frac{SP - CP}{CP}\right) \times 100$$

(iv) Loss percentage $= \frac{\text{loss}}{\text{cost price}} \times 100 = \left(\frac{CP - SP}{CP}\right) \times 100$

(v) $SP = \left(\frac{100 + \text{gain\%}}{100} \times CP\right) = \left(\frac{100 - \text{loss\%}}{100} \times CP\right)$

(vi) $CP = \left(\frac{100}{100 + \text{gain\%}} \times SP\right) = \left(\frac{100}{100 - \text{loss\%}} \times SP\right)$

(vii) SP = (100 + *k*)% of CP; when profit = *k*% of CP

(viii) SP = (100 - *k*)% of CP; when loss = *k*% of CP

Following examples will help us in better understanding of above Formulae:

QUESTIONS

Q1. A shopkeeper buys 500 oranges at 5 for ₹ 8 and sells at 2 for ₹5. Find :

(i) the cost price of each orange

(ii) the selling price of each orange

(iii) profit or loss on selling one orange

(iv) his total profit or loss on selling all the oranges.

Explanation:

(i) Since, the cost price of 5 oranges = ₹ 8

$\therefore$ Cost price of one (or each) orange = ₹ $\frac{8}{5}$ = ₹ 1.60

(ii) Since, the selling price of 2 oranges = ₹ 5

$\therefore$ The selling price of one orange = ₹ $\frac{5}{2}$ = ₹ 2.50

(iii) Since, SP is more than CP, there is a profit.

So profit on selling one orange = SP – CP = 2.5 – 1.6 = ₹ 0.90

(iv) Profit on selling all the oranges = ₹ (0.90 × 500) = ₹ 450

Hence, the total profit on selling all oranges = ₹ 450

Q2. A person buys 200 eggs at ₹ 2.40 per piece. Unfortunately 8 eggs got spoiled during transportation. The person sells the remaining eggs at ₹ 30 a dozen. Find his profit or loss.

(a) Profit of ₹ 40 (b) Loss of ₹ 40

(c) Loss of ₹ 20 (d) Neither profit nor loss

Ans: (d)

Explanation:

Cost price of all eggs = 200 × 2.4 = ₹ 480

Selling price of one egg

$= \frac{30}{12} = ₹ 2.5$

∴ Selling price of 192 eggs = 192 × 2.5 = ₹ 480

Having ruined 8 eggs, only 192 eggs remain. At present, as the overall selling price equals the overall cost price, the shopkeeper experiences neither a profit nor a loss.

Q3. By selling a washing machine for ₹ 46920, a shopkeeper suffers a loss of 8%. At what price should he sell it to gain 16%?

(a) ₹ 49160 (b) ₹ 59000

(c) ₹ 59160 (d) ₹ 59300

Ans: (c)

Explanation:

SP = (100 – *k*)% of CP; when loss

= *k*% of CP Loss is 8%

∴ SP = 92% of CP

$\Rightarrow 46920 = \frac{92}{100} \times CP$

$\Rightarrow$ CP = ₹ 51,000

Now, gain % = 16%

∴ New selling price = (100 + 16)% of CP

$\Rightarrow \frac{116}{100} \times 51000 = ₹ 59160$

Q4. Neeraj purchased a computer for ₹ 17,000. He had to sell it for ₹ 15,800. Find his profit or loss per cent.

(a) $8\frac{11}{17}\%$ (b) $7\frac{1}{17}\%$

(c) $7\frac{17}{21}\%$ (d) $8\frac{17}{21}\%$

Ans: (b)

Explanation:

Since, SP < CP, there will be loss

Loss percentage = $\frac{loss}{cost\ price} \times 100$

$= \frac{17000-15800}{17000} \times 100$

$= \frac{1200}{17000} \times 100$

$= 7\frac{1}{17}\%$

Q5. A dealer buys 200 quintals of sugar at ₹ 1100 a quintal. He spends ₹ 10,000 on transportation and storage. Then, he sells the wheat at ₹ 12 per kg. Find his profit or loss percentage.

(a) 3.43% (b) 5.45%

(c) 4.34% (d) 6.43%

Ans: (c)

Explanation:

CP = 1100 x 200 = ₹ 2,20,000

Transportation and storage cost = ₹ 10,000

Total CP = 2,20,000 + 10,000 = ₹ 2,30,000

Total SP = 12 x 200 x 100 = ₹ 2,40,000

∴ Now, since SP > CP, hence there will be profit.

Profit = SP – CP = ₹ (2,40,000 – 2,30,000) = ₹ 10,000

Profit percentage = $\frac{profit}{cost\ price} \times 100$

$\Rightarrow \frac{10,000}{2,30,000} \times 100 = 4.34\%$

Q6. A dealer sold 500 quintals of rice at a profit of 7%. If a quintal of rice cost him ₹ 2000, find his total profit and the selling price.

(a) 6,73,00; 11,70,000

(b) 70,000; 10,70,000

(c) 68,200; 10,80,000

(d) 77,200; 11,70,000

Ans: (b)

Explanation:

CP = 2000 x 500 = ₹ 10,00,000

Rate of Profit = 7%

Profit = 10,00,000 × $\frac{7}{100}$ = ₹ 70,000

∴ SP = CP + Profit = ₹ 10,00,000 + ₹ 70,000 = ₹ 10,70,000

Thus, profit = ₹ 70,000 and selling price = ₹ 10,70,000

Q7. Find the cost price of an article which is sold for ₹ 170 at a loss of 15%.

(a) 150 (b) 200

(c) 250 (d) 250

Ans: (b)

Explanation:

SP = ₹ 170

Loss = 15% Let CP = ₹ Y

$$SP = \left(\frac{100 - \text{loss}\%}{100} \times CP\right)$$

$$\Rightarrow 170 = \frac{100 - 15}{100} \times Y$$

$$\Rightarrow 170 = \frac{85}{100} \times Y$$

$\Rightarrow$ Y = 200

Therefore, cost price = ₹ 200

Q8. A person bought a car and sold it for Rs. 3,00,000. If he incurred a loss of 20%, then how much did he spend to buy the car? **(UPSC CSAT 2020)**

(a) Rs. 3,60,000 (b) Rs. 3,65,000

(c) Rs. 3,70,000 (d) Rs. 3,75,000

Ans: (d)

Explanation:

Let CP be Rs. 100*x*

SP of car = Rs 3,00,000

Loss percent = 20%

So loss = 20% of 100x = 20x

We know CP – SP = Loss

$\Rightarrow$ 100*x* – 3,00,000 = 20*x*

$\Rightarrow$ 80*x* = 300000

Or $x = \frac{300000}{80}$

Or 100*x* = Rs. 3,75,000

$\therefore$ Cost Price = Rs. 3,75,000

Q9. A shopkeeper sells an article at Rs. 40 and gets X % profit. However, when he sells it at Rs. 20, he faces the same percentage of loss. What is the original cost of the article ? **(UPSC CSAT 2018)**

(a) Rs. 10 (b) Rs. 20

(c) Rs. 30 (d) Rs. 40

Ans: (c)

Explanation:

We know that,

$$SP = \left(\frac{100 + \text{gain}\%}{100} \times CP\right), \text{ and}$$

$$SP = \left(\frac{100 - \text{loss}\%}{100} \times CP\right)$$

SP (case-I) = ₹ 40

SP (case-II) = ₹ 20

Profit % = X%

Loss % = X%

$\therefore$ Equation 1:

$$40 = \left(\frac{100 + X\%}{100} \times CP\right)$$

Equation 2:

$$20 = \left(\frac{100 - X\%}{100} \times CP\right)$$

Solving for CP and x

X = 33.33 %

And CP = ₹ 30

ELEMENTARY FORMULA ON PROFIT AND LOSS

FORMULA 1

If an item is sold for ₹ *a* and incurs a loss, but when sold for ₹ *b*, the gain equals the loss amount, then the original cost price can be stated as $\frac{a+b}{2}$.

EXAMPLES

E1. By selling an article for ₹ 100, some loss is incurred and by selling it for ₹ 150, there is a gain equal to the loss amount. Find the cost price of the article.

(a) 100 (b) 110

(c) 125 (d) None of the above

Ans: (c)

Explanation:

Cost price = ₹ $\frac{a+b}{2}$

Here, a = 100 and b = 150

$$\Rightarrow \frac{100+150}{2} = \frac{250}{2}$$

$\Rightarrow$ CP = ₹ 125

FORMULA 2

When the selling price of "a" item equals the cost price of "b" articles, the gain percentage or loss percentage is calculated as: $\left(\frac{b-a}{a} \times 100\right)\%$

if *b* > *a*, it is gain and if *a* > *b*, it is loss.

EXAMPLES

E2. The selling price of 25 articles is equal to the cost price of 40 articles. Then find the profit percentage.

(a) 18% (b) $18\frac{2}{3}\%$

(c) $18\frac{3}{2}\%$ (d) None of the above

Ans: (d)

Explanation:

Loss percentage = $\left(\frac{(b-a)}{a}\times 100\right)\%$

Here, b = 40 and a = 25

$\Rightarrow \frac{40-25}{25} \times 100$

$\Rightarrow \frac{15}{25} \times 100$

$\Rightarrow 60\%$

FORMULA 3

If the cost of making "*a*" item is ₹ *b*, and selling price of "*b*" item is ₹ *a*, then the percentage of profit or loss can be calculated as follows: $\left(\frac{a^2-b^2}{b^2}\times 100\right)\%$

(if $a > b$, it is gain, if $b > a$, it is loss)

EXAMPLES

E3. The cost price of 20 articles is ₹ 10 and selling price of 10 articles is ₹ 20. Then find the net profit percentage.

(a) 300% (b) 150%

(c) 400% (d) 350%

Ans: (a)

Explanation:

Profit percentage = $\left(\frac{(a^2-b^2)}{b^2}\times 100\right)\%$

Here, *a* = 20 and *b* = 10

$\Rightarrow \left(\frac{20^2-10^2}{10^2}\times 100\right)\%$

$\Rightarrow \left(\frac{400-100}{100}\times 100\right)\%$

$\Rightarrow \left(\frac{300}{100}\times 100\right)\%$

$\Rightarrow 300\%$

FORMULA 4

If you sell something and make a profit of a%, and then sell it for ₹ X more, the total profit becomes b%. The cost price of the item is figured out through this method.

CP = ₹ $\left(\frac{X}{b-a}\times 100\right)$

EXAMPLES

E4. When a bag is sold, it gains 8%. If it is sold for ₹20 more, then the gain 10%. Then, find the cost price of the bag.

(a) ₹ 700 (b) ₹ 800

(c) ₹ 900 (d) ₹ 1000

Ans: (d)

Explanation:

Cost price of the bag = ₹ $\left(\frac{X}{b-a}\times 100\right)$

Here, X = 20, *a* = 8 and *b* = 10

$\Rightarrow \left(\frac{20}{10-8}\times 100\right) \Rightarrow \left(\frac{20}{2}\times 100\right)$

$\Rightarrow$ ₹ 1000

FORMULA 5

When an article is sold and incurs a loss of *a*%, selling it for ₹ X more results in a gain of *b*%. The original cost price of the article can be calculated using this: ₹ $\left(\frac{X}{b+a}\times 100\right)\%$

EXAMPLES

E5. A set of bottles are sold at a loss of 5%. If it is sold for ₹150 more, then the gain is 10%. Then, find the cost price of the bottles.

(a) ₹ 1200 (b) ₹ 1300

(c) ₹ 1000 (d) ₹ 950

Ans: (c)

Explanation:

Cost price of the set of bottles

= ₹ $\left(\frac{X}{b+a}\times 100\right)$

Here, X = 150, *a* = 5 and *b* = 10

$\Rightarrow \left(\frac{150}{10+5}\times 100\right) \Rightarrow \left(\frac{150}{15}\times 100\right)$

$\Rightarrow$ ₹ 1000

DISCOUNT

A discount is a decrease in the original price of an item, which is known as the marked price. Discounts are always calculated based on the marked price of the product.

MARKED PRICE

The amount mentioned on the tag or label of a product is called the marked price or list price. For example, if you buy a pen for ₹20 and then decide to sell it with a marked price of ₹25, then ₹25 is the marked price or list price.

IMPORTANT FORMULAE

(i) Selling price = Marked price - Discount

(ii) Discount = Marked price - Selling price

(iii) Discount % $= \left(\frac{\text{Discount}}{\text{Marked price}}\right) \times 100$

If discount allowed is r% then

(iv) Selling price $= \left(\frac{100-r}{100} \times \text{marked price}\right)$

EXAMPLES

E6. Praveen had money to buy 10 glasses from a crockery shop. But the shopkeeper offered a very good discount. He could buy 16 glasses with the amount he had. What was the discount the shopkeeper offered?

(a) 30% (b) 33.5 %

(c) 35.5% (d) 37.5 %

Ans: (d)

Explanation:

Let us say Praveen had ₹ 100

Price of each glass before discount $= \frac{100}{10} = ₹ 10$

Price of each phone before discount $= \frac{100}{16} = ₹ 6.25$

Discount % $= \left(\frac{\text{Discount}}{\text{Marked price}}\right) \times 100$

$\Rightarrow \left(\frac{10-6.25}{10}\right) \times 100 \quad \Rightarrow \left(\frac{3.75}{10}\right) \times 100$

$\Rightarrow 0.375 \times 100 \quad \Rightarrow 37.5\ \%$

MARKUP PRICE

Markup involves incorporating additional charges into the overall cost of a product or service and generating a profit. Numerous businesses establish their pricing by computing the expenses associated with delivering goods and services and then marking up that amount by a percentage.

Therefore the **marked Price = cost price + % markup on cost price**

EXAMPLES

E7. If the marked price of an article is ₹ 550 and markup percentage is 12.5%, what is the cost price?

(a) ₹ 350.5 (b) ₹ 400

(c) ₹ 488.8 (d) ₹ 550

Ans: (c)

Explanation: MP = 112.5% of CP

$\therefore 550 = \frac{1125}{1000} \times CP$

$\Rightarrow 550 = \frac{9}{8} \times CP \qquad \Rightarrow CP = ₹ 488.8$

E8. If the cost price of an article is ₹ 500 and the percent markup is 20%. What is the marked price?

(a) ₹ 500 (b) ₹ 600

(c) ₹ 700 (d) ₹ 800

Ans: (b)

Explanation: MP = CP + % markup on CP

$\Rightarrow 500 + 500 \times \frac{20}{100}$

$\Rightarrow MP = ₹ 600$

SUCCESSIVE DISCOUNT

Suppose the marked price of an article is ₹ P and a discount of r_1% is given on it and on the reduced price, a further discount of r_2% is given. Then, it is said that the successive discount of r_1% and r_2% are given on the article.

SUCCESSIVE DISCOUNTS FOR TWO TERMS

Suppose there are two successive discounts on an item, r_1 % and r_2%. The selling price of the article after r_1% and r_2% discount

$= (100 - r_2)\%$ of $(100 - r_1)\% \times P$

$= \left(\frac{100-r_2}{100}\right) \times \left(\frac{100-r_1}{100}\right) \times P = \left(1-\frac{r_2}{100}\right) \times \left(1-\frac{r_1}{100}\right) \times P$

Or Net discount percentage $= \left(r_1 + r_2 - \frac{r_1 r_2}{100}\right)\%$

EXAMPLES

E9. A retail store offered a discount of 35% on every article purchased. Later they announced an additional discount of 45% on every article purchased. Find the total discount percentage availed by the customers.

(a) 50.50% (b) 64.25%

(c) 54.25% (d) 65%

Ans: (b)

Explanation:

1st discount offered = r_1 = 35%

2nd discount offered = r_2 = 55%

The formula for successive discount

$$=\left(r_1+r_2-\frac{r_1 r_2}{100}\right)\%$$

$$=\left(35+45-\frac{35\,X\,45}{100}\right)\%$$

$$=\left(80-\frac{1575}{100}\right)\%$$

= 64.25%

E10. In a store the successive discounts offered on a jacket are 10% and 20%. The store price of the bag is ₹ 150. Then calculate the selling price of the bag.

(a) ₹ 110 (b) ₹ 108

(c) ₹ 105 (d) ₹ 100

Ans: (b)

Explanation:

1st discount offered = r_1 4 = 10%

2nd discount offered = r_2 = 20%

Marked price = Rs. 150

Selling price of the bag $=\left(1-\frac{r_2}{100}\right)\times\left(1-\frac{r_1}{100}\right)\times P$

$$\Rightarrow\left(1-\frac{10}{100}\right)\times\left(1-\frac{20}{100}\right)\times 250$$

$\Rightarrow (1 - 0.1) \times (1 - 0.2) \times 150$

$\Rightarrow 0.8 \times 0.9 \times 150$

$\Rightarrow$ ₹ 108

SUCCESSIVE DISCOUNTS FOR THREE TERMS

Suppose there are three successive discounts on an item, r_1% , r_2% and r_2%. The selling price of the item after r_1%, r_2% and r_3% discount

$$=\left(\frac{100-r_3}{100}\right)\times\left(\frac{100-r_2}{100}\right)\left(\frac{100-r_1}{100}\right)\times P$$

$$=\left(1-\frac{r_3}{100}\right)\times\times\left(1-\frac{r_2}{100}\right)\times\left(1-\frac{r_1}{100}\right)P$$

EXAMPLES

E11. The successive discounts on a product in a store is given as 5%, 10% and 15%. Calculate overall discount.

(a) 27 % (b) 28.5 %

(c) 28 % (d) 27.3 %

Ans: (d)

Explanation:

1st discount offered = r_1 = 5%

2nd discount offered = r_2 = 10%

3rd discount offered = r_3 = 15%

Total discount on r_1 and r_2

$$=\left(r_1+r_2-\frac{r_1 r_2}{100}\right)\%$$

$$=\left(5+10-\frac{5x10}{100}\right)\%$$

$$=\left(15-\frac{50}{100}\right)\%$$

= 14.5 %

Overall discount on 14.5 % and r_3

$$=\left(14.5+r_3-\frac{14.5\,x\,r_3}{100}\right)\%$$

$$=\left(14.5+15-\frac{14.5\,x15}{100}\right)\%$$

$$=\left(29.5-\frac{217.5}{100}\right)\%$$

= 27.3 %

E12. The store price of the product is ₹ 2000. There are successive discounts on the product in the store as given as 10%, 15% and 20%. Calculate the selling price of the product.

(a) ₹ 1224 (b) ₹ 1200

(c) ₹ 1220 (d) ₹ 1230

Ans: (a)

Explanation:

1st discount offered = r_1 = 10%

2nd discount offered = r_2 = 15%

3rd discount offered = r_3 = 20%

Total discount on r_1 and r_2

$$=\left(r_1+r_2-\frac{r_1 r_2}{100}\right)\%$$

$$=\left(10+15-\frac{10x15}{100}\right)\% \quad =\left(25-\frac{150}{100}\right)\%$$

= (25 – 1.5)%

= 23.5 %

Overall discount on 23.5 % and r_3

$$=\left(23.5+r_3-\frac{23.5\ x r_3}{100}\right)\%$$

$$=\left(23.5+20-\frac{23.5\ x20}{100}\right)\%$$

$$=\left(43.5-\frac{470}{100}\right)\% \quad =(43.5-4.7)\%$$

= 38.8 %

∴ Discount = 38.8 % of ₹ 2000 = ₹ 776

Selling Price = Marked Price - Overall Discount

= ₹ 1224

SUBSTITUTING AN INACCURATE WEIGHT FOR THE ACTUAL WEIGHT

By employing an incorrect weight, denoted as *x* grams instead of the standard weight for a kilogram, and selling the item at its cost price, the shopkeeper experiences a percentage increase in profit

$$=\left[\frac{1000-x}{x}\times 100\right]\%$$

QUESTIONS

Q10. A dishonest dealer professes to sell his goods at cost price but he uses a weight of 750 gm for a one kg weight. Then, find his profit percentage

(a) $33\frac{1}{5}\%$ (b) $33\frac{1}{7}\%$

(c) $33\frac{1}{3}\%$ (d) $33\frac{1}{9}\%$

Ans: (c)

Explanation:

The profit percentage when a false weight is used

$$=\left[\frac{1000-x}{x}X100\right]\%$$

Here, x = 750

$$\Rightarrow\left[\frac{1000-750}{750}X100\right]\%$$

$$\Rightarrow\left[\frac{250}{750}X100\right]$$

$$\Rightarrow\frac{2500}{75}\%=\frac{100}{3}\%=33\frac{1}{3}\%$$

Q11. Rakesh had money to buy 8 mobile handsets of a specific company. But the retailer offered a very good discount on that particular handset. Rakesh could buy 10 mobile handsets with the amount he had. What was the discount the retailer offered?

(UPSC CSAT 2019)

(a) 15 % (b) 20 %

(c) 25 % (d) 30 %

Ans: (b)

Explanation:

Discount (D) = Marked price (MP) – Selling price (SP)

Let us say Rakesh had ₹ 100

Price of each phone before discount = $\frac{100}{8}$ = ₹ 12.5

Price of each phone before discount = $\frac{100}{10}$ = ₹ 10

$$\text{Discount \%} = \left(\frac{\text{Discount}}{\text{Marked price}}\right)\times 100$$

$$\Rightarrow\left(\frac{12.5-10}{12.5}\right)\times 100$$

$$\Rightarrow\left(\frac{2.5}{12.5}\right)\times 100 \Rightarrow 0.2 \text{ x } 100$$

$\Rightarrow$ 20 %

Q12. A person allows a 10 % discount for cash payment from the marked price of a toy and still he makes a 10 % gain. What is the cost price of the toy which is marked Rs. 770? **(UPSC CSAT 2016)**

(a) Rs. 610 (b) Rs. 620

(c) Rs. 630 (d) Rs. 640

Ans: (c)

Explanation:

MP = ₹ 770

CP = x

Discount = 10 % of MP

$\Rightarrow \frac{10}{100} * 770 \times = ₹ 77$

Price after Discount = 770 – 77 = 693 = SP

$\Rightarrow$ Gain = 693 – x = $\frac{10}{100} \times x$

$\Rightarrow 693 - x = 0.1x$

$\Rightarrow 1.1x = 693$

$\Rightarrow x = 630$

$\Rightarrow$ CP = ₹ 630

Q13. As a result of a 25% hike in the price of rice per kg, a person is able to purchase 6 kg less rice for Rs. 1,200. What was the original price of rice per kg?

(UPSC CSAT 2020)

(a) Rs. 30 (b) Rs. 40

(c) Rs. 50 (d) Rs. 60

Ans: (b)

Explanation:

Let the original price be Rs. 100x

Original quantity purchased for Rs. 1200

$= \frac{1200}{100x} \Rightarrow \frac{12}{x}$ kg

After 25% hike, the price will be

Rs. 125x

Quantity purchased for Rs. 1200 after the hike = $\frac{1200}{125x}$

$= \frac{48}{5x}$ kg

According to the question,

$\frac{12}{x} - \frac{48}{5x} = 6$

Or $30x = 60 - 48 = 12$

Or $x = \frac{2}{5} = 0.4$

So, original price = $100x = 100 \times 0.4$

$\Rightarrow$ Rs. 40 per kg

Method II: Expenditure = Price x Quantity

Expenditure here is constant.

Let the quantity be 100 kg at the price of Rs 100 per kg.

$\therefore$ Expenditure of 100 × 100 = Rs. 10000

Price of the commodity after increase by 25%

= Rs. 125 per kg.

New quantity = $\frac{10000}{125}$ = 80 kg.

Reduction in quantity = 100 – 80 = 20 kg.

The percentage reduction in quantity to keep the expenditure constant

$= \frac{20}{100} \times 100 = 20\%$

So, if the price is increased by 25%, the quantity purchased will decrease by 20%.

As per the question, 20% = 6 kg

So, 100% = 6 × 5 = 30 kg

Hence, original price = Expenditure/Original Quantity

$= \frac{1200}{30}$ = Rs. 40 per kg.

Q14. A shopkeeper has a faulty balance that displays 20% more weight than what it should weigh. If he sells his goods at the cost price using his faulty balance, what is his profit per cent?

(a) 25 (b) 20

(c) 33.33 (d) none

Ans: (b)

Explanation:

Shopkeeper cheats 20% in weight while selling.

Let original weight = X kg

Faulty weight is 20% more than the actual weight.

$\therefore$ faulty weight = X + 20 % of X

$\Rightarrow X + \frac{1}{5}X = \frac{6}{5}X$

Assume the cost price to be Rs. Y/kg.

$\therefore$ Total cost = Rs. XY

Selling price = Rs. $\frac{6}{5}$XY

Profit = $\frac{6}{5}$XY – XY = $\frac{XY}{5}$

Profit % = $\frac{Profit}{CP} \times 100$

$\Rightarrow \left(\frac{XY}{5} \div XY\right) \times 100$

$\Rightarrow \left(\frac{XY}{5} \times \frac{1}{XY}\right) \times 100$

$\Rightarrow \frac{1}{5} \times 100$

$\Rightarrow 20\%$

Q15. A deceitful sales person pretends to sell his goods at cost price but uses false weights and gains 13 $\frac{3}{9}$%. Find the false weight he is using instead of 1 kg weight.

(a) 762.45 gm (b) 850.54 gm

(c) 882.35 gm (d) 982.35 gm

Ans: (c)

Explanation:

Let the false weight be x gm.

percentage increase in profit

$$= \left[\frac{1000-x}{x} X\,100\right]\%$$

$$\Rightarrow \frac{120}{9} = \left[\frac{1000-x}{x}\right] \times 100$$

$$\Rightarrow \frac{120}{9}\, x = [1000 - x] \times 100$$

$\Rightarrow 120x = [1000 - x] \times 900$

$\Rightarrow x = 882.35$

$\therefore$ the shopkeeper is using weights of 882.35 gm instead of 1kg.

Q16. A shop owner offers the following discount options on an article to a customer:

1. **Successive discounts of 10% and 20%, and then pay a service tax of 10%.**
2. **Successive discounts of 20% and 10%, and then pay a service tax of 10%.**
3. **Pay a service tax of 10% first, then successive discounts of 20% and 10%.**

Which one of the following is correct?

(a) 1 only is the best option for the customer.

(b) 2 only is the best option for the customer.

(c) 3 only is the best option for the customer.

(d) All the options are equally good for the customer.

Ans: (d)

Explanation:

Let the base price be Rs. 100

Condition 1:

After 1st discount = 100 – 10% × 100 = Rs. 90

After 2nd discount = 90 – 20% × 90 = Rs. 72

After Service tax = 72 + 10% × 72 = Rs. 79.2

Condition 2:

After 1st discount = 100 – 20% × 100 = Rs. 80

After 2nd discount = 80 – 10% × 80 = Rs. 72

After Service tax = 72 + 10% × 72 = Rs. 79.2

Condition 3:

After Service tax = 100 + 10% × 100 = Rs. 110

After 1st discount = 110 – 20% × 110 = Rs. 88

After 2nd discount = 88 – 10% × 88 = Rs. 79.2

All the options are equally good for a customer.

Q17. Rakesh had money to buy 8 mobile handsets of a specific company. But the retailer offered a very good discount on that particular handset. Rakesh could buy 10 mobile handsets with the amount he had. What was the discount the retailer offered?

(a) 15% (b) 20%

(c) 25% (d) 30%

Ans: (b)

Explanation:

Let's assume the price of each mobile as 100 Rupees

Total amount needed to buy 8 mobiles = 800

But Rakesh got 10 mobiles for 800 itself, Which means

The actual price of 10 mobiles =1000 but he got it for 800

So, the discount offered = 200/1000 = 20%

Formulae summary

- Profit = Selling Price – Cost Price
- Loss = Cost Price – Selling Price
- Profit % = $\frac{(\text{Profit})}{(\text{Cost Price})} \times 100$
- Loss % = $\frac{(\text{Loss})}{(\text{Cost Price})} \times 100$
- Discount = Marked Price – Selling Price
- Discount % = $\frac{(\text{Discount})}{(\text{Marked Price})} \times 100$

PRACTICE QUESTIONS

1. **Shreya sold her watch for Rs 960 and incurred 40% loss. In order to obtain a profit of 30%. What should be the selling price?**
   (a) 2200 (b) 2180
   (c) 2080 (d) 2240
2. **Mohan purchased some lemons at the rate of 3 lemons for Rs 2. He sold all the lemons and earned 25% profits. At what rate did he sell all the lemon?**
   (a) 4 for Rs 3 (b) 5 for Rs 4
   (c) 6 for Rs 5 (d) 7 for Rs 6
3. **The profit earned by selling an article for Rs. 932 is equal to the loss incurred when the same article is sold for Rs. 348. What should be the sale price of the article for making a 50 percent profit?**
   (a) 580 (b) 640
   (c) 960 (d) 740
4. **A man purchased a book at 4/5th of its selling price and sold it at 10% more than its selling price. Find his gain percent.**
   (a) 35% (b) 33.3%
   (c) 45% (d) 37.5%
5. **A man purchases two articles A and B at a total cost of ₹780. He sells A with 20% profit and B at a loss of 20% and gets the same selling price for both articles. What is the purchasing price of A and B respectively?**
   (a) ₹300, ₹480 (b) ₹312, ₹468
   (c) ₹315, ₹465 (d) ₹255, ₹512
6. **After getting 2 successive discounts, a book with a list price of ₹200 is available at ₹150. If the second discount is 10 %, find the first discount percentage?**
   (a) 16.66% (b) 19%
   (c) 15% (d) 22.5%
7. **Shyam bought 800 pens at the rate of ₹3.75 per dozen. He sold 450 pens at the rate of 2 pens for ₹1 and the remaining pens at the rate of 5 pens for ₹2. Find the gain percentage of Shyam.**
   (a) 35% (b) 44%
   (c) 42% (d) 46%
8. **Guru sells 39 articles for the same money as he paid for 52. What is his gain percent?**
   (a) 12.5% (b) 25%
   (c) 33.33% (d) 40%
9. **When a producer allow a 30% commission on the retail price of his product, he earns a profit of 5%. What would be his profit % if the commission is reduced by 18%?**
   (a) 24% (b) 32%
   (c) 23% (d) 30%
10. **A shopkeeper marks his goods 20% above the cost price but allows a 25 % discount. His net gain/loss in percentage is?**
    (a) 5% loss (b) 5%profit
    (c) 10% loss (d) no profit no loss
11. **A man sold an article for ₹6300 after giving a discount of 10% on the labelled price. He would have earned a profit of 26%, had there been no discount. What was the actual % of profit earned?**
    (a) 13.4% (b) 14.2%
    (c) 12.6% (d) 13.8%
12. **On selling a pen at 8% loss and a book at 15% gain, Ram gains ₹9. If he sells the pens at 8% gain and the book at 10% gain, then he gains ₹13. The actual price of the book is**
    (a) ₹80 (b) ₹88
    (c) ₹85 (d) ₹82
13. **A shopkeeper sold two clock sets for ₹888 each, gaining 10% on one, and lossing 10% on the other. Find his gain or loss percentage.**
    (a) 5% (b) 10%
    (c) 2% (d) 1%
14. **A shopkeeper offers his customers 10% discount and still makes a profit of 20%. What is the actual cost to him of an article marked ₹280?**
    (a) ₹210 (b) ₹185
    (c) ₹200 (d) ₹225
15. **A dishonest shopkeeper sells his goods at the cost price but still earns a profit of 20% by underweighting. What weight does he use for a kilogram?**
    (a) 850gm (b) 833.33gm
    (c) 830gm (d) 840gm
16. **A dealer buys an article marked at ₹40000 with 20% and 10% off. He spends ₹1200 on its repairs and sells it for ₹30000. What is his gain or loss percent?**
    (a) 10% gain (b) 10% loss
    (c) No gain no loss (d) None of these

17. An article was sold at a loss of 10%. If it was sold for Rs 56 more, there would have been a gain of 4%. Find the initial selling price of the article.

(a) 280 (b) 320

(c) 360 (d) 420

18. Rakesh buys 12 dozen Banana for Rs 432, 12 Bananas are found rotten and the rest are sold at 5 Bananas for 20Rs. Find his percentage gain or loss.

(a) $13\frac{1}{3}\%$ (b) $11\frac{9}{11}\%$

(c) $22\frac{2}{9}\%$ (d) $9\frac{1}{11}\%$

19. The marked price of an article is Rs. 1500. Two successive discounts of D% and 20 % is given on an article and it is sold for Rs 900. Find the value of D.

(a) 30% (b) 25%

(c) 20% (d) 15%

20. By selling an article for Rs. 7800 a man allows a 8% discount and earns 19.6 % profit. If he did not give any discount what should be his profit percent?

(a) 27.6% (b) 28%

(c) 30% (d) 32.4%

ANSWERS

1. (c)	2. (c)	3. (c)	4. (d)	5. (b)	6. (a)	7. (d)	8. (c)	9. (b)	10. (c)
11. (a)	12. (b)	13. (d)	14. (a)	15. (b)	16. (c)	17. (c)	18. (c)	19. (b)	20. (c)

EXPLANATIONS

1. (c)

Explanation:

She incurred a loss of 40%

Selling Price = 60% of CP = 0.6 CP = 960

$CP = \frac{960}{0.6} = 1600$

In order to obtain a profit of 30%, selling price(SP) must be 130% of CP or 1.3CP

SP = 1.3 × 1600 = 2080

2. (c)

Explanation:

Cost price of 3 lemons = Rs 2

He sold them at 25% profit

Selling price = 1.25 × CP = 1.25 × 2 = Rs 2.5

He sold 3 lemons for Rs 2.5. i.e., 6 lemons for Rs 5

3. (c)

Explanation:

Profit = SP – CP

Loss = CP – SP

The profit earned by selling an article for Rs. 932 is equal to the loss incurred when the same article is sold for Rs. 348

932 – CP = CP – 348

1280 = 2CP $\Rightarrow$ CP = 1280/2 = 640

Selling price(SP) for 50% profit = 1.5 CP

SP = 1.5 × 640 = Rs 960.

4. (d)

Explanation:

Let sellig price = Rs x

Then, cost price = $\frac{4x}{5}$

Selling Price is 110% of $x = \frac{11x}{10}$

Gain = $\frac{11x}{10} - \frac{4x}{5} = \frac{3x}{10}$

gain % = $\frac{3x/10}{4x/5} \times 100 = 37.5\%$

5. (b)

Explanation:

Let the cost price(CP) of article A be Rs x

then cost price of B be = (780 – x)

Now, Selling price of A = $\frac{120x}{100} = \frac{6x}{5}$

Selling price of B = $(780 - x)\frac{80}{100} = (780 - x)\frac{4}{5}$

According to the question

$\frac{6x}{5} = (780 - x)\frac{4}{5}$

10x = 3120

x = 312

Cost price of article A = Rs 312

Cost price of article B = 780 – 312 = Rs 468

6. (a)

Explanation:

Let the first discount be x%

Second discount percent = 10% (given)

Price after 2 successive discounts = 90% of (100 – x)% of 200 = 150

$$=\left(\frac{90}{100}\right)\left(\frac{100-x}{100}\right)*200=150$$

$= 9(100 - x) = 750$

$x = 16.66\%$

7. (d)

Explanation:

Cost price(CP) of total pens = $\left(\frac{3.75}{12}\right)800$ = ₹250

total selling price (SP) = (1/2) × 450 + (2/5) × 350 = ₹365

total gain = (365 – 250) = ₹115

Gain % = $\left(\frac{115}{250}\right)$ × 100 = 46%

8. (c)

Explanation:

The selling price (SP) of 39 articles is equal to the Cost price (CP) of 52 articles

SP × 39 = CP × 52

$$\frac{SP}{CP}=\frac{52}{39}$$

$$\frac{SP}{CP}=\frac{4}{3}$$

Gain percent = $\frac{SP-CP}{CP}*100=\frac{4-3}{3}*100=33.33\%$

9. (b)

Explanation:

Let the retail price be ₹100

Commission is 30%

Selling price = (100 – 30) = ₹70

Profit = 5% = $\frac{SP-CP}{CP}*100\%$

Cost price (CP) = $\left(\frac{70}{105}\right)$ × 100 = ₹ 200/3

Now new commission is 12% = ₹12

New Selling price = (100 – 12) = 88

gain= (88 – 200/3) = 64/3

gain % = $\left(\frac{64}{3}\right)$ × $\left(\frac{3}{200}\right)$100 % = 32%

10. (c)

Explanation:

Let the cost price be CP = 100x

He marks his goods 20% above CP

MRP(Marked Price) = 120% of CP = 1.2 × 100x = 120x

He allows a discount of 25%

Selling price = 75% of MRP = 0.75 × 120x = 90x

Loss= CP – SP = 100x – 90x = 10x

Loss percentage = $\frac{10x}{100x}*100=10\%$

11. (a)

Explanation:

Let the market price be ₹x

Selling price = 90%of $x = \frac{90x}{100}=\frac{9x}{10}$

According to the question

$\frac{9x}{10}$ = 6300 $\Rightarrow$ x = 7000

Then, cost price $\left(\frac{7000}{126}\right)*100=\frac{50000}{9}=5555.5$

Profit % = $\left(\frac{sp-cp}{cp}\times 100\right)$

Actual% Profit = (6300 – 5555.5/5555.5) × 100

= 13.4%

12. (b)

Explanation:

Let the cost price of a pen and a book be ₹x and ₹y respectively.

Then, (92% of x) + (115% of y) = $x + y + 9$

$(0.92x) + (1.15y) = x + y + 9$

$(0.15y) - (0.08\ x) = 9$ ...(i)

and (108% of x) + (110% of y) = $x + y + 13$

$(1.08\ x) + (1.10y) = x + y + 13$

$(0.08x) + (0.10y) = 13$ ...(ii)

Adding equation (i) and (ii)

$0.25y = 22 \Rightarrow y$ = ₹88

13. (d)

Explanation:

When selling price of two articles is same

And gain% = loss% = x%

then there will always be loss

overall loss% $=\frac{(x)^2}{100}\%=\frac{(10^2)}{100}=1\%$

14. (a)

Explanation:

Marked price = ₹280

discount % = 10

discount = $\frac{(280\times10)}{100}=28$

selling price = market price – discount = 280 – 28 = ₹252

profit = 20%

cost price = $\left(\frac{100}{120}\right)$ × 252 = ₹210

15. (b)

Explanation:

Profit = 20%

$\frac{\text{true weight}}{\text{false weight}}=\frac{100+\text{gain\%}}{100+x}$

Here, selling price = cost price

i.e. x = 0 (loss%)

false weight = $\left(\frac{1000\times100}{120}\right)$ = 833.33gm

16. (c)

Explanation:

Marked price = ₹40000

discounts = 20% and 10%

cost price = (80% of 90% of ₹40000) + ₹1200

= $\left(\frac{80}{100}\right)\times\left(\frac{90}{100}\right)\times40000+1200$

=28800 + 1200 = ₹30000

Selling price also = ₹30000

So, no loss no gain in the transaction

17. (c)

Explanation:

Let the cost price of the article be 'x'

It was sold at 10% loss, Selling price = 90% of CP = 0.9x

If it was sold at Rs 56 more it would have been a gain of 4%,

Selling price = 104% of CP = 1.04x

$1.04x = 0.9x + 56$

$0.14x = 56$

$x=\frac{56}{0.14}=400$

Selling price of the article = 0.9x = 0.9 × 400 = 360

18. (c)

Explanation:

CP of 12 dozen Banana = 432

12 Banana (1 dozen) were rotten, he sold only 11 dozen

He sold 5 Banana's for 20Rs

SP of 11 dozen (11 × 12 = 132) Banana's = $\frac{20*132}{5}=528Rs$

Gain = 528 – 432 = 96

$Gain\ \%=\frac{96}{432}*100=22\frac{2}{9}\%$

19. (b)

Explanation:

Marked price = 1500

After giving successive discounts of D% and 20%, the article is sold at Rs. 900

We have

$1500*\frac{(100-D)}{100}*\frac{(100-20)}{100}=900$

$(100-D)=\frac{900*100}{15*80}=75$

D = 100 – 75 = 25%

20. (c)

Explanation:

Discount = D = 8%

Profit percentage = P = 19.6%

Selling Price(SP) = (100 + P)% CP ...(i)

where P = percentage profit and CP = Cost price

Selling Price = (100 – D)% of MRP ...(ii)

Where D = discount offered in percentage and MRP = marked price

equating (i) and (ii) we get

(100 + P)% CP = (100 – D)% MRP = SP

$\frac{CP}{MRP}=\frac{(100-D)\%}{(100+P)\%}$

$\frac{CP}{MRP}=\frac{100-8}{100+19.6}=\frac{92}{119.6}$

Profit percentage = $\frac{27.6}{92}*100=30\%$

Simple and Compound Interest

10

INTRODUCTION

Interest is the cost of using money over time or the return earned on an investment. It is a fundamental concept in finance and plays a crucial role in various financial transactions. Interest can be earned (when you lend money or invest) or paid (when you borrow money or take out a loan). It represents the compensation for the risk taken, the opportunity cost of lending money, or the cost of borrowing.

KEY TERMS

1. **Principal (P)**

   Definition: The initial amount of money invested or borrowed.

   Application: The principal is used as the base for calculating interest.

2. **Rate of Interest (R)**

   Definition: The percentage charged or earned on the principal per time period.

   Application: A crucial factor in determining the amount of interest calculations.

3. **Time (T)**

   Definition: The duration for which the money is invested or borrowed, expressed in years.

   Application: Time plays a significant role in determining the total interest accrued.

4. **Total Amount (A)**

   Definition: The sum of the principal and the interest earned or paid.

   Application: Represents the final amount after interest calculations.

5. **Compounding Period (n)**

   Definition: The number of times interest is compounded per year.

   Application: Affects the frequency with which interest is added to the principal.

INTEREST IS PREVALENT IN VARIOUS FINANCIAL ACTIVITIES, INCLUDING

- **Loans:** Borrowers pay interest on the amount they borrow from a lender. The interest is a percentage of the principal and is determined by the interest rate and the loan term.
- **Investments:** Investors earn interest on the money they have invested. The interest earned depends on the rate of return and the investment duration.
- **Savings Accounts:** Banks pay interest to individuals who deposit money into savings accounts. The interest serves as a reward for keeping money in the account.
- **Bonds:** Bondholders receive interest payments from the issuer at regular intervals. This is a form of income for bond investors.

Interest can be broadly categorized into two main types: **simple interest** and **compound interest.**

SIMPLE INTEREST (SI)

Simple interest is a method of calculating the interest on a principal amount over time. Simple interest is calculated solely on the initial principal amount, without taking into account any interest that accumulates over time.

The formula for calculating simple interest is:

Simple Interest (SI) = $\frac{P \times R \times T}{100}$ Where,

P is the principal amount (the initial amount of money).

R is the rate of interest (in %) per time period.

T is the time the money is invested or borrowed for, expressed in the same time units as the rate.

The resulting value represents the interest earned or paid over the given time period. The total amount after interest can be calculated by adding the simple interest to the principal:

Total Amount = Principal Amount + Simple Interest

Let's look at an example: Suppose you invest ₹1,000 at a simple interest rate of 5% per year for 3 years. Using the simple interest formula:

Simple Interest = $\frac{1000 \times 5 \times 3}{100} = ₹150$

So, the simple interest earned over the 3-year period is ₹150. The total amount after interest would be ₹1,000 + ₹150 = ₹1,150.

TRICKS TO SOLVE QUESTIONS ON SI

Trick 1: Change of rate of interest and time when interest is calculated for different specified time.

If R is rate of interest per annum and T is time period in years

- For calculating SI half-yearly: New Interest rate = $\frac{R}{2}$ %; New Time period = 2T
- For calculating SI quarterly: New Interest rate = $\frac{R}{4}$ %; New Time period = 4T
- For calculating SI monthly: New Interest rate = $\frac{R}{12}$ %; New Time period = 12T

Trick 2: When a sum of money becomes 'n' times of itself in T years at a rate of R% per annum then

$$R = \frac{100(n-1)}{T}\%$$

Trick 3: If an amount P_1 is lent out at simple interest of R_1% per annum and another amount P_2 at simple interest rate of R_2% per annum, then the rate of interest for the whole sum can be given by

$$R = \frac{P_1R_1 + P_2R_2}{P_1 + P_2}$$

QUESTIONS

Q1. Mrs. Patel borrowed a sum of ₹ 10,000 with a simple interest rate of 3% per annum for a duration of two years. The interest amount she will pay over the two-year period is equivalent to 3% of her monthly income. What is Mrs. Patel's monthly salary?

(a) ₹ 30,000 (b) ₹ 16,000
(c) ₹ 20,000 (d) ₹ 12,000

Ans: (c)

Explanation:

According to question

3% of Monthly Income = $\frac{PRT}{100}$

$\frac{(10{,}000 \times 3 \times 2)}{100}$ = 3% of Salary then Salary

= ₹ 20,000

Q2: The difference between the simple interest received from two banks on ₹ 500 for 2 years is ₹ 2.50. What is the difference between their rates?

(UPSC CSAT 2010)

(a) 0.25% (b) 0.5%
(c) 1% (d) 2.5%

Ans: (a)

Explanation:

$$I_1 = \frac{P_1 \times R_1 \times T_1}{100};\ I_2 = \frac{P_2 \times R_2 \times T_2}{100}$$

$$I_2 - I_1 = \frac{P_2 \times R_2 \times T_2}{100} - \frac{P_1 \times R_1 \times T_1}{100}$$

Here, $I_2 - I_1 = 2.50$, $P_1 = P_2 = 500$, $T_1 = T_2 = 2$

$$I_2 - I_2 = \frac{500 \times R_2 \times 2}{100} - \frac{500 \times R_1 \times 2}{100} = 2.50$$

$$10\,(R_2 - R_1) = 2.50 \Rightarrow (R_2 - R_1) = 0.25\%$$

COMPOUND INTEREST(CI)

Compound interest is a method of calculating interest where interest is not only earned on the initial principal amount but also on the accumulated interest from previous periods. This results in exponential growth over time.

- The compound interest formula is given by:

$$A = P \times \left(1 + \frac{r}{n}\right)^{nt}$$

- **Compound Interest (CI) = A – P**

Where:

A is the future value of the investment or loan, including interest.

P is the principal amount (initial investment or loan amount).

r is the annual interest rate (as a decimal).

n is the number of times interest is compounded per year.

t is the time the money is invested or borrowed for, in years.

Let's say you invest ₹1,000 at an annual interest rate of 5%, compounded quarterly (n = 4), for 3 years. Using the compound interest formula:

$$A = 1000 \times \left(1 + \frac{0.05}{4}\right)^{4\times3} = 1607.5$$

This is the future value (A) of the investment after 3 years.

Key Characteristics of CI

1. **Exponential Growth:** Compound interest leads to exponential growth of the invested or borrowed amount over time. The interest earned or paid in each compounding period contributes to the subsequent periods.

2. **Impact of Compounding Frequency (n):** The more frequently interest is compounded, the greater the effect on the total amount. Continuous compounding (where interest is compounded infinitely often) results in the maximum impact.
3. **Time Horizon:** Compound interest has a compounding effect over time. The longer the money is invested or borrowed, the more significant the impact of compounding.
4. **Comparisons with Simple Interest:** Compound interest generally results in a higher total amount compared to simple interest, especially over longer periods.

SIMPLE INTEREST (SI) VS COMPOUND INTEREST (CI)

In Simple Interest

- The interest remains constant over each year.
- The graph of SI would represent a straight line, indicating a linear growth pattern.
- The total amount (principal + interest) increases gradually in a linear fashion.

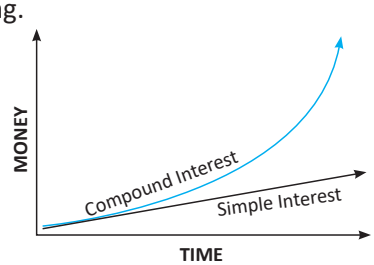
In Compound Interest

- The interest compounds over each year, leading to exponential growth.
- The graph would show a steeper curve compared to the simple interest graph.
- The total amount grows more rapidly due to the compounding effect.

Here's a simple description of how the graph in each case might look:

Simple Interest Graph: A straight line ascending gradually from the initial principal to the final total amount.

Compound Interest Graph: A curved line ascending more steeply, reflecting the exponential growth due to compounding.



The exact shape of the curves would depend on the specific values of the principal, interest rate, and time, but in general, the compound interest graph would exhibit a more pronounced upward curve compared to the linear growth of the simple interest graph.

FORMULAE TO SOLVE QUESTIONS ON CI

Formula 1: When the interest rates vary for different years, calculating compound interest can become a bit more complex. In such cases, you would need to use a modified formula that accounts for the different rates each year. The general formula for compound interest when rates vary is as follows:

$$A = P \times \left(1+\frac{r_1}{n_1}\right)^{n_1 \times t_1} \times \left(1+\frac{r_2}{n_2}\right)^{n_2 \times t_2} \times \ldots \times \left(1+\frac{r_n}{n_n}\right)^{n_n \times t_n}$$

- r_i is the interest rate(in decimal) for the i^{th} period.
- n_i is the number of times interest is compounded per year for the i^{th} period.
- t_i is the time (in years) for which the interest is applied at the i^{th} period.

Formula 2: Exponential decay: Exponential decay is a mathematical model that describes the process of reducing an amount by a consistent percentage rate over a series of time intervals. The general formula for exponential decay is:

$A = P \times (1 - r)^t$

Where:

A is the final amount after t time periods.

P is the initial amount (the starting value).

r is the decay rate per time period as a decimal (the % decrease expressed as a decimal).

t is the number of time periods.

Formula 3: Difference between CI and SI (P is principal and r is annual rate expressed as decimal)

- For the first compounding period (for first year in general), the SI and CI are equal.
- The difference between SI and CI for first two years, is equal to one year's interest on the interest for one year = Pr^2
- The difference between SI and CI for the first three years = $Pr^2(r + 3)$.

Formula 4: A sum of money placed at compound interest becomes x time in 'a' years and y times in 'b' years. These two sums can be related by the following formula:

$(x)^{\frac{1}{a}} = (y)^{\frac{1}{b}}$

Rule of 72

This rule provides a quick estimate of how long it takes for an investment to double at a fixed annual rate of interest. Divide 72 by the annual interest rate to get an approximate number of years for doubling. For example, if the interest rate is 6%, it would take approximately $\frac{72}{6}$ = 12 years to double.

QUESTIONS

Q3. If Sarah invests ₹ 90,000 at the rate of 10% per annum compounded annually, how much interest will she earn at the end of 3 years?

(a) ₹ 1,19,790 (b) ₹ 29,790

(c) ₹ 29,282 (d) ₹ 1,19,282

Ans: (b)

Explanation: P = ₹ 90000

n = 3 [Since Interest is compounded annually]

r = 10% p.a.

Since $A = P\left(1+\frac{r}{100}\right)^n$ $\therefore A = ₹\ 90000\left(1+\frac{10}{100}\right)^3$

$= ₹\ 90000\left(1+\frac{1}{10}\right)^3 = ₹\ 90000\left(\frac{11}{10}\right)^3$

$= ₹\ 90000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10}$

$= ₹\ 90 \times 11 \times 11 \times 11 = ₹\ 119790$

Now, C.I.= A – P = ₹ 119790 – ₹ 90000 = ₹ 29,790.

Q4. John is planning to save for his daughter's education by depositing ₹20,000 in a savings account with compound interest. The bank offers an annual interest rate of 20%, compounded half-yearly. If John keeps the money in the account for 2 years, what will be the interest accumulated at the end of this period?

(a) ₹ 9,790 (b) ₹ 29,790

(c) ₹ 9,282 (d) ₹ 29,282

Ans: (c)

Explanation:

Principal (P) = ₹ 20000

Rate (r) = 20% per annum = $\frac{20}{2}$ % or 10% per half year

Time(n) = 2 years = 4 half years

Since, $A = P\left(1+\frac{r}{100}\right)^n$

$\therefore$ Amount $= ₹\ 20000 \times \left(1+\frac{10}{100}\right)^4$

$= ₹\ 20000 \times \left(1+\frac{1}{10}\right)^4 = ₹\ 20000 \times \left(\frac{11}{10}\right)^4$

$= ₹\ 20000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10}$

$= ₹\ 2 \times 11 \times 11 \times 11 \times 11 = ₹\ 29282$

$\therefore$ C.I.= A – P

$\therefore$ Compound Interest = ₹ (29282 – 20000)

= ₹ 9282.

Q5. Emily wants to accumulate ₹1290 as compound interest in her savings account after 2 years. The bank offers an annual interest rate of 15%. Considering this, what initial amount should Emily deposit into her account?

(a) ₹ 3,000 (b) ₹ 4,000

(c) ₹ 5,000 (d) ₹ 6,000

Ans: (b)

Explanation:

First Method:

Let the sum deposited be ₹ 100

Then, Amount $= ₹\ 100 \times \left(1+\frac{15}{100}\right)^2$

[Since, Amount $= P\left(1+\frac{r}{100}\right)^n$]

Amount $= ₹\ 100 \times \left(1+\frac{3}{20}\right)^2$

$= ₹\ 100 \times \left(\frac{23}{20}\right)^2 = ₹\ \frac{529}{4}$

$\therefore$ Compound Interest $= \frac{529}{4} - 100 = ₹\ \frac{129}{4}$

If C.I. is ₹ $\frac{129}{4}$, then sum deposited = ₹ 100

If C.I. is ₹ 1, then sum deposited = ₹ $100 \times \frac{4}{129}$

If C.I. is ₹ 1290, then sum deposited

$= ₹\ 100 \times \frac{4}{129} \times 1290 = ₹\ 100 \times 4 \times 10 = ₹\ 4,000$

Hence, the sum deposited is ₹ 4,000.

Alternate Method :

Amount $= P\left(1+\frac{r}{100}\right)^n = P\left(1+\frac{15}{100}\right)^2$

$= P\left(1+\frac{3}{20}\right)^2 = P\left(\frac{23}{20}\right)^2 = P \times \frac{23}{20} \times \frac{23}{20}$

$\therefore$ C.I.= $P \times \frac{23}{20} \times \frac{23}{20} - P$

$= P\left(\frac{529}{400} - 1\right)$

But, C.I. = ₹ 1290

$\therefore P\left(\frac{529}{400} - 1\right) = 1290$

or $P\left(\frac{529-400}{400}\right) = 1290$

or $P\left(\frac{129}{400}\right)=1290$

or $P = \frac{1290\times400}{129}$

or $P = 10 \times 400 = 4000$

Hence, Principal = ₹ 4,000.

Q6. On credit, Shyam purchases a bicycle with a price tag of ₹ 4000. If he settles the amount after two years, with an interest rate of 5% for the first year and 15% for the second year, what will be the total cost incurred?

(a) ₹ 4,000 (b) ₹ 4,410

(c) ₹ 4,830 (d) ₹ 5,290

Ans: (c)

Explanation:

Here, $P = 4000$, $R_1 = 5\%$, $R_2 = 15\%$

Amount after 2 years $=P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)$

$= 4000 \times \left(1+\frac{5}{100}\right)\left(1+\frac{15}{100}\right)$

$= 4000 \times \frac{105}{100} \times \frac{115}{100}$

$= 4000 \times \frac{21}{20} \times \frac{23}{20} = ₹ 4830$

Thus, the bicycle will cost ₹ 4830 to Shyam.

Q7. The population of a town was 62500 two years ago. Experiencing an annual decline of 4% due to migration from the cities, what is the current population of the town?

(a) 57,000 (b) 57,500

(c) 57,600 (d) 57,800

Ans: (c)

Explanation:

Population two years ago = 62500

Rate of decrease of population = 4% per annum.

Present population $= 62500 \times \left(1-\frac{4}{100}\right)^2$

$= 62500 \times \left(1-\frac{1}{25}\right)^2$

$= 62500 \times \left(\frac{24}{25}\right)^2$

$= 62500 \times \frac{24}{25} \times \frac{24}{25}$

$= 57600.$

Hence, present population = 57600.

Q8. Starting with an initial count of 10000 bacteria in a culture, the bacterial population undergoes a 10% increase in the first hour, followed by a 10% decrease in the second hour, and concludes with another 10% increase in the third hour. What is the final count of bacteria after the three-hour period?

(a) 11,000 (b) 12,000

(c) 10,890 (d) 10,750

Ans: (c)

Explanation: P = Original count of bacteria = 10000

Bacteria count after 3 hours

$=10000\times\left(1+\frac{10}{100}\right)\times\left(1-\frac{10}{100}\right)\times\left(1+\frac{10}{100}\right)$

$= 10000 \times \frac{11}{10} \times \frac{9}{10} \times \frac{11}{10}$

$= 10 \times 11 \times 9 \times 11 = 10890$

INSTALLMENTS

The concept of installments, in general, involves breaking down a larger payment into smaller, more manageable parts spread over time. Installments are commonly used in various financial transactions, purchases, and loan agreements. Here are key aspects of installments.

1. **Definition:** Installments are a series of regular, fixed payments made by a borrower or buyer to a lender or seller. These payments are typically made at scheduled intervals until the total amount is paid off.
2. **Purpose:** The primary purpose of installment payments is to make larger expenses more affordable and manageable for individuals or businesses. Instead of paying a significant amount upfront, the total cost is divided into smaller, periodic payments.
3. **Common Usage:** Installments are used in a variety of financial transactions, including:
   - **Consumer Purchases:** Buying goods and services, such as appliances, electronics, or furniture, with payment plans.
   - **Loans:** Repaying borrowed funds over time, such as personal loans, auto loans, or mortgages.
   - **Credit Cards:** Paying off credit card balances through monthly installments.
   - **Online Shopping:** Many online retailers offer installment payment options for purchases.

4. **Fixed or Variable Payments:** Installment payments can be fixed, with the same amount paid at each interval, or variable, where the amount may change based on certain factors, such as interest rate fluctuations.
5. **Interest Rates:** In many installment plans, especially for loans, interest may be charged on the outstanding balance. The interest rate is a percentage of the remaining amount and contributes to the total cost of the purchase or loan.
6. **Scheduled Intervals:** Installments are typically paid at regular intervals, such as monthly, quarterly, or annually, depending on the terms of the agreement.

PAYMENTS IN INSTALLMENTS CALCULATION

Let us take a situation where the interest is paid monthly, and the principal along with interest is repaid over four months. Let P be the principal amount (the total loan amount), R be the annual interest rate, t be the time in years, n be the total number of months and 'x' denotes installment variable.

Here's the Calculation for Each Month

End of the First Month:

Amount to be paid = Installment(x) + amount of interest for remaining 3 months.

End of the Second Month:

Amount to be paid = Installment(x) + amount of interest for remaining 2 months.

End of the Third Month:

Amount to be paid = Installment(x) + amount of interest for remaining 1 month.

End of the Fourth Month:

Amount to be paid = Installment(x) + No interest

Each month, the amount to be paid includes the installment (x) plus the proportional interest for the remaining months, based on the total loan period of 4 months. The sum of all these amounts will be equal to the principal borrowed and interest given on that for 4 months.

Let us generalize this concept of payment of installments under both Simple and Compound interest then:

1. **Installments Paid With Simple Interest:**

   General formula for installment calculation under SI:

$$x=\frac{100P}{100n+\frac{n(n-1)r}{2}}$$

   (**Note:** If you calculating monthly installments, replace "n" by number of months, and "r" by "r/12"

2. **Installments Paid With Compound Interest:**

   To calculate the installments paid with compound interest, we use the following formula:

$$P\left(1+\frac{R}{100}\right)^n=x\left(1+\frac{R}{100}\right)^{n-1}+x\left(1+\frac{R}{100}\right)^{n-2}+x\left(1+\frac{R}{100}\right)^{n-3}+x\left(1+\frac{R}{100}\right)^{n-4}+\ldots$$

QUESTIONS

Q9. Suppose Sarah takes out a loan of ₹ 16,550. The loan agreement stipulates that she must repay the borrowed amount in three equal installments at a compound interest rate of 10% per annum. What would be the annual installment amount that Sarah needs to pay to settle the loan?

(a) ₹ 6656 (b) ₹ 6655

(c) ₹ 5517 (d) ₹ 7172

Ans: (b)

Explanation:

Principal =Rs. 16,550, Rate = 10% per annum

Time = 3 years

For Installment at CI,

$$P\left(1+\frac{R}{100}\right)^n=x\left(1+\frac{R}{100}\right)^{n-1}+x\left(1+\frac{R}{100}\right)^{n-2}+x\left(1+\frac{R}{100}\right)^{n-3}+x\left(1+\frac{R}{100}\right)^{n-4}+\ldots$$

$$16550=\frac{x}{1+\frac{R}{100}}+\frac{x}{\left\{1+\frac{R}{100}\right\}^2}+\frac{x}{\left\{1+\frac{R}{100}\right\}^3}$$

$$16550=\frac{x}{1+\frac{10}{100}}+\frac{x}{\left\{1+\frac{10}{100}\right\}^2}+\frac{x}{\left\{1+\frac{10}{100}\right\}^3}$$

$$16550 = \frac{x}{1+\frac{1}{10}} + \frac{x}{\left\{1+\frac{1}{10}\right\}^2} + \frac{x}{\left\{1+\frac{1}{10}\right\}^3}$$

$$16550 = \frac{x}{\frac{11}{10}} + \frac{x}{\left\{\frac{11}{10}\right\}^2} + \frac{x}{\left\{\frac{11}{10}\right\}^3}$$

$$16550 = \frac{10x}{11} + \frac{100x}{121} + \frac{1000x}{1331}$$

$$16550 = \frac{1210x + 1100x + 1000x}{1331}$$

$$16550 = \frac{3310x}{1331}$$

$$x = 6655$$

Each installment = ₹ 6655

Q10. A person bought a refrigerator worth Rs. 22800 with 12.5% interest compounded yearly. At the end of the first year he paid Rs. 8650 and at the end of second year Rs. 9125. How much will he have to pay at the end of third year to clear the debt? **(UPSC CSAT 2018)**

(a) Rs. 9990 (b) Rs. 10000

(c) Rs. 10590 (d) Rs. 11250

Ans: (d)

Explanation:

Using the Formula Simple Interest

$= \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$

1st year's Interest = 22800 × 12.5/100 = 2850

Amount at the end of I Year = 22800 + 2850 = 25650.

Payment made for first year = 25650 – 8650 = 17000

Interest on 17,000: 17000 × 12.5/100 = 2125

Total Amount after Second year:

17000 + 2125 = 19125

Payment made at the end of second year: 9125.

Amount Remaining: 19125 – 9125= 10000.

Interest at the end of third year:

$10000 \times \frac{12.5}{100} = 1250$

Total amount to be paid: 10000 + 1250 = 11250.

Formulae Summary

- Amount = Principal + Interest
- Simple Interest = $\frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$

Compound Interest $= P \times \left(1+\frac{R}{100}\right)^T - P$

Where,

- P = Principal
- R = Rate
- T = Time

- **Doubling of money** – Rule of 72 i.e. **Rate × Time = 72** (approx.)
- **Tripling of money** – Rule of 114 i.e. **Rate × Time = 114** (approx.)
- **Quadrupling of money** – Rule of 144 i.e. **Rate × Time = 144** (approx.)

PRACTICE QUESTIONS

1. **In what time (in years) will the simple interest on ₹800 at 5% per annum be the same as the simple interest on ₹2000 for 5 years at 4% per annum?**
   (a) 10 (b) 2
   (c) 6 (d) 5
2. **Rohit created a fixed deposit of ₹30,000 in the State Bank of India. If the rate of interest is 15% Simple interest per annum, charged half yearly then what amount will he get after 54 months?**
   (a) ₹ 50100 (b) ₹ 45020
   (c) ₹ 50250 (d) ₹ 40150
3. **A sum of money becomes four times at the simple interest rate of 5% per annum. At what rate the sum would have become ten times in the same time?**
   (a) 10 % (b) 25 %
   (c) 20 % (d) 15 %
4. **Aman took a loan of ₹20000 at 8% simple interest per year. At the end of second year he paid back a certain amount and at the end of fifth year he paid back ₹17980 and cleared the loan. What is the amount he pays back after the second year?**
   (a) ₹ 8700 (b) ₹ 7700
   (c) ₹ 8500 (d) ₹ 9600
5. **A sum of ₹ 3100 was lent partly at 5% and partly at 8% simple interest. The total interest received after 3 year was ₹600. The ratio of money lent at 5% to that of 8% is?**
   (a) 16 : 15 (b) 11 : 12
   (c) 21 : 12 (d) 8 : 9
6. **The compound interest on ₹48000 compounded semi-annually for $1\frac{1}{2}$ years at the rate of 10% per annum is-**
   (a) ₹ 7566 (b) ₹ 7832
   (c) ₹ 2763 (d) ₹ 3763
7. **The amount and the compound interest on ₹200000 compounded quarterly for 9 months at the rate of 4% per annum is-**
   (a) ₹106060 and ₹6060
   (b) ₹206060.2 and ₹ 6060.2
   (c) ₹106060.2 and ₹6060.2
   (d) ₹206060 and ₹6060
8. **The simple interest on a certain sum of money for 2 years at 12% per annum is half the compound interest on ₹8000 for 2 years at 10% per annum. What is the sum placed on simple interest?**
   (a) ₹ 1550 (b) ₹ 1650
   (c) ₹ 1750 (d) ₹3500
9. **A fund house gives interest which compounds annually. Rakesh deposited ₹ 1000 and received ₹ 1210 at the end of the second year. The rate of compound interest per annum is-**
   (a) 10% (b) 15%
   (c) 11.5% (d) 20.5%
10. **A sum compounded annually becomes 27/8 times of itself in 3 years. Then, the rate of interest per annum is-**
    (a) 35% (b) 25%
    (c) 50% (d) 30%
11. **Pradeep borrowed ₹9600 from Gopal at 9% per annum simple interest for 3 years. He then added some more money to the borrowed sum and lent it to Rohit for 4 years at 9% per annum simple interest. If Pradeep gains ₹1440 in the whole transaction, how much money (in ₹) did he add from his side?**
    (a) ₹ 2600 (b) ₹ 1600
    (c) ₹ 2500 (d) ₹ 2400
12. **A man borrows ₹4000 at 8% per annum on compound interest. At the end of every year, he pays ₹1500 as part payment of loan and interest. How much does he still owe to the bank after three such annual payments?**
    (a) ₹ 179 (b) ₹ 200
    (c) ₹ 169.25 (d) none of these
13. **The difference between the compound interest and the simple interest on a certain sum at 25% per annum for 2 years is ₹ 90. Then, sum is:**
    (a) ₹1250 (b) ₹1600
    (c) ₹1450 (d) ₹1440
14. **One year ago, the ratio between father's and daughter's age was 4:1. The ratio of their ages 4 years hence will be 3:1. Find the ratio of their age 4 years ago.?**
    (a) 8 : 15 (b) 7 : 11
    (c) 37 : 7 (d) 4 : 7
15. **P, Q, R together contributes ₹60,000 for a business. P contributes ₹5000 more than Q and Q contributes ₹8000 more than R. Out of a total profit of ₹57,000, the share of Q is?**
    (a) ₹ 12000 (b) ₹ 13500
    (c) ₹ 16700 (d) ₹ 19950

ANSWERS

1. (*a*)	2. (*c*)	3. (*d*)	4. (*a*)	5. (*a*)	6. (*a*)	7. (*b*)	8. (*d*)	9. (*a*)	10. (*c*)
11. (*b*)	12. (*c*)	13. (*d*)	14. (*c*)	15. (*d*)					

EXPLANATIONS

1. (a)

Explanation:

Here, Principal = ₹2000, Time = 5 year, and Rate = 4%

$\therefore$ Simple interest = $\frac{P*R*T}{100} = \frac{2000*5*4}{100}$ = ₹400

Now, Simple interest = ₹400, Principal = ₹800, Rate = 5%

Then, Time = $\frac{100*SI}{P*R} = \frac{100*400}{800*5}$ = 10 years

Therefore in 10 years simple interest on ₹800 at 5% per annum will be the same as the simple interest on ₹2000 for 5 years at 4% per annum.

2. (c)

Explanation:

Now given that P = ₹ 30000, R = 15%, T = 54 months

It is given that interest is charged half yearly

R = $\frac{15}{2}$ and T = $\frac{54}{12}$ × 2 = 9 half years

Now, SI = $\frac{P*R*T}{100}$

SI = $\frac{30000*7.5*9}{100}$ = ₹ 20250

Now, Amount = Principal + Interest

Amount = ₹ 30000 + ₹ 20250

Amount = ₹50,250

Therefore, Rohit will get ₹ 50250 after 54 months.

3. (d)

Explanation:

Given that Amount = 4P

R = 5%

Time =T

SI + P = 4P

SI = 3P

Simple interest = $\frac{P*R*T}{100}$

3P = $\frac{P*5*T}{100}$ T = 60

Given that time is same, Amount = 10P

SI = 10P – P = 9P

9P = $\frac{P*R*60}{100}$ R = 15

Hence, the correct answer is 15%.

4. (a)

Explanation:

Here, Principal = ₹20000

Rate of interest = 8%

The interest for 2 year = $\frac{20000*8*2}{100}$ = ₹ 3200

Assume that he paid ₹x at the end of the second year.

$\therefore$ Interest will be calculated on

₹ (20000 - x + 3200)

Interest for 3 year = $\frac{(23200-x)*8*3}{100}$

$= \frac{6}{25}(23200-x)$

Amount = P + SI

$\Rightarrow \frac{6}{25}(23200-x) + 23200 - x = 17980$

$\Rightarrow 6(23200 - x) + 25(23200 - x) = 449500$

$\Rightarrow 31(23200 - x) = 449500$

$\Rightarrow x = 8700$

Therefore, Aman paid ₹8700 after the second year.

5. (a)

Explanation:

Let the sum lent at the rate 5% be ₹ x.

Then, sum lent at rate 8% = ₹ $(3100 - x)$

$\therefore$ Total Simple interest = 600

$\therefore \frac{x*5*3}{100} + \frac{(3100-x)*8*3}{100} = 600$

$\Rightarrow \frac{15x}{100} + \frac{74400}{100} - \frac{24x}{100} = 600$

$\Rightarrow x$ = ₹1600

Amount lent at rate 8 % = (3100 - 1600) = ₹1500

$\therefore$ Required ratio = $\frac{1600}{1500} = \frac{16}{15}$ = 16 : 15

6. (a)

Explanation:

Given. P = ₹ 48,000

Rate of interest = 10 % per annum

And time = $1\frac{1}{2}$ yr = $\frac{3}{2}$ yr

Since, the interest is compounded half- yearly

Then, rate(R) = $\frac{10}{2}$ % half -yearly

And time (n) = $2 \times \frac{3}{2} = 3$ half-yearly

$\therefore A = 48000\left(1+\frac{5}{100}\right)^3 \left[A = P\left(1+\frac{R}{100}\right)^n\right]$

$= 48000\left(1+\frac{1}{20}\right)^3$

$= 480000\left(\frac{21}{20}\right)^3$

$= 48000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} = ₹55566$

Now, compound interest = A – P

= ₹55566 – ₹48000 = ₹7566

7. (b)

Explanation:

Given, P = 200000, rate of interest = 4% per annum

And time = 9 months

Since, the interest is compounded quarterly.

Then, rate (R) = $\frac{4}{4}$ = 1% quarterly

And time (n) = 9 months = $\frac{9}{12} \times 4 = 3$ quarter yearly

$\therefore$ Amount, $A = P\left(1+\frac{R}{100}\right)^n = 200000\left(+1\frac{1}{100}\right)^3$

$= 200000\left(\frac{101}{100}\right)^3 = 200000 \times \frac{101}{100} \times \frac{101}{100} \times \frac{101}{100}$

= ₹206060.20

$\therefore$ Compound interest

= A – P = ₹206060.20 – ₹200000 = ₹ 6060.20

8. (d)

Explanation:

Let the principal amount be ₹P

By given condition, SI = $\frac{1}{2}$ CI

$\Rightarrow \frac{P*12*2}{100} = \frac{1}{2}\left[8000\left(1+\frac{10}{100}\right)^2 - 8000\right]$

$\left[\therefore A = P\left(1+\frac{R}{100}\right)^n\right]$

$\Rightarrow \frac{24P}{100} = \frac{1}{2}\left[8000 \times \frac{121}{100} - 8000\right]$

$\Rightarrow \frac{24P}{100} = \frac{1}{2}\left[9680 - 8000\right]$

$\Rightarrow \frac{24P}{100} = 840$

$\therefore P = \frac{840*100}{24} = ₹3500$

9. (a)

Explanation:

Given, Principal (P) = ₹ 1000

Amount (A) received after 2 yr = ₹ 1210

Let rate of interest = R% per annum

$\therefore A = P\left(1+\frac{R}{100}\right)^n$

$\therefore 1210 = 1000\left(1+\frac{R}{100}\right)^2$

$\Rightarrow \frac{1210}{1000} = \left(\frac{100+R}{100}\right)^2$

$\Rightarrow \frac{11}{10} = \frac{100+R}{100}$ $\Rightarrow 100 + R = 110$

$\therefore R = 110 - 100 = 10$

$\therefore$ Rate of interest = 10%

10. (c)

Explanation:

Let principal be ₹ x and the rate be R% per annum.

$\therefore$ Amount, $A = \frac{27}{8}x$ and $n = 3$ year

$\left[\therefore A = P\left(1+\frac{R}{100}\right)^n\right]$

$\therefore \frac{27}{8}x = x\left(1+\frac{R}{100}\right)^3$

$\Rightarrow \frac{27}{8} = \left(1+\frac{R}{100}\right)^3$ $\Rightarrow \frac{3}{2} = \left(1+\frac{R}{100}\right)$

$\Rightarrow \frac{R}{100} = \frac{1}{2}$

$\therefore R = 50$ %

11. (b)

Explanation

Let the money added be ₹ P, Then,

$\Rightarrow \frac{((9600+P)*9*4)}{100} - \frac{(9600*9*3)}{100} = 1440$

$\Rightarrow (9600 + P) \times 36 - (9600 \times 27) = 1440 \times 100$

$\Rightarrow (9600 + P) \times 4 - (9600 \times 3) = 16000$

$\Rightarrow 9600 + 4P = 16000$

$\Rightarrow 4P = 6400$

$\therefore P = 1600$

So, money added is ₹ 1600.

12. (c)

Explanation

Amount at the end of 1 year

$= 4000\left(1+\frac{8}{100}\right) = ₹\ 4320$

∴ Amount due after 1 year

= (4320 – 1500) = ₹ 2820

Amount at the end of second year

$= 2820\left(1+\frac{8}{100}\right) = ₹\ 3045.60$

∴ Amount due after second year

= (3045.60 – 1500) = ₹ 1545.60

Amount at the end of third year

$= 1545.60\left(1+\frac{8}{100}\right) = ₹\ 1669.25$

Amount due after the payment of third installment is

= (1669.25 – 1500) = ₹ 169.25

13. (d)

Explanation

Let the sum be ₹100.

Case-I: Then, simple interest on ₹100 for 2 year

$=\left(\frac{100*25*2}{100}\right) = ₹50$

Case-II: Amount when ₹100 is borrowed for 2 years on compound interest

$= 100\left(1+\frac{25}{100}\right)^2$ [$\therefore A = P\left(1+\frac{R}{100}\right)^n$]

$= 100 \times \frac{5}{4} \times \frac{5}{4} = ₹\left(\frac{625}{4}\right)$

$\therefore$ Compound interest for 2 yr $= \left(\frac{625}{4} - 100\right)$

$= ₹\left(\frac{225}{4}\right)$

Difference between CI and SI $= \left(\frac{225}{4} - 50\right) = ₹\left(\frac{25}{4}\right)$

If the difference between CI and SI is ₹ $\left(\frac{25}{4}\right)$, then the sum = ₹ 100

If the difference between CI and SI is ₹ 90, then the sum $= ₹\left(100 \times \frac{4}{25} \times 90\right) = ₹1440$

Here, the sum is ₹ 1440.

Shortcut method

Difference (D) between SI and CI for 2 years is given by $D = \frac{PR^2}{100^2}$

14. (c)

Explanation

Let the present age of father be 'x' and daughter be 'y'

According to the question $\frac{x-1}{y-1} = \frac{4}{1}$

$\Rightarrow x - 1 = 4y - 4$

$\Rightarrow x = 4y - 3$ ...(i)

Also, $\frac{x+4}{y+4} = \frac{3}{1}$

$\Rightarrow x + 4 = 3y + 12$

$\Rightarrow x = 3y + 8$ ...(ii)

From equation (i) & (ii) we get

$4y - 3 = 3y + 8$

$\Rightarrow y = 11$

So, $x = 41$

Now the ratio 4 years before is $\frac{41-4}{11-4} = \frac{37}{7}$

Required ratio is 37:7

15. (d)

Explanation:

Let the contribution of R be x.

Then, Q contribute = x + 8000

and P contribute = x + 8000 + 5000 = x + 13000.

So, Total contribution = $x + x$ + 8000 + x + 13000

= 60000 (given)

$\Rightarrow$ 3x = 39000 $\Rightarrow$ x = 13000

P : Q : R = 26000 : 21000 : 13000 = 26 : 21 : 13

Now, Q's share = 57,000 × $\frac{21}{60}$ = ₹19950.

Time, Speed and Distance

11

INTRODUCTION

The chapter on Time, Speed, and Distance explores fundamental concepts vital for understanding motion and travel. It delves into the relationships between the three key elements: time taken for a journey, the speed at which the journey is made, and the distance covered. Through examples, we will learn to calculate one of these elements when the other two are known, apply these concepts to real-life situations, and solve various types of problems that we may encounter in exams or in everyday life.

CONCEPT OF MOTION

When an object travels from point A to point B, covering a distance (D), it takes a certain amount of time (T) to do so at a specific speed (S).

Expressing the relationship among T, S, and D: **T × S = D,** meaning Time multiplied by Speed equals Distance or **Time x Speed = Distance**

From above relation we can deduce that:

- **Distance = Speed × Time,**
- **Time** $= \frac{\textbf{Distance}}{\textbf{speed}}$
- **Speed** $= \frac{\textbf{Distance}}{\textbf{time}}$

Note: To solve the problem, all the units involved in the calculation must be uniform i.e., either all of them be in meters and seconds or in kilometers and hours etc.

CONVERSION OF UNITS

- 1 km/hr $= \frac{5}{18}$ m/sec and 1 m/sec $= \frac{18}{5}$ km/hr

  [1 km =1000m, 1 hr = 60 min, 1 min = 60 sec]
- 1 mile =1609.30 m =1.6093 km and
- 1 km = 0.621mile, 1 yard = 0.9144 m and 1 m =1.0936 yards and 1 m = 39.4 inches

Average Speed

1. If a certain distance is covered at a speed of x km/h and then same distance is covered at a speed of y km/h, then the average speed for the whole journey $= \frac{2xy}{x+y}$.

To understand this let's look at example.

EXAMPLES

E1. If half the journey is travelled at a speed of 15 km/h and next half of the journey at a speed of 10 km/h what is the average speed of the whole journey:

(a) 12 km/h (b) 12.5 km/h

(c) 25 km/h (d) 17 km/h

Ans: (a)

Explanation:

In the Question

x = 15 km/h and y = 10 km/h

So the average speed

$$= \frac{2xy}{x+y} = \frac{2.15.10}{15+10} = \frac{300}{25} = 12 \text{ km/h}$$

2. If a person or a motor car covers three equal distances at the speed of x km/h, y km/h and z km/h respectively, then the average speed for the entire journey $= \frac{3xyz}{xy+yx+zx}$

Following example will us in better understanding:

EXAMPLES

E2. A bus covers the first 160 km at a speed of 120 km/h, another 160 km at 140 km/h and the last 160 km at 80 km/h. Find the average speed of the bus for the entire journey.

(a) 104 km/h (b) 107.23 km/h

(c) 105.1 km/h (d) 106 km/h

Ans: (b)

Explanation:

Speed of the bus in 1st 160 km =

x = 120 km/h

Speed of the bus in 2nd 160 km = y = 140 km/h

Speed of the bus in last 160 km = z = 160 km/h

Average speed $= \frac{3xyz}{xy+yx+zx}$

$$= \frac{3.120.140.160}{120.140+140.160+160.120} = \frac{4032000}{37600} = 107.23 \text{km/h}$$

3. If a person covers A km at a speed of x km/h, B km at a speed of y km/h and C km at a speed of z km/h, then average speed during the whole journey $= \frac{A+B+C}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}$. Following example will help us in better understanding:

EXAMPLES

E3. A person covers 20 km at a speed of 10 km/h, 60 km at a speed of 24 km/h and 10 km at a speed of 20 km/h. Find the average speed for the entire journey.

(a) 18 km/h (b) 15 km/h

(c) 8 km/h (d) 21 km/h

Ans: (a)

Explanation:

In this question $A = 20$, $B = 60$ and $C = 10$

$x = 10$, $y = 24$ and $z = 20$

So, the average speed

$$= \frac{A+B+C}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}} = \frac{20+60+10}{\frac{20}{10}+\frac{60}{24}+\frac{10}{20}} = \frac{90}{5} = 18\text{km/h}$$

RELATIVE MOTION WITH TWO OR MORE BODIES

❑ **When two bodies are moving in the same direction**

If the speeds of bodies A and B are S_A and S_B, then their relative speed is $S_A - S_B$ or $S_B - S_A$. In simpler terms, when they're moving together, the relative speed is the difference between their speeds, always treated as positive.

❑ **When two bodies are moving in the opposite direction:**

If the speeds of bodies A and B are S_A and S_B, then their relative speed is $S_A + S_B$. Put simply, when they're moving towards each other, the relative speed is the sum of their speeds.

Following example will help us in understanding these concepts:

EXAMPLES

E4. Two places P and Q are 800 km apart from each other. Two persons, Aman and Bharat start from P towards Q at an interval of 2 hours. Whereas Aman leaves P for Q before Bharat. The speeds of Aman and Bharat are 40 km/h and 60 km/h respectively. Bharat overtakes (or catches or meets) Aman at M, which is on the way from P to Q.

(i) How long will Bharat take to overtake?

(ii) What is the distance from P, where Bharat overtakes Aman (i.e., PM)?

(iii) What is the ratio of time taken by Aman and Bharat to meet at M?

(iv) What is the extra time required by Aman to reach Q?

(v) How many hours later Aman will reach Q than that of Bharat?

(vi) After how many hours Aman and Bharat will be separated by 50 km before M, when both are moving?

(vii) How many hours does Bharat require to advance himself, by 100 km, in comparison to Aman?

Explanation:

(i) Since Aman moves 2 hours earlier than Bharat at the speed of 40 km/h, so in 2 hours Aman will cover 80 km. Thus, Aman will be 80 km away from Bharat.

Now, since they are moving in the same direction therefore their relative speed = 60 – 40 = 20 km/h

It means either they reduce (or create) the difference of 20 km in each hour between themselves. Thus, the required time to overtake or reduce the difference by

$$80\text{km} = \frac{\text{difference in distance}}{\text{relative speed}} = \frac{80}{20} = 4\text{hours}$$

(ii) The distance between P and M

PM = (Time required to overtake × Speed of the faster body) = 4 × 60 = 240 km or Bharat has to move for 4 hours at a speed of 60 km/h to reach at point M so the PM = 60 x 4 = 240 km.

(iii) PM = 240 km

Time taken by Aman to reach M = $\frac{240}{40} = 6\text{hours}$

Time taken by Bharat to reach M = $\frac{240}{60} = 4\text{hours}$

So the ratio of time taken by Aman and Bharat to meet at M = 6 : 4 = 3 : 2

(iv) Time taken by Aman to reach Q = $\frac{800}{40} = 20\text{hr}$

Time taken by Bharat to reach Q

$= \frac{800}{60} = 13\text{hr}\,20\text{minutes}$

the extra time required by Aman to reach Q = 20 hr – 13 hr 20 minutes = 6 hr 40 minutes

(v) Since Aman leaves 2 hours earlier, he will reach Q only 4 hours 40 minutes late.

(vi) When Bharat starts to move towards Q the difference between Aman and Bharat is 80 km.

The required difference between Aman and Bharat = 50 km

Hence, they have to reduce it by 30 km.

Their relative speed = 20 km/hr

So the time take to reduce the difference by 30 km

$= \frac{30}{20} = \frac{3}{2}$ hr or 90 minutes

(vii) When Bharat starts to follow Aman (towards Q) Aman was 80 km ahead of Bharat.

Also Bharat wants to overtake Aman and further go ahead of Aman by 100 km.

Thus, the net difference (required) = 180 km.

$\therefore$ Required time

$= \frac{(\text{distance advanced + required difference})}{\text{relative speed}}$

$= \frac{(100+80)}{20} = \frac{180}{20} = 9$ hrs.

E5. The distance between two places P and Q is 700 km. Two persons A and B started towards Q and P from P and Q simultaneously. The speed of A is 30 km/h and speed of B is 40 km/h. They meet at a point M which lies on the way from P to Q.

(i) How long will they take to meet each other at M?

(ii) What is the ratio of PM : QM ?

(iii) What is the extra time needed by A to reach at Q than to reach at P by B?

(iv) What is the ratio of time taken by A and B to reach their respective destinations after meeting at M?

Explanation:

(i) Since, they are coming towards each other from opposite ends, therefore the relative speed will be the sum of their speeds = 30 + 40 = 70 km/h.

Thus, the required time to meet at M = Time required to cover 700 km with a speed 70 km/h

Time taken = $\frac{700}{70} = 10$ hours

(ii) PM = distance travelled by A in 10 hours = 300 km

MQ = distance travelled by B in 10 hours = 400 km

PM : MQ = 300 : 400 = 3 : 4

(iii) Time taken by A to reach

Q = $\frac{700}{30} = 23$ hours 20 minutes

Time taken by B to reach

A = $\frac{700}{40} = 17$ hours 30 minutes

Difference = 5 hours 50 minutes

(iv) Time required to A to cover MQ = $\frac{400}{30}$ hours = 13 hours 20 minutes= 800 minutes

Time required to B to cover PM

$= \frac{300}{40}$ hours = 7 hours 30 minutes = 450 minutes

the ratio of time taken by A and B to reach their respective destinations after meeting at M = 800 : 450 = 16 : 9

MOTION IN A STRAIGHT LINE

This idea is like adding more information to what we already learned about how things move in relation to each other.

(a) When Two Things Start Moving Closer To Each Other.

Suppose there's a starting gap (let's call it D) between two things, A and B. Regardless of how fast A and B are (their speeds S_A and S_B), they both need to cover that initial distance (D) to meet for the first time. The meeting distance doesn't change based on how fast each of them is; they just have to cover that distance.

(i) They meet by covering distances in proportion to their individual speeds.

(ii) If the initial gap between two bodies, A and B, is D, then A and B collectively need to cover a distance of D for their first meeting.

(iii) In the following meetings (like second, third, fourth, and so on), A and B need to cover an additional distance of 2D compared to the previous meeting. For example, for the fourth meeting, they have to cover $D + (3 \times 2D) = 7D$ distance together. Likewise, for the seventh meeting, they need to cover $D + (6 \times 2D) = 13D$ units of distance together. So, with each subsequent meeting, the distance they cover is 2D more than the previous one.

(b) When two bodies start moving towards the same direction

Suppose two bodies, A and B, start at the same end of a track. They have speeds S_A and S_B, and the track has a length D. For their first meeting, A and B together need to cover a distance of 2D, regardless of how fast they are.

(i) When they start moving, the first meeting requires covering a distance of 2D, given that the initial distance between two specific points is D. The faster body reaches the opposite end before the slower one, starts heading back, and they meet somewhere in between, each covering distances based on their respective speeds

(ii) In each following meeting, they need to collectively cover an additional 2D distance compared to the previous one. Therefore, for the nth meeting, they have to cover a total distance of (n × 2D) units together.

(iii) Their speeds and distances are always in the same ratio at any moment.

Following example will help us in better understanding:

EXAMPLES

E6. The distance between two points P and Q is 100 m. A is initially at P and B is at Q. The speeds of A and B is 20 m/s and 30 m/s. They move between P and Q to and fro :

(i) Find the time required for the first meeting.

(ii) Distance covered by A till the third meeting.

(iii) Distance covered by B till the fifth meeting.

(iv) The distance between P and the place of the fourth meeting.

(v) The distance between Q and the place of the fifth meeting.

(vi) The ratio of distances covered by each one till the third meeting.

Explanation: We can draw a table for the given question:

No. of meeting	Distance covered together	Time (second)	Distance covered by A	Distance covered by B	Distance between P and point of meeting	Distance between Q and point of meeting
1st	100m	2	40m	60m	40m	60m
2nd	300m	6	120m	180m	80m	20m
3rd	500m	10	200m	300m	At P	At P
4th	700m	14	280m	420m	80m	60m
5th	900m	18	360m	540m	40m	20m

(i) 2 second

(ii) 200 m (Distance = Speed x Time = 20 x 10 = 200 m)

(iii) 540 m (D = 18 × 30 = 540 m)

(iv) 80 m

(v) 60 m

(vi) 2 : 3 is always equal to the ratio of respective speeds.

E7. The distance between P and Q is 100 m and the speeds of A and B are 20 m/s and 30 m/s respectively. Initially A and B are at P. They move to and fro between P and Q.

(i) The distance covered by A i n the first meeting.

(ii) The time required for the second meeting.

(iii) The distance covered by B to meet for the third time.

(iv) The ratio of distances covered by A and B till the fourth meeting.

(v) The distance between P and the place of the fifth meeting.

(vi) The distance between Q and the point of the third meeting.

Explanation: We can draw a table for the given question:

No. of meeting	Distance covered together	Time (second)	Distance covered by A	Distance covered by B	Distance between P and point of meeting	Distance between Q and point of meeting
1st	200m	4	80m	120m	80m	20m
2nd	400m	8	160m	240m	40m	60m
3rd	600m	12	240m	360m	40	60
4th	800m	16	320m	480m	80m	20m
5th	1000m	20	400m	600m	At P	100m

(i) The distance covered by A in the first meeting = 80 m

(ii) Time required for the second meeting = 400 /50 = 8 s

(iii) The distance covered by B for the third time meeting = 12 x 30 = 360 m

(iv) A : B = 320 : 480 = 2 : 3

(v) 0m

(vi) 60m

QUESTIONS

Q1. Kush covered half of his journey at 20 km/h and the rest at x km/h, then his average speed is 24 km/h. What is the value of x?

(a) 30 (b) 32

(c) 36 (d) 40

Ans: (a)

Explanation:

Average speed = $\frac{2xy}{x+y}$ implies

$24 = \frac{2.20.x}{x+20}$ or $24(x + 20) = 40x$

Or $24x + 480 = 40x$

0r $16x = 480$ or $x = 30$

Q2. A person *X* starts from Lucknow and another person Y starts from Kanpur to meet each other. Speed of *X* is 15 km/h, while the speed of *Y* is 25 km/h. If the distance between Lucknow and Kanpur is 120 km and both X and Y start their journey at the same time, when will they meet?

(a) 1 h later (b) 2 h later

(c) $\frac{1}{2}$ h later (d) 3 h later

Ans: (d)

Explanation:

X and Y are starting from opposite points so their relative speed = 15 + 25 = 40 km/h

They need to cover 120 km distance to meet each other

So the time taken by them to meet each other = $\frac{120}{40}$ = 3hr

Q3. Aarav and Bhanu travel the same distance at 9 km/h and 10 km/h respectively. If A takes 20 minutes longer than B, the distance travelled by each is:

(a) 16 (b) 20

(c) 30 (d) none of these

Ans: (c)

Explanation:

Let's take the distance travelled by them = *x* km

Speed of Aarav = 9 km/h

Time taken by Aarav to travel *x* km = $\frac{x}{9}$hours Speed of Bhanu = 10 km/h

Time taken by Bhanu to travel x km = $\frac{x}{10}$hours

Now according to the question

$\frac{x}{9}-\frac{x}{10}=\frac{20}{60} or \frac{x}{90}=\frac{1}{3} or x=30$

Q4. A car travels from a place X to place Y at an average speed of v km/hr, from Y to X at an average speed of 2v km/hr, again from X to Y at an average speed of 3v km/hr and again from Y to X at an average speed of 4v km/hr. Then the average speed of the car for the entire journey **(UPSC CSAT 2020)**

(a) is less than v km/hr

(b) lies between v and 2v km/hr

(c) lies between 2v and 3v km/hr

(d) lies between 3v and 4v km/hr

Ans: (b)

Explanation:

Let the distance between *X* and *Y* be 12 km and *v* = 1 km/hr. (for simplification)

Time taken in 1st journey = Distance/Speed = 12/v = 12/1 = 12 hrs

Time taken in 2nd journey = 12/2v = 12/2 = 6 hrs

Time taken in 3rd Journey = 12/3v = 12/3 = 4 hrs

Time taken in 4th Journey = 12/4v = 12/4 = 3 hrs

Now, total distance = 12 × 4 = 48 kms

Average Speed = Total Distance/Total Time = 48 / (12 + 6 + 4 + 3) = 48/25 (this value lies between 1 and 2)

So, the average speed of the car for the entire journey lies between v and 2v.

Q5. In a race, a competitor has to collect 6 apples which are kept in a straight line on a track and a bucket is placed at the beginning of the track which is the starting point. The condition is that the competitor can pick only one apple at a time, run back with it and drop it in the bucket, how much total distance he has to run if the bucket is 5 metres from the first apple and all other apples are placed 3 metres apart? **(UPSC CSAT 2016)**

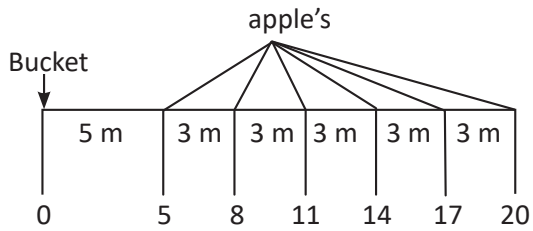
(a) 40 m (b) 50 m

(c) 75 m (d) 150 m

Ans: (d)

Explanation:

6 apples are placed 3 metres apart in a straight line and the bucket is 5 metres away from the first apple as shown in figure:



The competitor can pick one apple at a time and run back to drop in the bucket.

Distance travelled in picking the 1st apple and run back to drop it in the bucket

= 5 + 5 = 10 m

Distance travelled in picking the 2nd apple and run back to drop it in the bucket

= 2(5 + 3) = 16 m

Distance travelled in picking the 3rd apple and run back to drop it in the bucket

= 2(5 + 6) = 22 m

Distance travelled in picking the 4th apple and run back to drop it in the bucket

= 2(5 + 9) = 28 m

Distance travelled in picking the 5th apple and run back to drop it in the bucket

= 2(5 + 12) = 34

Distance travelled in picking the 6th apple and run back to drop it in the bucket

= 2(5 + 15) = 40

So, total distance travelled

= 10 + 16 + 22 + 28 + 34 + 40 = 150 m

TRAIN -RELATED CONCEPTS

(i) The relative speed of two trains(or objects) is the sum of their individual speeds when they move in opposite directions.

(ii) The difference of their speeds is equal to their relative speed when both trains are moving in the same direction.

(iii) The sum of their individual lengths is always equal to the distance they need to cover to cross each other.

(iv) The length of the train and the bridge, platform, etc. are always added together to get the distance they need to cover to cross the bridge, platform, etc.

(v) The length of the train is the only factor that determines the distance they need to cover for objects like poles, trees, etc.

QUESTIONS

Q6. A train crosses a tree in 10 seconds. If the length of the train is 200 m, then find the speed of the train.

(a) 75 km/h (b) 65 km/h

(c) 72 km/h (d) 80 km/h

Ans: (c)

Explanation:

Distance = Length of train

= Speed of train × Time

200 = Speed × 10

$\Rightarrow$ Speed = 20 m/s

Speed = $20 \times \frac{18}{5}$ = 72 km/h

Q7. A train of length 300 m crosses a bridge of length 200 m in 20 seconds. What is the speed of the train?

(a) 75 km/h (b) 65 km/h

(c) 72 km/h (d) 90 km/h

Ans: (d)

Explanation:

Length of train + Length of bridge

= Speed of train × Time 300 + 200 = 20 × Speed

Speed = $\frac{500}{20}$ = 25 m/s = 90 km/h

Q8. A train crosses a man coming from the opposite direction in 6.5 seconds. If the speed of man is 10 m/s and the speed of the train is 30 m/s, the length of the train is:

(a) 260 m (b) 265 m

(c) 270 m (d) 290 m

Ans: (a)

Explanation:

Length of train = Time × Relative speed

= 6.5 × (10 + 30) = 6.5 × 40 =260 m

Q9. Two trains coming from the opposite sides cross each other in 10 seconds if the lengths of first train and second train be 250 m and 150 m respectively, also the speed of first train be 54 km/h, the speed of second train:

(a) 115 km/h (b) 108km/h

(c) 120 km/h (d) 90 km/h

Ans: (b)

Explanation:

Speed of first train = 54 km/h = 15 m/s

Now, Time = Sum of length of the two trains/Sum of their speeds

$10 = \frac{250+150}{10+x}$

$\Rightarrow x$ = 30 m/s = 108 km/h

Q10. A freight train left Delhi for Mumbai at an average speed of 40 km/hr. Two hours later, an express train left Delhi for Mumbai, following the freight train on a parallel track at an average speed of 60 km/hr. How far from Delhi would the express train meet the freight train? **(UPSC CSAT 2017)**

(a) 480 km (b) 260 km

(c) 240 km (d) 120 km

Ans: (c)

Explanation:

Average speed of freight train = 40 kmph.

Distance travelled in 2 hrs. = 80 kms.

Speed of express train = 60 kmph.

Relative speed = 60 – 40 = 20 kmph.

Time taken to cover 80 km with the relative speed of 20 kmph = $\frac{80}{20}$=4 hrs

Actual distance = 4 × 60 = 240 km.

Q11. A train 200 metres long is moving at the rate of 40 kmph. In how many seconds will it cross a man standing near the railway line? **(UPSC CSAT 2018)**

(a) 12 (b) 15

(c) 16 (d) 18

Ans: (d)

Explanation:

Speed of the train = 40 km/hr or 40,000 m in 60 minutes

Or 40,000 m in 3600 second

So speed of train = 400/36 m/s

Distance = 200 m

Time = $\frac{\text{Time}}{\text{Distance}}$

So time taken by train = 200/(400/36) = 18 seconds

BOATS AND RIVERS (OR STREAMS) RELATED CONCEPT

(i) The downstream speed is the sum of the speeds of the boat and the river when they move in the same direction.

(ii) The upstream speed is the difference of the speeds of the boat and the river when they move in opposite directions. If B is the speed of the boat in still water and C is the speed of the current of the river, then

- Downstream Speed(D) = (B+C)
- Upstream Speed(U) = (B-C), Where B > C
- Speed of boat in still water (B) = $\frac{(D+U)}{2}$
- Speed of current (C) = $\frac{(D-U)}{2}$

(iii) When the boat travels the same distance with and against the water flow, then the time consumed by boat in upstream/time consumed by boat in downstream = $\frac{\text{Upstream Speed}}{\text{Downstream speed}}$

QUESTIONS

Directions (Q12 - Q14): A boat can move at 10 km/h in still water (i.e., when water is not flowing). The speed of the stream of the river is 2 km/h. A boat takes 80 minutes to go from point A to another point B and return to the same point.

Q12. What is the distance between the two points?

(a) 7.5 km (b) 6.5 km

(c) 6.4 km (d) 7.8 km

Ans: (c)

Explanation:

Downstream speed of boat

= 10 + 2 = 12 km/h

Upstream speed of boat = 10 - 2 = 8 km/h

Therefore,

$$\frac{\text{Downstream speed}}{\text{Upstream speed}} = \frac{\text{Upstream time}}{\text{Downstream time}}$$

$$\frac{12}{8} = \frac{3}{2} = \frac{\text{Time taken in upstream direction}}{\text{Time taken in downstream direction}}$$

Time taken in downstream = $\frac{2}{2+3} \times 80 = 32$ min

Time taken in upstream = $\frac{3}{2+3} \times 80 = 48$ min

Distance between two points = downstream speed × downstream time

$= \frac{12 \times 32}{60} = 6.4$ km

Distance between two points = upstream speed × upstream time

$= \frac{8 \times 48}{60} = 6.4$ km

Q13. What is the ratio of downstream speed and upstream speed?

(a) 3:5 (b) 3:2

(c) 3:7 (d) 3:4

Ans: (b)

Explanation:

Downstream speed of boat

= 10 + 2 = 12 km/h

Upstream speed of boat = 10 - 2 = 8 km/h

12 : 8 = 3:2

Q14. What is the ratio of time taken in downstream speed to the upstream speed?

(a) 2:3 (b) 3:2

(c) 3:7 (d) 3:4

Ans: (a)

Explanation:

Downstream speed of boat

= 10 + 2 = 12 km/h

Upstream speed of boat = 10 - 2 = 8 km/h

Therefore,

$$\frac{\text{Downstream speed}}{\text{Upstream speed}} = \frac{\text{Upstream time}}{\text{Downstream time}}$$

$$\frac{12}{8} = \frac{3}{2} = \frac{\text{Time taken in upstream direction}}{\text{Time taken in downstream direction}}$$

Time taken in downstream = $\frac{2}{2+3} \times 80 = 32$ min

Time taken in upstream = $\frac{3}{2+3} \times 80 = 48$ min

32 : 48 $\Rightarrow$ 2:3

Q15. A man takes half time rowing a certain distance downstream than upstream. What is the ratio of the speed of man in still water to the speed of current? (UPSC CSAT 2019)

(a) 1 : 2 (b) 2 : 1

(c) 1 : 3 (d) 3 : 1

Ans: (d)

Explanation:

Let the speed of man in still water be x units and the speed of current be y units.

Let the total distance be D.

If the time taken is half, then the speed must be double, as the distance is constant.

So, $x + y = 2(x - y)$

Or $x = 3y$

$\therefore x : y = 3 : 1$

RACES

RACE IN STRAIGHT LINE

Startup or Head Start

When runners are about to start a race, if one runner lets the other runner go ahead before the race officially begins, it's called giving a "head start" or "startup." Imagine runner A and runner B. If A lets B start a little ahead of them, we say that A has given B a "head start" in the race. This head start could be a certain distance, like a few meters. So, we might say that A has given B a "head start" of, for example, 5 meters. It's like a friendly way of making the race more interesting by giving someone a bit of an advantage before everyone starts running together.

Dead Heat

When the runners reach the finish line at the same time, we say they finish the race in a "dead heat." Basically, it means they both cross the finish line together, so there's no clear winner between them.

(i) If someone says that A can give B a head start of x meters in a y-meter race, it means that in the y-meter race, B runs x meters less than A, but they both finish the race in the same amount of time.

(ii) A beats B by t seconds in a y-meter race, it means A is the winner and B is the loser. When A crosses the finish line, B is still some distance behind. So, B takes t seconds to cover that remaining distance. This allows us to calculate B's speed – how fast B is running – by dividing the remaining distance by the time it takes to cover it.

RACE ON CIRCULAR TRACK

(i) If objects are moving in opposite directions, their relative speed is the sum of their individual speeds.

(ii) When two objects are moving in the same direction, their relative speed is equal to the difference between their individual speeds.

First Meeting

(i) A and B are two runners. The duration it takes for them to meet each other for the first time

$$= \frac{\text{Length of the circular track}}{\text{Relative speed}}$$

(ii) If there are multiple runners, assuming A is the fastest, the time it takes for A to first meet B is denoted as t_{AB}, for A to first meet C is denoted as t_{AC}, and for A to first meet D is denoted as t_{AD}, and so forth. The collective time it takes for all runners to meet A for the first time is determined by the Least Common Multiple (LCM) of t_{AB}, t_{AC}, t_{AD} and so on.

First Meeting at the Starting Point

If A requires time t_A to complete one round, B requires time t_B, C requires time t_C, and so forth, then the time it takes for them to meet again at the starting point is equal to the Least Common Multiple (LCM) of t_A, t_B, t_C, and so on.

QUESTIONS

Q16. A runs $\frac{7}{4}$ times as fast as B. If A gives B a start of 300 m, how far must the winning post be if both A and B have to end the race at same time?

(a) 1400 m (b) 700 m

(c) 350 m (d) 210 m

Ans: (b)

Explanation:

Let the speed of *A* and *B* is 7*x* and 4*x* respectively.

Their relative speed = 7*x* – 4*x* = 3*x*.

So the time taken by *A* to catch *B* = $\frac{300}{3x} = \frac{100}{x}$

So the distance travelled by A in time $\frac{100}{x} = \left(\frac{100}{x}\right) \times 7x$ = 700m

Thus, the winning post is 700 m away from the starting point.

Q17. A beats B by 100 m in a race of 1200 m and B beats C by 200 m in a race of 1600 m. Approximately how many meters can A beat C in a race of 9600 m?

(a) 1600 m (b) 1800 m

(c) 1900 m (d) 2400 m

Ans: (c)

Explanation:

Ratio of speeds of A:B = 12:11 and ratio of speeds of *B*:*C* = 8:7

Therefore ratio of speeds of *A*:*B*:*C* = 96:88:77 (If *a*:*b* and b:c are given then a : b : c = (*a*.*b*) : (*b*.*b*) : (*b*.*c*))

It implies that when A runs 96 meters, C runs 77m.

If A runs 9600m then C runs 7700 meters

So in a 9600 m race A will beat C by 1900 m.

Directions (Q18 - Q19): In a one km race, A can give B a start of 20 m and also in a half km race C beats A by 50m.

Q18. If A, B and C run a 2 km race, what is the difference between the distances covered by the two losers, when the winner finishes the race?

(a) 64 m (b) 38 m

(c) 32 m (d) 58 m

Ans: (b)

Explanation:

It is clear from the direction that C is the fastest so C will win the race and.

Now according to the question when C runs 1 km it beats A by 50 meters.

It implies when C runs 1000 m in that time A runs 950 m.

When C runs 2000 m then A runs 1900 m.

Now it is given that A is 20m ahead in a race of 1000m

It implies when A runs 1000 m B runs 980 m.

Now when A runs 1900 m the B runs

$\frac{980}{1000}\times 1900 = 1862\,m$

Then the difference between A and B (two losers)

= 1900 – 1862 = 38 m.

Q19. *B* and *C* run a half km race, who should give a start to the slower runner and of how many metres so that they both finish the race at the same time?

(a) C, 59.5 m (b) B, 34 m

(c) C, 48 m (d) B, 56.5 m

Ans: (a)

Explanation:

In a 1 km race C can give B a start of 119 m. Therefore in a half km race C can give B a start of 59.5 m.

Q20. In a 500 metres race, B starts 45 metres ahead of A, but A wins the race while B is still 35 m behind. What is the ratio of the speeds of A and B assuming that both start at the same time? (UPSC CSAT 2015)

(a) 25 : 21 (b) 25 : 20

(c) 5 : 3 (d) 5 : 7

Ans: (a)

Explanation:

Distance covered by *A* & *B* in given time is for *A* = 500 m and for *B* = 500 – 45 – 35 = 420 m

Distance ratio = 500 : 420 = 25 : 21

speed = Distance / time

$\Rightarrow$ speeds ratio = 25 : 21 (Time is constant)

Q21. When a runner crossed the 12 km mark, she was informed that she had completed only 80% of the race. How many kilometres was the runner supposed to run in this event? (UPSC CSAT 2019)

(a) 14 (b) 15

(c) 16 (d) 16.5

Ans: (b)

Explanation:

Let the total distance in the event = *x* km

Now according to question 80% of *x* = 12 km

$\frac{80}{100}x = 12 \text{ or } x = \frac{1200}{80} = 15\ km$

Q22. Two persons, A and B, are running on a circular track. At the start, B is ahead of A and their positions make an angle of 30 degrees at the centre of the circle. When A reaches the point diametrically opposite to his starting point, he meets B. What is the ratio of speeds of A and B, if they are running with uniform speeds? (UPSC CSAT 2017)

(a) 6 : 5 (b) 4 : 3

(c) 6 : 1 (d) 4 : 2

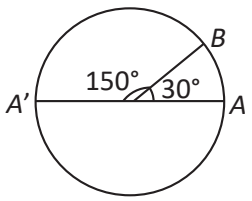
Ans: (a)

Explanation:

Let the speed of A and B be x and y units respectively.

Let the radius of the circular track is r and after time 't' both A and B meet at A'.

At the start, B is ahead of A and their positions make an angle of 30° at the centre of the circle.



Distance travelled by A to reach the diametrically opposite end *A'* = πr

Distance travelled by B to reach the diametrically opposite end *A'* = $\frac{150}{180}\pi r = \frac{5}{6}\pi r$

We know that, distance = speed × time

$\pi r = xt \quad \frac{5}{6}\pi r = yt$

From these equations we get $\frac{x}{y} = \frac{6}{5}$ Hence *x* : *y* = 6 : 5

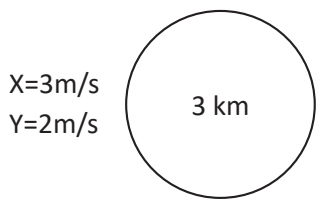
Q23. *X* and *Y* run a 3 km race along a circular course of length 300m. Their speeds are in the ratio 3:2. If they start together in the same direction, how many times would the first one pass the other (the start-off is not counted as passing)?

(a) 2 (b) 3

(c) 4 (d) 5

Ans: (b)

Explanation:



Let the speed of x be 3 m/sec and Y be 2 m/sec.

So, their relative speed = 3 – 2 = 1 m /sec

X will cross Y when he covers an extra 300 m.

Thus, the time taken by X to cross Y = Distance/Relative Speed = 300/1 = 300 seconds

Therefore X will cross Y every 300 seconds, or 300/60 = 5 minutes.

Now, the time taken for X to complete the entire race = Total Distance/Speed

= 3000/3 = 1000 seconds.

So, X will cross Y 3 times during the entire race i.e. after 300, 600, and 900 seconds.

Q24. A man started from home at 14:30 hours and drove to the village, arriving there when the village clock indicated 15:15 hours. After staying for 25 minutes, he drove back by a different route of length 1.25 times the first route at a rate twice as fast, reaching home at 16:00 hours. As compared to the clock at home, the village clock is

(a) 10 minutes slow (b) 5 minutes slow

(c) 10 minutes fast (d) 5 minutes fast

Ans: (d)

Explanation:

It is given that the total time taken by the man to come back home is 90 minutes, Out of which he stayed in the village for 25 minutes.

So, his total travelling time = 90 – 25 = 65 minutes The return route was 1.25 times the initial route.

So, time taken must have increased by 25% too. So, if the initial time taken was 1t, now it must be 1.25t.

But it is also given that while returning he drove twice as fast. So, time taken must have been halved.

So, time taken while returning back = $(1.25t/2)$ $=0.625\ t$

Total time taken = $t + 0.625t$ = 65 min

Or, 1.625t = 65 minutes t = 40 minutes

So, the man took 40 minutes to reach the village.

So, the actual time at that moment = 14:30 + 40 minutes = 15:10 hours

It is clear that the village clock is 5 minutes fast.

Q25. A person X from a place A and another person Y from a place B set out at the same time to walk towards each other. The places are separated by a distance of 15 km. X walks with a uniform speed of 1.5 km/hr and Y walks with a uniform speed of 1 km/hr in the first hour, with a uniform speed of 1.25 km/hr in the second hour and with a uniform speed of 1.5 km/hr in the third hour and so on. Which of the following is/are correct?

1. They take 5 hours to meet.
2. They meet midway between A and B.

Select the correct answer using the code given below:

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

Ans: (c)

Explanation:

It is given that the distance between places A and B = 15 km.

Speed of X = 1.5 km/hr

So, Distance covered by X in 5 hours = 7.5 km

We need to find the distance covered by Y in 5 hours.

Speed of Y in 1st hour = 1 km/hr

So, Distance covered by Y in 1st hour = 1 km

Similarly, Distance covered by Y in 2nd hour = 1.25 km

Distance covered by Y in 3rd hour = 1.5 km

Distance covered by Y in 4th hour = 1.75 km

Distance covered by Y in 5th hour = 2 km

So, the total distance covered by Y in 5 hours = 1 + 1.25 + 1.5 + 1.75 + 2 = 7.5 km

Hence, both the given statements are true.

Q26. X, Y and Z are three contestants in a race of 1000 m. Assume that all run with different uniform speeds. X gives Y a start of 40 m and X gives Z a start of 64 m. If Y and Z were to compete in a race of 1000 m, how many metres would Y give to Z?

(a) 20 (b) 25

(c) 30 (d) 35

Ans: (b)

Explanation:

In the 1000 m race, *X* gives *Y* 40 m start. That means Y starts the race 40 m ahead of *X*.

In the 1000 m race, *X* gives Z a 64 m start. That means Z starts the race 64 m ahead of *X*.

So, this means Y gives a lead of 24 m to Z in a 1000 – 40 = 960 m race.

So, by unitary method, the lead given by Y in 1000 m = (24/960)×1000 = 25 m.

PRACTICE QUESTIONS

1. **Arvind can cover a certain distance in 42 minutes when he travels at a speed of 60 km/hour. Find the speed to cover the same distance in 35 minutes.**
   (a) 72 km/hr (b) 64 km/hr
   (c) 68 km/hr (d) 76 km/hr
2. **Speed of a man is 5 km/hr. If he takes 5 minutes rest for every km, then how much time will he take to cover a distance of 200 km?**
   (a) 40 hr. 40 min (b) 40 hr. 35 min
   (c) 56 hr. 35min (d) 60hr. 30 min
3. **A driver of a tourist bus starts half an hour later than the scheduled time. If he wants to reach the destination on time, he has to increase the speed by 1/3rd. If the destination is 130 km away, find the original speed of the bus.**
   (a) 72 km/hr. (b) 65 km/hr.
   (c) 68 km/hr. (d) 76 km/hr.
4. **A person can cover a journey in 4 hours. If he decreases his speed by 1/12, he covers 15 km less. Find his original speed.**
   (a) 45 km/hr. (b) 65 km/hr.
   (c) 50 km/hr. (d) 64 km/hr.
5. **If a man walks from P to Q and rides from Q to P, it takes 6 hours 34 minutes. But if he rides both ways, he saves 1 hour 26 minutes. Then how much time will he take if he walks both ways?**
   (a) 7:32 hr. (b) 7:42 hr
   (c) 6:56 hr. (d) 8:00 hr.
6. **If Naveen goes halfway through the journey at 60 kmph and remaining at 80 kmph, he takes 7 hours. Find the total distance travelled by him?**
   (a) 600 km (b) 560 km
   (c) 480 km (d) 580 km
7. **A man goes 50 km by Rickshaw, 60 km by Bike and 60 km by Bus, he takes 15 hours to cover the whole distance. If speed of bus is 4 times that of bike and 3 times that of Rickshaw. Find the speed of Rickshaw.**
   (a) 6 kmph (b) 8 kmph
   (c) 10 kmph (d) 12 kmph
8. **A policeman runs after a thief who has a 200-meter headstart. The policeman can run 1 km in 8 minutes and the thief can run 1 km in 10 minutes. Find the total distance traveled by the thief before getting caught.**
   (a) 640 m (b) 750m
   (c) 800 m (d) 850 m
9. **A Deer saw a Cheetah at a distance of 300 m and started running at the speed of 27 km/hr. After 2 minutes, the Cheetah started to run after the deer at the speed of 36 km/hr. find the distance traveled by the Cheetah before he catches the deer.**
   (a) 4.8 km (b) 5.6 km
   (c) 5.2 km (d) 4.2 km
10. **A monkey tries to climb on a greased pole of height 40 meter. In 1 minute, he climbs 4 meters, in 2nd minute he slips 2 meters and this trend continues. How much time will the monkey take to climb the pole?**
    (a) 35 min (b) 36 min
    (c) 37 min (d) 34 min
11. **The speed of an engine is 30 kmph without any wagon. The decrease in speed is directly proportional to the square root of the number of wagons attached. If 9 wagons are attached, the speed will reduce to 21 kmph. Find the maximum number of wagons the engine can move.**
    (a) 88 (b) 99
    (c) 100 (d) 96
12. **A person can cover 240 km in 6 hours partly by Bus and partly by car. If he had travelled all the way by car, he would have saved half of the time taken by bus and reached his destination 2 hours earlier. Find the speed of the Bus.**
    (a) 24 kmph (b) 30 kmph
    (c) 42kmph (d) 48 kmph
13. **A Car and a Bus start from Bengaluru at 08:00 am. The car reached Mysuru at 10:30 am, it stopped for 1 hour and returned on the same way. After half an hour, the car meets the same Bus. Find the time at which the Bus reached Mysuru.**
    (a) 12:00 pm (b) 1:00pm
    (c) 12:30pm (d) 2:00 pm.
14. **A Car travelling 400 km covers the first 100 km at the rate of 100 kmph, the second 100 km at 120 kmph, the third 100 km at the rate of 150 kmph. and the last 100 km at the rate of 200 kmph. Find the average speed of the car.**
    (a) 142.5kmph. (b) 133.3 kmph.
    (c) 148.67 kmph. (d) 152.5 kmph.

15. **A person travels a certain distance at a uniform speed in 5 hours. In return journey, he increases his speed by 5 kmph and covers the same distance in 4 hours. What was his initial speed?**

(a) 30 kmph. (b) 25 kmph.

(c) 20 kmph. (d) 36 kmph.

16. **Ajay and Vijay travel the same distance at the speeds of 20 kmph and 25 kmph respectively. If Ajay takes 30 min longer than Vijay, Find the distance traveled by Vijay.**

(a) 50km (b) 48 km

(c) 68 km. (d) 64 km

17. **There are 40 trees planted with a constant distance between them. A bike takes 36 seconds to cover the distance from the 1st to the 13th tree. Find the total time it will take to reach the last tree.**

(a) 120 sec (b) 117 sec

(c) 104 sec (d) 123 sec

18. **A man can row at a speed of 6 kmph in still water to a certain upstream point and back to the starting point in a river that flows at 2 kmph. Find his average speed for the total journey.**

(a) $6\frac{2}{3}$kmph (b) $4\frac{1}{3}$kmph

(c) $4\frac{2}{3}$kmph (d) $5\frac{1}{3}$kmph

19. **The driver of a car sees a bus 80 m ahead of him. After 18 seconds, the bus is 100 meters behind. If the speed of the car is 45 kmph, what is the speed of the bus?**

(a) 13 kmph (b) 12 kmph

(c) 10 kmph (d) 9 kmph

20. **A boat travels upstream from Q to P and downstream from P to Q in 8 hours. If the speed of the boat in still water is 12 kmph. and the speed of the current is 3 km/ hr. the distance between P and Q (in km) is**

(a) 30km (b) 42km

(c) 45 km (d) 36km

ANSWERS

1. (*a*)	2. (*c*)	3. (*b*)	4. (*a*)	5. (*d*)	6. (*c*)	7. (*c*)	8. (*c*)	9. (*a*)	10. (*c*)
11. (*b*)	12. (*b*)	13. (*b*)	14. (*b*)	15. (*c*)	16. (*a*)	17. (*b*)	18. (*d*)	19. (*d*)	20. (*c*)

EXPLANATIONS

1. (a)

Explanation:

Ratio of time = 42 : 35 = 6 : 5

Ratio of speed = 5 : 6

Ratio of speeds = 5*x* : 6*x*

5*x* = 60 km/hr.

x = 12 km/hr.

6*x* = 6 × 12 = 72km/hr.

2. (c)

Explanation:

Time required to cover 200 km (without rest)

$= \frac{200}{5}$ = 40 hrs.

Total time he took for rest = 199×5 = 995 minutes = 16 hrs 35 mins.

Total time = 40hrs + 16 hrs 35 mins. = 56 hrs 35 minutes

3. (b)

Explanation:

The ratio of original and increased speed

$= \frac{1}{(1+3)} = 3:4$

The ratio of original and increased time

= 4: 3 or 4*x* : 3*x*

Difference of time = 4*x* – 3*x* = 30min (given)

x = 30 min

Original time = 4*x* = 4 × 30 min = 120 min = 2 hours

Distance of the destination = 130 km

Speed = Distance/time = 130/2 = 65 km/hr.

4. (a)

Explanation:

Ratio of original and decreased speed = 12 : 11

Ratio of original and decreased distance = 12 : 11 or 12x : 11x (speed is directly proportional to distance)

Difference in distance covered = $12x - 11x = x = 15$km (given)

Original distance = $12 \times 15 = 180$ Km.

Time taken = 4 hours

Original speed = distance/time = 180/4 = 45km/hr.

5. (d)

Explanation:

Walk + Ride = 6hrs: 34 minutes (given)

Ride + Ride = 5hrs: 08 minutes

$2 \times$ Ride = 5:08 hours $\Rightarrow$ Ride = 2:34 hours

Walk + 2:34 = 6:34

Walk = 6:34 – 2:34 = 4 hrs.

Time taken if he walks both ways = $2 \times$ Walk = $2 \times 4 = 8$ hrs.

6. (c)

Explanation:

Ratio of speed = 60: 80 = 3:4

Ratio of time = 4: 3 or $4x : 3x$

Total journey time = $4x + 3x = 7$ hrs.

$x = 1$ hr.

Actual time = 4 hrs. and 3 hrs.

Distance = speed × time

Total distance = $4 \times 60 + 3 \times 80 = 480$ km.

7. (c)

Explanation:

Ratio of speed of bus and bike = 4:1 or 12:3

Ratio of speed of Bus and rickshaw = 3:1 or 12:4

Ratio of speed of Bus: Rickshaw: Bike = 12 : 4 : 3

Actual speeds of Bus, Rickshaw, Bike are 12x, 4x, and 3x respectively.

Time = Distance/ speed

Time taken by Rickshaw = 50/4x = 12.5/x kmph

Time taken by Bus = 60/12x = 5/x kmph

Time taken by Bike = 60/3x = 20/x kmph

Total time = $\frac{12.5}{x} + \frac{5}{x} + \frac{20}{x} = \frac{37.5}{x} = 15\text{hours}$

$x = \frac{37.5}{15} = 2.5$ hrs.

Speed of rickshaw = $4x = 4 \times 2.5 = 10$ kmph.

8. (c)

Explanation:

Policeman can run 1 km in 8 minutes

Speed of policeman = 1000/8 = 125 meters/min

Thief can run 1 km in 10 minutes

Speed of thief = 1000/10 = 100 meters/min.

Relative speed = 125 – 100 = 25 meters/min.

Time taken by the policeman to catch the thief = 200/25 = 8 minutes

Distance covered by the thief before getting caught = $100 \times 8 = 800$ meters

9. (a)

Explanation:

The initial distance between Cheetah and Deer = 300 meters

Speed of Deer = 27 kmph = $27 \times 1000/60$ = 450 meters/min.

Speed of Cheetah = 36 kmph = $36 \times 1000/60$ = 600 meters/min.

Distance covered by deer in 2 minutes = 2×450 = 900 meters

Relative speed = 600 – 450 = 150 meters/min.

Time required by Cheetah to catch the deer = distance/time = $\frac{(300 + 900)}{150}$

= 8 minutes

Distance traveled by cheetah = $8 \times 600 = 4800$ meters. = 4.8 Km.

10. (c)

Explanation:

Height covered in 1st min = 4 meter

Height covered in 2nd min = –2 meter

Net height covered after 2 minutes = 2 meters

$\text{Time required to cover } 36 \text{ meters} = \frac{36}{2} * 2 = 36 \text{minutes}$

In 37th minute the monkey will climb 4 m and completely climb the pole of 40m

11. (b)

Explanation:

Speed of engine = 30 kmph

Speed of engine with 9 wagons = 21 kmph

Reduced speed = 30 – 21 = 9 kmph.

Reduction in speed is directly proportional to the square root of the number of wagons attached.

$9 = k\sqrt{w}$ (w = number of wagons)

$9 = k\sqrt{9} \Rightarrow k = 3$

To find maximum number of wagons that an engine can move:

$30 = k\sqrt{w} = 3\sqrt{w}$

$10 = \sqrt{w} \Rightarrow w = 100$

If the number of wagons is 100, the speed will be 0 or the engine will come to stand still.

Therefore, the maximum number of wagons the engine can carry = 100 – 1 = 99

12. (b)

Explanation:

Total time of journey = 6 hours

Let the time traveled by Bus be 'B' and the time traveled by car will be '(6 – B)'

If he had traveled all the way by car, he would have saved half of the time taken by bus and reached his destination 2 hours earlier

$B \times \frac{1}{2} = 2 \Rightarrow B = 4$ hrs.

Time traveled by Car = 6–B = 2 hrs.

If he had traveled all the way by Car, he could have reached his destination 2 hour early i.e.,6–2 = 4 hrs.

Speed of Car = $\frac{240}{4}$ = 60kmph

Actual time traveled by car = 2 hours

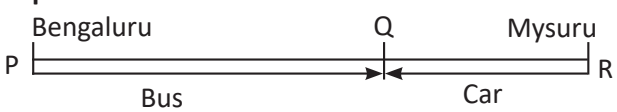
Distance covered by car = 2 × 60 = 120km

Distance covered by Bus = 240 – 120 = 120km

Speed of Bus = 120/4 = 30Kmph

13. (b)

Explanation:



Car started at 8:00 a.m. and reached at 10:30 a.m.

Time taken by car to reach Mysuru = 2:30 hrs.

It stops for one hour i.e., till 11:30 a.m. and returns in the same way. It meets the Bus after 30 min.

The Car meets the Bus at 11:30 + 00:30 = 12 p.m.

Let Q be the point where the car meets the bus

Time taken by Bus to cover the distance

PQ = 12:00 – 8:00 = 4 hrs.

Time required by Car to cover the distance

QP = 2:30 – 00:30 = 2hrs

Ratio of time taken by Car and Bus to cover the same distance = 4:2 = 2:1

If Car takes 2:30 hrs. then Bus requires 5:00 hrs. to reach Mysuru.

The Bus will reach at 1:00 p.m.

14. (b)

Explanation:

Avg speed = $\frac{\text{Total distance}}{\text{total time}}$;

$\left(\text{Time} = \frac{\text{Distance}}{\text{Speed}}\right)$

$$Avg\ speed = \frac{D}{\frac{D1}{S1} + \frac{D2}{S2} + \frac{D3}{S3} + \frac{D4}{S4}}$$

$$= \frac{400}{\frac{100}{100} + \frac{100}{120} + \frac{100}{150} + \frac{100}{200}}$$

$$= \frac{400 * 600}{600 + 500 + 400 + 300}$$

= 133.33 kmph

15. (c)

Explanation:

Ratio of initial and final time = 5 : 4

Ratio of initial and final speeds = 4 : 5

Let the actual speed be $4x : 5x$

Difference in speeds $= 5x - 4x = 5$kmph

$x = 5$ kmph

initial speed $= 4x = 4 \times 5 = 20$kmph

16. (a)

Explanation:

Ratio of speeds of Ajay and Vijay = 20 : 25 = 4:5

Ratio of time taken by Ajay and Vijay = 5 : 4

Let the actual times taken by Ajay and Vijay be 5x and 4x respectively.

The difference in time $= 5x - 4x = 30$min $\quad x = 30$ min.

time taken by Vijay $= 4x = 4 \times 30$min = 2hr

Distance covered by Vijay = 2 × 25 = 50 km

17. (b)

Explanation:

From the 1st tree to the 13th tree, there are 12 spaces between them.

Time required to cover 12 spaces = 36 seconds

Time required to cover 1 space = $\frac{36}{12}$ = 3 seconds

Time required to reach 40th tree (to cover 39 spaces) = 39 × 3 = 117 seconds

18. (d)

Explanation:

Upstream speed = 6 – 2 = 4kmph

Downstream speed = 6 + 2 = 8kmph

Let the distance be 'D' km

Avg speed = Total distance /Total time

$$\text{Avg speed} = \frac{D+D}{\frac{D}{4}+\frac{D}{8}}$$

$$= \frac{2D*8}{2D+D} = \frac{16D}{3D} = \frac{16}{3} = 5\frac{1}{3}\text{kmph}$$

19. (d)

Explanation:

Total distance covered by the car= 80+100 = 180 m

Time taken = 18 sec

Relative speed = Total distance/time = $\frac{180}{18}$ = 10 m/s

= 10 * (18/5) km/h = 36 km/h

Relative speed = Speed of car – speed of bus

$\Rightarrow$ 36 = 45 – Speed of bus

$\Rightarrow$ Speed of bus = 45–36 = 9 km/h

20. (c)

Explanation:

Downstream speed = 12 + 3 = 15kmph

Upstream speed = 12 – 3 = 9kmph

Let the distance between P and Q be 'D' km

Total time = 8 hrs.

$$\frac{D}{15}+\frac{D}{9}=8 \qquad \frac{3D+5D}{45}=8$$

$$D = \frac{360}{8} = 45 \text{ km}$$

Time and Work

12

INTRODUCTION

The chapter on Time and Work is designed to guide you through understanding how different tasks can be completed efficiently by one or more people or machines. It focuses on calculating the amount of work done and the time taken to complete it. By the end of this chapter, you'll be equipped to analyse and approach various time and work-related Problems.

CONCEPT OF EFFICIENCY

If someone can finish a job in 2 days, it means they do half of the work each day, making their daily efficiency 50%. Efficiency is about how much work someone can do in a day. So, if a person takes 4 days to finish a task, their daily efficiency is 25%. This means they complete one-fourth of the work each day.

To understand efficiency better, think of it as how much of the total work someone can do in a single day. If a person can finish a task in n days, their daily efficiency is $\frac{1}{n}$.

For example, if they can do 1/3 of the work in a day, it means they can finish the entire job in 3 days. It's all about breaking down the work into what can be done in one day.

If someone can finish a task in n days, their one day's work is $\frac{1}{n}$ and their percentage efficiency is $\frac{1}{n} \times 100$. following example will help us in better understanding

EXAMPLES

E1. A can do a job in 10 days and B can do the same job in 15 days, in how many days working together can they complete the job?

(a) 5 (b) 10

(c) 6 (d) 12

Ans: (c)

Explanation:

A's 1 day's work = $\frac{1}{10}$,

B's 1 day's work = $\frac{1}{15}$

$\therefore$ (A + B) 's 1 day's works = $\frac{1}{10} + \frac{1}{15} = \frac{5}{30} = \frac{1}{6}$

$\therefore$ Time taken by both to finish the whole work $= \frac{1}{\frac{1}{6}} =$ 6 days

THE CONNECTION BETWEEN TIME AND EFFICIENCY

The relationship between efficiency and time, measured in days, hours, minutes, etc., is inverse.

For instance, if A is twice as efficient as B, it indicates that A completes the same task in half the time than B.

Note:

The ratio of the time it takes to complete a task is equivalent to the ratio of the reciprocals of efficiency, and the same is true in the opposite direction. Following example will help us in better understanding.

QUESTIONS

Q1. P is twice as efficient as Q, P takes 30 days to do a job, then in how many days Q can finish the same job?

(a) 45 (b) 40

(c) 50 (d) 60

Ans: (d)

Explanation:

Ratio of efficiency of $P : Q = 2 : 1$

$\therefore$ Ratio of required days of $P : Q = \frac{1}{2} : \frac{1}{1} = 1 : 2$

Now since P takes 30 days, so Q will take 60 days to finish the same job.

Q2. A can do a job in 8 days, B can do the same job in 12 days and C can do the same job in 24 days. In how many days will they finish the work together?

(a) 5 (b) 4

(c) 6 (d) 12

Ans: (b)

Explanation:

A's 1 day's work = $\frac{1}{8}$,

B's 1 day's work = $\frac{1}{12}$

C's 1 day's work = $\frac{1}{24}$

$\therefore$ (A + B + C) 's 1 day's works = $\frac{1}{8} + \frac{1}{12} + \frac{1}{24} = \frac{6}{24} = \frac{1}{4}$

Total time taken = $\frac{1}{\frac{1}{4}} = 4$ days

Q3. Arun and Riya can complete a work, working independently in 12 days and 24 days, respectively. How Long will it take for both to complete the work, working together?

(a) 5 (b) 8

(c) 6 (d) 12

Ans: (b)

Explanation:

Arun's 1 day's work = $\frac{1}{12}$ of the entire work

Riya's 1 day's work = $\frac{1}{24}$ of the entire work

When both work together, work completed in a day

$= \frac{1}{12} + \frac{1}{24} = \frac{3}{24} = \frac{1}{8}$ of the entire work

$\therefore$ Time taken by both to finish the whole work

$= \frac{1}{\frac{1}{8}} = 8$ days

Q4. If A and B, working together can finish a piece of work in 20 days, B and C in 24 days, C and A in 30 days, how much time will they take to complete the work, if A, B and C work together?

(a) 5 (b) 4

(c) 16 (d) 12

Ans: (c)

Explanation:

(A + B) 's 1 day's work

$= \frac{1}{20}$ of total work ...(i)

(B + C) 's 1 day's work

$= \frac{1}{24}$ of total work ...(ii)

(C+ A) 's 1 day's work = $\frac{1}{30}$ of total work ...(iii)

Adding eq (i), (ii) and (iii) we get

$2(A + B + C) = \frac{1}{20} + \frac{1}{24} + \frac{1}{30} = \frac{15}{120} = \frac{1}{8}$

Hence, $(A + B + C)$'s 1 day's work

$= \frac{1}{8 \times 2} = \frac{1}{16}$ of total work

Time taken to complete the work = $\frac{1}{\frac{1}{16}} = 16$ days

Q5. *P* is thrice as efficient as *Q* and is therefore able to finish a piece of work in 40 days less than *Q*. Find the time in which *P* and *Q* can complete the work individually.

(a) 20, 60 (b) 45, 20

(c) 50, 30 (d) 60, 10

Ans: (a)

Explanation:

Efficiency of P and Q = 3:1

Required number of days of P and Q = 1:3

If P requires x days then Q requires 3x days

$3x - x = 40$

$2x = 40 \Rightarrow x = 20$

$\therefore$ 3x = 3 × 20 = 60

Q6. A person X can complete 20% of work in 8 days and another person Y can complete 25% of the same work in 6 days. If they work together, in how many days will 40% of the work be completed? (UPSC CSAT 2020)

(a) 6 (b) 8

(c) 10 (d) 12

Ans: (a)

Explanation:

No. of days taken by X to complete 20% of work is = 8 days

So, no. of days taken by X to complete the 100% work = 8 × 5 = 40 days.

Work done by X in 1 day = $\frac{1}{40}$

No. of days taken by Y to complete 25% of work is = 6 days

So, no. of days taken by Y to complete the 100% work = 6 × 4 = 24 days

Work done by *Y* in 1 day = $\frac{1}{24}$

Work done by *X* and Y together in 1 day = $\frac{1}{24} + \frac{1}{40} = \frac{1}{15}$

Hence no. of days taken by X and Y together to complete the work = 15 days

No of days to complete the work by *X* and *Y* together to complete 40% of work

= 40% of 15 = $\frac{(15 \times 40)}{100} = 6$ days

CONCEPT OF NEGATIVE WORK

In this case one person works but another destroys it or cancels it. For example Ram can write 20 pages per hour but his younger sister Riya erases 10 pages per hour which Ram writes. It means effectively Ram writes 10 pages per hour since each hour his sister erases 10 pages out of 20 written pages. Take another example :

EXAMPLES

E2. A Bucket can be filled in 15 minutes but there is a leakage in it which can empty the full bucket in 60 minutes. In how many minutes can it be filled?

(a) 6 (b) 24

(c) 20 (d) 18

Ans: (c)

Explanation:

In 1 minute bucket is filling = $\frac{1}{15}$ part

In 1 minute bucket is emptying = $\frac{1}{60}$ part

$\therefore$ In 1 minute, effective filing of bucket

$= \frac{1}{15} - \frac{1}{60} = \frac{3}{60} = \frac{1}{20}$ part

Since $\frac{1}{20}$ part of the bucket is filled in 1 minute.

$\therefore$ Bucket will be filled in $\frac{1}{\frac{1}{20}}$ =20 minutes.

INVERSE PROPORTION (OR PRODUCT CONSTANCY)

Alright, think about when you're working on a project. If you work faster, it takes less time, and if you work slower, it takes more time – that's what we mean by "inverse proportion."

When more than one person or machine is working on a project, we figure out the work rate by considering how many people or machines are working and for how long. We express the amount of work in terms like "man-days" or "man-hours," which basically means how much work can be done in a day or an hour by one person.

QUESTIONS

Q7. If 40 persons can do a piece of work in 4 days then calculate the number of persons required to complete the work in 16 days

(a) 10 (b) 24

(c) 20 (d) 18

Ans: (a)

Explanation:

Number of persons × days = work

40 × 4 = 160 man-days

Now, $x \times 16 = 160$ man-days

$\Rightarrow x = 10$

Q8. If 60 workers can do a piece of work in 10 days working 14 hours a day, then how many workers are required to complete the work working 12 hours a day in 14 days?

(a) 30 (b) 50

(c) 20 (d) 18

Ans: (b)

Explanation:

M: No. of Men, D: No. of days and H: No. of hours worked Since Total Work is constant,

$M_1 \times D_1 \times H_1 = M_2 \times D_2 \times H_2$

$60 \times 10 \times 14 = M_2 \times 14 \times 12$

$\Rightarrow M_2 = 50$ workers

Q9. If 15 persons can do a piece of work in 10 days then calculate the number of persons required to complete the work in 30 days

(a) 5 (b) 3

(c) 2 (d) 1

Ans: (a)

Explanation:

Number of persons × days = work

15 × 10 = 150 man-days

Now, $x \times 30 = 150$ man-days

$\Rightarrow x = 5$

RELATION BETWEEN EFFICIENCIES

Imagine you have a task that takes 2 men to complete in 3 days. On the other hand, 12 boys can finish the same task in just 3 days. Now, to figure out how much work can be done in 1 day, we calculate "man-days" and "boy-days."

So, 2 men × 3 days = 6 man-days. This means 6 men can finish the job in 1 day. Similarly, 12 boys × 3 days = 36 boy-days. This implies 36 boys can finish the same job in 1 day.

Here's the interesting part: to finish the work in just 1 day, you need 6 men or 36 boys. So, the efficiency of 6 men is equal to the efficiency of 36 boys. In other words, 1 man is as efficient as 6 boys.

EXAMPLES

E3. 12 men and 16 women can do a job in 15 days. In how many days can 6 men and 8 women finish the same job working together?

(a) 10 (b) 24

(c) 20 (d) 30

Ans: (d)

Explanation:

Since 12M + 16W = 15 days

2(6M + 8W) = 15 days

$\Rightarrow$ (6M + 8W) = 30 Days

Now, the number of people working is now only half of what it used to be. Because of this change, we need to double the number of days to get the job done. So, the total time needed is now 30 days.

WORK AND WAGES

A certain number of M_1 men finish a task W_1 in D_1 days, putting in T_1 hours of work each day, and they earn a wage of Rs. R_1. Similarly, another group of M_2 men complete a task W_2 in D_2 days, working T_2 hours per day, and they earn Rs. R_2 as their wage.

$$\frac{M_1 \times D_1 \times T_1}{W_1 \times R_1} = \frac{M_2 \times D_2 \times T_2}{W_2 \times R_2}$$

QUESTIONS

Q10. If 18 persons can build a wall 140 m long in 42 days, then the number of persons who can build a wall 100 m long in 9 days:

(a) 50 (b) 40

(c) 60 (d) 30

Ans: (c)

Explanation:

$M_1 = 18$, $W_1 = 140$, $D_1 = 42$

$M_2 = ?$, $W_2 = 100$, $D_2 = 9$

Using the formula:

$$\frac{M_1 \times D_1}{W_1} = \frac{M_2 \times D_2}{W_2} \Rightarrow \frac{18 \times 42}{140} = \frac{M_2 \times 9}{100}$$

Solving, we get $M_2 = 60$

Q11. 12 Men and 6 women can do a piece of work in 14 days, while 8 men and 9 women can do the work in 16 days. If a man gets Rs 54 per day, what should be the wage of a woman per day (assuming no gender wage gap)?

(a) 30 (b) 40

(c) 48 (d) 54

Ans: (a)

Explanation:

According to question:

In one day

$12M + 6W = \frac{1}{14}$ and $8M + 9W = \frac{1}{16}$

By solving these equations $M = \frac{1}{224}$ and

$W = \frac{1}{336}$

Now ratio of efficiencies of M and W $\frac{M}{W} = \frac{3}{2}$

So the Woman's wage = $\frac{2}{3} \times 45$ = Rs 30

Q12. *A*, *B*, *C* working independently can do a piece of work in 8, 16 and 12 days respectively. *A* alone works on Monday, B alone works on Tuesday, *C* alone works on Wednesday; A alone, again works on Thursday and so on consider the following statements:

1. The work will be finished on Thursday.

2. The work will be finished in 10 days.

Which of the above statements is/are correct?

(UPSC CSAT 2023)

(a) 1 Only (b) 2 Only

(c) Both 1 and 2 (d) Neither 1 nor 2

Ans: (a)

Explanation:

A, B, C working independently can do a piece of work in 8, 16 and 12 days respectively.

Let the total amount of work be LCM (8, 16, 12) = 48 units (for simplification)

Efficiency of A = $\frac{48}{8}$ = 6 units/day

Efficiency of B = $\frac{48}{16}$ = 3 units/day

Efficiency of C = $\frac{48}{12}$ = 4 units/day

The amount of work done in 3 days (Monday + Tuesday + Wednesday) = 6 + 3 + 4 = 13 units

Thereafter, the amount of work done in subsequent 3 days (Thursday + Friday + Saturday) = 6 + 3 + 4 = 13 units

Thereafter, the amount of work done in subsequent 3 days (Sunday + Monday + Tuesday) = 6 + 3 + 4 = 13 units

Thereafter, the amount of work done in subsequent 2 days (Wednesday + Thursday) = 6 + 3 = 9 units

So, amount of work done in 11 days = 13 + 13 + 13 + 9 = 48 units

So, work will get finished in 11 days on Thursday.

It means that Statement 1 is correct, but Statement 2 is incorrect.

Q13. Four friends A, B, C and D need to cross a bridge. A maximum of two persons can cross it at a time. It is night and they just have one lamp. Persons that cross the bridge must carry the lamp to find the way. A pair must walk together at the speed of the slower person. After crossing the bridge, the person having faster speed in the pair will return with the lamp each time to accompany another person in the group. Finally, the lamp has to be returned at the original place and the person who returns the lamp has to cross the bridge, the time taken by them is as follows : A : 1 minutes, B : 2 minutes, C : 7 minutes and D : 10 minutes. What is the total minimum time required by all the friends to cross the bridge? **(UPSC CSAT 2016)**

(a) 23 minutes (b) 22 minutes

(c) 21 minutes (d) 20 minutes

Ans: (a)

Explanation:

A + *B* = 2 minutes + 1 returning minute = 3 minutes.

A + *C* = 7 + 1 = 8 minutes

A + *D* = 10 + 1 = 11 minutes

A = 1 minute.

∴ Minimum time required by all the friends to cross the bridge= 3 + 8 + 11 + 1 = 23 minutes.

Q14. Ram and Shyam work on a job together for four days and complete 60% of it. Ram takes leave then and Shyam works for eight more days to complete the job. How long would Ram take to complete the entire job alone? **(UPSC CSAT 2016)**

(a) 6 days (b) 8 days

(c) 10 days (d) 11 days

Ans: (c)

Explanation:

Let's take 100 units of work.

In 4 days Ram and Shyam together finish 60 units of work.

Remaining 40 units are finished by Ram alone in 8 days

No. of units done by Ram in 1 day = 5 units.

So in 4 days Ram finished 20 units out of 60 units

No. of units finished by Shyam in 4 days

= 60 - 20 = 40.

No. of units finished by Shyam in 1 day = 10 units.

No. of days taken by Shyam to finish all 100 units = 10 days.

PIPES AND CISTERNS

Pipes and Cisterns play a role in filling or emptying tanks, reservoirs, or cisterns.

An Inlet refers to a pipe or tap connected to a tank that fills it, while **an Outlet** is a pipe or tap linked to a tank that empties it. All the concepts of efficiency and negative work are very useful here. Following example will help us in better understanding.

QUESTIONS

Q15. A tank is half full. Pipe A can fill the tank in 14 min. Pipe B can empty it in 21 min. If both pipes are open, then how long will it take to empty the tank?

(a) 21 min (b) 25 min

(c) 20 min (d) 42 min

Ans: (a)

Explanation:

Pipe A can fill $\frac{1}{14}$ of the tank in 1 min.

Pipe B can empty $\frac{1}{21}$ of the tank in 1 min.

So, In 1 minute, $\frac{1}{14}-\frac{1}{21}=\frac{1}{42}$ of the tank is empty.

In 42 minutes the whole tank is empty.

Since tank is half full, So $\frac{1}{2}\times 42=21$ minutes.

Q16. A tank can be filled by pipe A in 8 minutes and pipe B can empty that tank in 12 minutes when operated independently. Then, at what time will the tank be filled, if both pipes are working together?

(a) 21 min (b) 25 min

(c) 20 min (d) 24 min

Ans: (d)

Explanation:

Part filled by pipe *A* in 1 min = $\frac{1}{12}$

Part emptied by pipe *B* in 1 min = $\frac{1}{8}$

Part filled by both pipe operating together in 1 minute

$= \frac{1}{8} - \frac{1}{12} = \frac{1}{24}$

Total time taken = $\frac{1}{\frac{1}{24}}$ = 24 minute

Q17. A pipe can fill a tank in 16 hours. Due to a leak in the bottom,it is filled in 20 hours ,if the tank is full, then how long will it take to empty from the leak?

(a) 21 hours (b) 80 hours

(c) 20 hours (d) 40 hours

Ans: (b)

Explanation:

Part filled in 1 hour = $\frac{1}{16}$

Part emptied due to leak in 1 hour = $\frac{1}{20}$

Then, $\frac{1}{16} - \frac{1}{20} = \frac{5-4}{80} = \frac{1}{80}$

Time taken = $\frac{1}{\frac{1}{80}}$ = 80 hours.

Q18. Two pipes A and B can independently fill a tank completely in 20 and 30 minutes respectively. If both the pipes are opened simultaneously, how much time will they take to fill the tank completely?

(UPSC CSAT 2015)

(a) 10 minutes (b) 12 minutes

(c) 15 minutes (d) 25 minutes

Ans: (b)

Explanation:

Pipe A can fill $\frac{1}{20}$ of the tank in 1 min.

Pipe B can fill $\frac{1}{30}$ of the tank in 1 min.

In 1 min $\frac{1}{20} + \frac{1}{30} = \frac{5}{60}$ of the tank.

In 1 minutes these pipes can fill $\frac{1}{12}$ of the tank.

Time taken to both the pipes to fill the tank = 12 minutes.

Q19. A man completes 7/8th of a job in 21 days. How many more days will it take him to finish the job if the quantum of work is further increased by 50%.

(UPSC CSAT 2021)

(a) 24 (b) 21

(c) 18 (d) 15

Ans: (d)

Explanation:

Time taken to complete $\frac{7}{8}$ of a job = 21 days.

Quantum of work further increased by 50%

Time required to complete the job =

21/(⅞)= 24 days.

So time taken by the man to complete 50% work,

t1 = $\frac{24}{2}$ = 12 days

Also, remaining work after 21 days = $1 - \frac{7}{8} = \frac{1}{8}$

Time required to complete $\frac{1}{8}$ of the job,

t2 = Time required to complete the whole work - time taken to complete $\frac{7}{8}$ of the job i.e. = 24 – 21 = 3 days.

Now the number of days required = *t*1 + *t*2 = 12 + 3 = 15 days.

PRACTICE QUESTIONS

1. **P and Q do a piece of work in 24 days, Q and R do the same work in 30 days, and R and P do it in 40 days. In how many days, Q alone can complete the same work?**
   (a) 40 days (b) 30 days
   (c) 35 days (d) 36 days

2. **The capacity of *Q* is 2.5 of *P*, Capacity of *R* is 0.75 times of *P*. If *P*, *Q* and *R* works alone, then the ratio of time taken by each will be:**
   (a) 15:8:20 (b) 12:10:15
   (c) 10:8:13 (d) 15:6:20

3. **Jay is twice as fast as Veeru and is therefore able to finish work in 20 days less than Veeru. Find the time in which they can do it working together.**
   (a) $11\frac{1}{3}$ days (b) $12\frac{2}{3}$ days
   (c) $11\frac{2}{13}$ days (d) $13\frac{1}{3}$ days

4. **In a hostel, the food is required for 400 students for 100 days. After 20 days, 100 new students came to the hostel. For how many days the remaining food will last?**
   (a) 72 days (b) 70 days
   (c) 60 days (d) 64 days

5. **24 men can complete l/ 3rd of work in 9 days. In how many days 18 men can complete the same work?**
   (a) 28 days (b) 32 days
   (c) 36 days (d) 30days

6. **2 men or 3 women or 4 boys can do a piece of work in 78 days. Then in how many days will 1 man, 1 woman and 1 child working together complete the work?**
   (a) 72 days (b) 76 days
   (c) 68 days (d) 70 days

7. **A certain number of men can do a piece of work in 42 days. If there were 4 men less, it could be finished in 14 days more. How many men were there?**
   (a) 14 (b) 16
   (c) 18 (d) 24

8. **60 men can complete a job in 80 days. However, after 48 days, some men out of the assigned 60 men left the job. The remaining people took another 80 days to complete the job. The number of men who left the job is:**
   (a) 26 (b) 24
   (c) 30 (d) 36

9. ***A*, *B* and *C* can do a work in 9, 12 and 18 days respectively. On the first day, A works alone and on 2nd day, A is associated with B and C, and this pattern continues. Then how many days are required to complete the work?**
   (a) $4\frac{7}{9}$ days (b) $5\frac{2}{3}$ days
   (c) $6\frac{4}{9}$ days (d) $6\frac{7}{9}$ days

10. ***P* can do a piece of work in 8 days; *Q* and *R* together can do it in 6 days, while *P* and *R* together can do it in 4 days. How long will R alone take to complete the work?**
    (a) 7 days (b) 9 days
    (c) 8 days (d) 10 days

11. ***P*, *Q* and *R* together can finish a piece of work in 10 days. *P* and *R* together work twice as much as *Q*. *P* and *Q* together work thrice as much as *R*. In how many days, *Q* alone can do the same work?**
    (a) 25 days (b) 30 days
    (c) 28 days (d) 24 days

12. ***P* alone takes 16 days more to complete the work then both *P* and *Q* working together. If *Q* works alone, he takes 4 days more to complete the work, then *P* and *Q* working together. In how many days *P* and *Q* finish that work together?**
    (a) 8 days (b) 10 days
    (c) 14 days (d) 18 days

13. **4 women can complete a piece of work in 24 days and 6 men can complete the same work in 18 days. In how many days 4 men and 4 women complete that work?**
    (a) $13\frac{3}{4}$ days (b) $11\frac{11}{15}$ days
    (c) $12\frac{12}{17}$ days (d) $11\frac{8}{15}$ days

14. **20 women completed a work in 24 days. The same work can be completed in 16 days by 10 men. How many days will be required to complete when 6 women and 3 men are employed to do the same work?**
    (a) 26 days (b) 28 days
    (c) 32 days (d) 36 days

15. If 5 men can complete a piece of work in 6 days by working 7 hours a day, then in how many days can 7 men do the same work by working 6 hours a day?

(a) 5 days (b) 4 days

(c) $4\frac{1}{2}$ days (d) $4\frac{1}{3}$ days

16. *A*, *B* and *C* can finish a work in 8, 12 and 15 days respectively. On the first day, *A* is associated with *B* & on 2nd day *A* is associated with C and this process continues. Then in how many days total work will be completed?

(a) $4\frac{12}{13}$ days (b) $4\frac{13}{14}$ days

(c) $4\frac{24}{25}$ days (d) $4\frac{23}{24}$ days

17. *P* and *Q* can do a work in 10 days and 15 days respectively. They start the work together but after 4 days, *P* left the work and the remaining work is done by *Q* alone. Find the total number of days required to complete the work.

(a) 7 days (b) 9 days

(c) 12 days (d) 11 days

18. A contractor undertook road construction work which has to be completed in 45 days. He employed 100 men in the beginning and 100 more after 40 days and completed the construction at the right time. If he had not employed the additional men in how many days total work would have been finished?

(a) 48 days (b) 50 days

(c) 52 days (d) 60 days

19. *P* can complete work in 7 More hours than *Q*. While *P* does the same work in 16 more hours than *R*. If *P* and *Q* complete the work in the same time in which *R* alone does. Then in what time *P* will do the same work alone?

(a) 20 hours (b) 18 hours

(c) 32 hours (d) 28 hours

20. Mohan is 50% more efficient than Prakash. While Mohan takes 15 days less than Prakash to finish the work alone. Then in how many days they can finish 50% of more work together?

(a) 27 days (b) 18 days

(c) 15 days (d) 24 days

ANSWERS

1. (*a*)	2. (*d*)	3. (*d*)	4. (*d*)	5. (*c*)	6. (*a*)	7. (*b*)	8. (*d*)	9. (*b*)	10. (*c*)
11. (*b*)	12. (*a*)	13. (*c*)	14. (*c*)	15. (*a*)	16. (*c*)	17. (*b*)	18. (*b*)	19. (*d*)	20. (*a*)

EXPLANATIONS

1. (a)

Explanation:

Time taken by $P + Q$ to complete the work = 24 days

Time Taken by $Q + R$ to complete the work = 30 days

Time Taken by $R + P$ to complete the work = 40 days

Let the Total Work be 120 units (LCM of 24, 30, 40)

One day work of $P + Q = \frac{120}{24} = 5$ units

One day work of $Q + R = \frac{120}{30} = 4$ units

One day work of $R + P = \frac{120}{40} = 3$ units

We have one day work of $P + Q + Q + R + R + P$

$= 5 + 4 + 3$ units

$2(P + Q + R) = 12$ units $\Rightarrow P + Q + R = 6$ units/day

We have, $R + P = 3$ units

$Q + 3 = 6 \Rightarrow Q = 3$ units/day

Time required by Q alone to complete the work $= \frac{120}{3} = 40$ days

2. (d)

Explanation:

$\frac{P}{R} = \frac{1}{0.75} = \frac{4}{3}$

Let, P = 4 units and C = 3 units

Q = 2.5P = 2.5 × 4 = 10units

Let, the total work be 60 units (LCM of 3, 4 and 10)

Time taken by *P* to complete the work will be $\frac{60}{4} = 15$

Time taken by *Q* to complete the work will be $\frac{60}{10} = 6$

Time taken by *R* to complete the work will be $\frac{60}{3} = 20$

Ratio of time taken by P, Q, R = 15:6:20

3. (d)

Explanation:

Ratio of efficiency of Jay and Veeru = 2:1

then, Ratio of time taken by Jay and Veeru

= 1:2 or 1x : 2x

Here, difference = 2x – 1x = 20 (given)

x = 20 days

Actual time taken by Jay and Veeru are 20 and 40 days respectively

Let the total work be 40 units (LCM of 20, 40)

time required to complete the work if they work together $= \frac{40}{(2+1)} = 13\frac{1}{3}$ days

4. (d)

Explanation:

Let's assume each student consumes 1 unit of food per day

Food present in the hostel

= 400 × 100 = 40, 000 units

Food remaining after 20 days

= 40, 000 – 20 × 400 = 32000 units

Number of students present in the hostel after 20 days = 400 + 100 = 500

The remaining food will last for = $\frac{32000}{500}$ = 64 days

5. (c)

Explanation:

24 men completes 1/3rd work in 9 days

Therefore, total time required by 24 men to complete the whole work = 9 × 3 = 27 days

We have, $\frac{M_1D_1}{W} = \frac{M_2D_2}{W}$

Time required by 18 men to complete the same work:

24 × 27 = 18 × D (Since work is same)

$D = 24 \times \frac{27}{18}$ = 36 days

6. (a)

Explanation:

2M = 3W = 4C

Ratio of efficiency

$\frac{M}{W} = \frac{3}{2}$ and $\frac{M}{C} = \frac{2}{1}$

Multiplying the first ratio with 2 and second ratio with 3

M : W : C = 6 : 4 : 3

Let one day work of 1 man, 1 woman and 1 child be 6 units, 4 units and 3 units respectively

2 Men can do the whole work in 78 days

Total work = 2 × 6 × 78 = 936 units

Time required to complete the work if 1 man 1 woman and 1 child work together = $\frac{936}{(6+4+3)}$

$\frac{936}{13}$ = 72 days

7. (b)

Explanation:

Let the number of men be 'x'

$x \times 42 = (x - 4) \times 56$

$42x = 56x - 224$

$14x = 224 \quad x = 16$

8. (d)

Explanation:

60 men can complete the job in 80 days.

Let the number of men who left the job be 'x'

Let one person does 1 unit of work every day

Total work = 60 × 80 = 4800 units

Work completed after 48 days = 60 × 48 = 2880 units

Remaining work = 4800 – 2880 = 1920units

$1920 = (60 - x) \times 80$

$1920 = 4800 - 80x$

$80x = 2880 \Rightarrow x = 36$

9. (b)

Explanation:

A, B, and C can do the work in 9, 12, and 18 days respectively

Let the total work be 36 units (LCM of 9, 12, 18)

One day work of A = $\frac{36}{9}$ = 4 units

One day work of B = $\frac{36}{12}$ = 3 units

One day work of C = $\frac{36}{18}$ = 2 units

On the first day, A alone works and on the 2nd day, A is associated with B and C

Work completed in 2 days = A + (A + B + C) = 4 + (4 + 3 + 2) = 13 units

2 – day work = 13 units

4 – day work = 13 × 2 = 26 units

Remaining work = 36 – 26 = 10 units

On 5th day, A will complete 4 units of work

Remaining work will be 6 units

On 6th day, A + B + C will be working together

Time required to complete 6 units of work

$= \frac{6}{(4+3+2)} = \frac{6}{9}$ = 2/3day

So, total time required = $5\frac{2}{3}$ days

10. (c)

Explanation:

Time taken by P to complete the work = 8 days

Time taken by Q + R to complete the work = 6 days

Time taken by P + R to complete the work = 4 days

Let the total work be 24 units (LCM of 8, 6, 4)

One day work of $P = \frac{24}{8}$ = 3 units

One day work of $Q + R = \frac{24}{6}$ = 4 units

One day work of $P + R = \frac{24}{4}$ = 6 units.

3 + R = 6 units

R = 3 units

Time required by R to complete the same work $= \frac{24}{3}$ = 8 days

11. (b)

Explanation:

Ratio of efficiency

$(P + R)$: Q = 2:1 (total 3 units)

$(P + Q)$: R = 3:1 (total 4 units)

Total work should be the same. therefore, multiplying the first ration with 4 and second ratio with 3

$(P + R)$: Q = 8:4 (total 12 units)

$(P + Q)$: R = 9:3 (total 12 units)

Q = 4 units/day , R = 3 units/day

$P + R$ = 8, P + 3 = 8, P = 5 units/day

$P + Q + R$ can finish the work in 10 days

Total work = (5 + 3 + 4) × 10 = 120 units

Time required by Q to complete the work $= \frac{120}{4}$ = 30 days

12. (a)

Explanation:

Let time taken by P and Q working together be 'x' days

Time taken by P alone will be 'x + 16' days

Time taken by Q alone will be 'x + 4' days

Let the Total work be "$(x + 16)$ $(x + 4)$" units

Work done by P in 1 day will be

$= (x + 16)(x + 4)/(x + 16) = (x + 4)$ units

Work done by Q in 1 day will be

$= (x + 16)(x + 4)/(x + 4) = (x + 16)$ units

Work done by P and Q working together in one day

$= (x + 16) + (x + 4) = (2x + 20)$ units

Time taken by P and Q working together

= Total work /work done in one day

$$x = \frac{(x+16)(x+4)}{(2x+20)}$$

$2x^2 + 20x = x^2 + 20x + 64$

$x^2 = 64 \Rightarrow x = 8$

13. (c)

Explanation:

4 women can complete the work in 24 days and 6 men can complete the work in 18 days.

4W × 24 = 6M × 18

$$\frac{M}{W} = \frac{8}{9}$$

Let, One day work of a Man (M) = 8 units and that of a Woman(W) = 9 units

4 women can complete a work in 24 days

Total work = 4 × 9 × 24 = 864 units

Time required to complete the work by 4 men and 4 women = 864/(4 × 9 + 4 × 8)

$\frac{864}{68} = 12\frac{12}{17}$ days

14. (c)

Explanation:

20W × 24 = 10M × 16

$$\frac{M}{W} = \frac{2*24}{16} \Rightarrow \frac{M}{W} = \frac{3}{1}$$

20 women can complete the work in 24 days

Total work = 20 × 1 × 24 = 480 units

Time required by 6 women and 3 men to complete the work = 480/(6 × 1 + 3 × 3) = 32 days

15. (a)

Explanation:

We know

$$\frac{M_1 D_1 H_1}{W} = \frac{M_2 D_2 H_2}{W}$$

Since work is same

5 × 6 × 7 = 7 × D × 6

$$D = \frac{5*6*7}{6*7}$$

D = 5 days

16. (c)

Explanation:

A, B and C can do work in 8, 12 and 15 days respectively.

Let the total work be = 120 units (LCM of 8, 12, 15)

One day work of A $= \frac{120}{8}$ = 15 units

One day work of B = $\frac{120}{12}$ = 10 units

One day work of C = $\frac{120}{15}$ = 8 units

On first day, A is associated with B & on 2nd day, A is associated with C

Therefore, work completed in 2 days

= (A + B) + (A + C) = (15 + 10) + (15 + 8) = 48 units

2 – day work = 48 units

4 – day work = 96 units

Remaining work = 120 – 96 = 24 units

On 5th day (A + B) will be working and they complete 25 units of work in one day

Time required to complete 24 units of work = 24/25th day

Total time = $4+\frac{24}{25}$ = $4\frac{24}{25}$ days

17. (b)

Explanation:

P and Q can do a work in 10 days and 15 days respectively

Let the total work be 30 units (LCM of 10, 15)

One day work of P = $\frac{30}{10}$ = 3 units

One day work of Q = $\frac{30}{15}$ = 2 units

If they start working together, one day work will be 3 + 2 = 5 units

Work completed in 4 days = 5 × 4 = 20 units

Remaining work = 30 – 20 = 10 units

Time required by Q alone to complete the remaining work = $\frac{10}{2}$ = 5 days

Total time = 4 + 5 = 9 days

18. (b)

Explanation:

Let us assume each man does 1 unit of work in one day

Initially there were 100 men and after 40 days there were 200 men

The work was completed in 45 days.

Total work = 40 × 100 + 5 × 200 = 5000 units

Time required to complete the work if no additional men were hired = $\frac{5000}{100}$ = 50 days

19. (d)

Explanation:

Let time taken by R or '*P* and *Q*' working together be '*x*' hours

Time taken by P alone to complete the work will be 'x + 16' hours

Time taken by Q alone will be 'x + 9' hour [difference between P and Q is 7 hours]

Let the Total work be "(*x* + 16) (x + 9)" units

Work done by P in 1 hour will be

= (*x* + 16) (*x* + 9)/(*x* + 16) = (x + 9) units

Work done by Q in 1 hour will be

= (*x* + 16) (*x* + 9)/(*x* + 9) = (*x* + 16) units

Work done by P and Q working together in one hour = (*x* + 16) + (*x* + 9) = (2*x* + 25) units

Time taken by P and Q working together

= Total work/work done in one hour.

$$x=\frac{(x+16)(x+9)}{(2x+25)}$$

$2x^2 + 25x = x^2 + 25x + 144$

$x^2 = 144$

x = 12 hours

Time taken by P alone to complete the work will be

= *x* + 16 = 12 + 16 = 28 hours

20. (a)

Explanation:

Ratio of efficiency of Mohan and Prakash

= 150:100 = 3:2

Ratio of time of Mohan and Prakash = 2*x* : 3*x*

The difference in time = 3*x* – 2*x* = 15 days

x = 15 days.

Actual time taken by Mohan and Prakash = 30 and 45 days respectively.

Total work = 30 × 3 or 2 × 45 = 90 units

150% of total work = 1.5 × 90 = 135 units

Time required to complete 150% of the work when they work together = 135/(3 + 2) = 27 days

Permutations and Combinations

13

INTRODUCTION

A permutation is an arrangement of objects in a specific order. Whereas a combination is a selection of objects without regard to the order. Both are important concepts in combinatorics (mathematics of counting and arranging), and an increasing number of questions appear in the examination from both concepts.

FUNDAMENTAL PRINCIPLE OF COUNTING (FPC)

(a) Multiplication Rule

If there are two tasks such that one of them can be completed in 'm' different ways following which another task can be completed in 'n' different ways, then the two tasks in succession can be completed in (m × n) ways. This rule can be further extended to any number of tasks.

(b) Addition Rule

If there are two events that cannot happen at the same time (mutually exclusive events), and if one event can occur in 'm' ways and another event can occur in 'n' ways, then there are (m+n) ways for either of the two events to occur.

UNDERSTANDING THE PRINCIPLE

The FPC provides us with a systematic way to count and calculate the total number of outcomes in a multi-stage process.

Step 1: Identifying Independent Choices

Identifying the independent choices or stages within a problem. Independent choices are those where the outcome of one choice doesn't affect the outcomes of others.

Step 2: Counting Each Choice

For each choice, determine the number of options available.

Step 3: Multiplying Choices

To find the total number of outcomes multiply the number of choices at each stage.

Let's see the principle in action. Imagine you're planning a trip with the following choices:

1. **Destinations:** You want to visit London or New York or Sydney (3 cities).
2. **Modes of Transport:** You can choose to travel by plane, train, or boat between each city (3 modes).
3. **Accommodations:** You can stay in hotels, hostels, or vacation rentals in each city (3 options).

Using FPC we can easily find the number of different unique travel itineraries possible.

- 3 (destinations) × 3 (modes of transport) × 3 (accommodations) = 27 different travel plans!

EXAMPLES

E1. The number of 5 digit numbers, with digits from the set {1,2,3,4,5} and repetition of digits is not allowed, is

(a) 60 (b) 80

(c) 120 (d) 140

Ans: (c)

Explanation:

Making a 5-digit number is equivalent to filling 5 boxes

Places:	1	2	3	4	5
Number of Choices	5	4	3	2	1

For the 1st box, there are 5 choices (1 through 5).

For the 2nd box, after using one digit for the first position, 4 choices are left (since no repetition is allowed).

For the 3rd box, 3 choices are left.

For the 4th box, 2 choices are left.

For the 5th box, 1 choice is left.

Total number of unique 5-digit numbers

Possible = 5 (choices for the 1st box) × 4 (choices for the 2nd box) × 3 (choices for the 3rd box) × 2 (choices for the 4th box) × 1 (choice for the 5th box) = 120 different 5-digit numbers.

E2. In an exam comprising 5 true-false questions, if a student answers to each question randomly, how many distinct answer sheets (sequences containing T and F) can be produced?

(a) 25 (b) 32

(c) 31 (d) 35

Ans: (b)

Explanation:

For every question, there are 2 choices (T or F). Now, to find the total number of different answer sheets(sequences of T and F), multiply these choices together for all 5 questions

$2 \times 2 \times 2 \times$ ..... 5 times

$= 2^5$ ways = 32 ways

E3. A suitcase has a number lock with 3 rings each marked with digits {0,1,2,3,4,5,6,7,8,9}. It can only be unlocked with a particular sequence set by its owner. A thief tries to open the lock by guessing the combination, in how many ways it is possible to make an unsuccessful attempt to open the lock.

(a) 100 (b) 1000

(c) 999 (d) None of these

Ans: (c)

Explanation:

Number of ways of marking each ring

= 10 different numbers.

So, total number of ways of setting a combination

= 10(choices for 1st ring) ×

10(choices for 2nd ring) × 10(choices for 3rd ring)

$= 10^3 = 1000$

But out of these 1000 attempt, one attempt is successful. Therefore, number of unsuccessful attempt = 1000 – 1 = 999

PERMUTATION VS COMBINATION

Permutation refers to the arrangement or ordering of a set of objects in a specific sequence or order.

While **combination** refers to the selection of a subset of objects from a larger set, where the order of selection does not matter.

Consider a set of three objects: A, B, and C. We want to count the number of ways to select two objects from this set.

Permutations: If we are interested in the permutations of two objects from this set, **we consider the order**. The permutations would be:

- AB (A selected first, B selected second)
- BA (B selected first, A selected second)
- AC (A selected first, C selected second)
- CA (C selected first, A selected second)
- BC (B selected first, C selected second)
- CB (C selected first, B selected second)

Combinations: If we are interested in the combinations of two objects, **we don't consider the order**. The combinations would be: AB, AC and BC (Here AB and BA are considered the same as the order doesn't matter)

FACTORIAL NOTATIONS

The continued product of first '*n*' natural numbers is called 'n-factorial' and is denoted as *n*!.

- **n! = n(n –1)(n –2).....3.2.1**

For example 5! = 1.2.3.4.5 = 120; 6! = 1.2.3.4.5.6 = 720

Points to Remember

- 0! = 1
- Factorials of only natural numbers are defined.
- n! = n (n – 1)! i.e. 6! = 6 × 5! or 8! = 8 × 7!
- Remember : 0 ! = 1, 1 ! = 1, 2 ! = 2, 3 ! = 6, 4 ! = 24, 5! = 120, 6 ! = 720

LINEAR PERMUTATIONS

The number of permutations of n different things taken r at a time is ${}^nP_r$, where

$${}^nP_r = \frac{n!}{(n-r)!} = n\,(n-1)\,(n-2)\ldots(n-r+1)$$

where 'n' is the total number of items, '*r*' is the number of items you're arranging.

Suppose you have a set of 7 books, and you want to arrange 3 of them on a bookshelf in a specific order. In how many different ways can you arrange these 3 books?

Using the permutation formula: ${}^7P_3 = \frac{7!}{4!} = \frac{7\times6\times5\times4!}{4!} = 210.$ or simply as $7 \times 6 \times 5 = 210$.

Note: The number of all permutations of 'n' distinct objects taken all at a time without repetition = ${}^nP_n = n!$

THE M^N vs N^M DILEMMA

These kinds of problems occur with permutations of different objects in which objects can be repeated.

A classic problem where this dilemma occurs is:

There are 'm' men and '*n*' monkeys ($n > m$). If a man can have any number of monkeys, in how many ways can every monkey be assigned a master?

For each monkey, there are 'm' choices for a master (since a man can have any number of monkeys). Since there are 'n' monkeys, the total number of ways to assign monkeys to men is **(m × m × m × ….. n times) = m^n.**

Another way to approach this problem is, that men can select a monkey in '*n*' ways. Since there are '*m*' men, the total number of ways in which masters can get a monkey is **(n × n × n × …….× m times) = n^m.**

To solve this confusion we must think of this problem as: 'a man can have any number of monkeys', while 'a monkey can't have any number of masters'.

So, the second approach is wrong as in that case 'one monkey can go to more than one master'.

STRING METHOD

If you want to consider arrangements where some specific items are together, you can treat these specific items as a single unit. This reduces the problem of arranging the rest of the items, including the combined unit.

Let's understand this with an example:

Suppose you have 5 friends, *A*, *B*, *C*, *D*, and *E*, and you want to arrange them in a row of chairs. However, you specifically want A and B to sit together.

Treat AB as a Single Unit (tied with a string): Consider *A* and *B* together as a single unit, denoted as AB. Now, you have four "units" to arrange: *AB*, *C*, *D*, and *E*.

Arrangements within the AB Pair: Within the *AB* pair, you can arrange *A* and B in two ways: AB or BA.

Total Arrangements

- Now, arrange the four units (AB, C, D, E) in a row. The number of arrangements is 4! = 24.
- However, within the AB pair, you have 2!(=2) ways to arrange A and B.
- So, the total number of arrangements is 24 × 2 = 48

Note:

1. Number of permutations of '*n*' different things, taken all at a time, when '*m*' specified things always come together is **$(n - m + 1)! \times m!$**
2. If you want to arrange items such that some specific items are not together, you can use the complementary counting approach. Instead of directly calculating the arrangements where the items are not together, you calculate the total number of arrangements and then subtract the arrangements where the items are together.

   Hence, the number of permutations of 'n' different things, taken all at a time, when '*m*' specified things never come together is **$n! - (n - m + 1)! \times m!$**

CIRCULAR PERMUTATIONS

Circular permutation deals with the arrangement of objects, individuals, or elements in a circular or closed-loop fashion. Unlike linear permutation, circular permutation involves arranging objects in a circle. This means that there are no distinct endpoints (starting points).

Thus, you can rotate the entire circle without changing the relative order of the objects. This leads to different starting points but the same arrangement in terms of object order. Therefore, you need to divide by the number of objects to account for this rotational symmetry.

Imagine you have 5 friends, and you want to arrange them in a circle for a group photo. In how many different ways can you arrange your friends around the circle?

Let's say these friends are A,B,C,D and E. Let's consider one of the arrangements as an example:

ABCDE. You can rotate this arrangement to get the following equivalent arrangements:

BCDEA, CDEAB, DEABC, EABCD

Similar to this arrangement each arrangement gets counted 5 times.

So, the number of distinct circular arrangements is:

$$\frac{120 \text{ (total arrangements)}}{5 \text{ (rotational symmetry)}} = 24$$

Circular Permutation Formula: The number of ways to arrange 'n' distinct objects in a circle is given by

(n - 1)! when rotations are considered identical. Remember that here clockwise and anti-clockwise arrangements are considered different.

Note: If we arrange flowers or garland beads in a necklace then there is no distinction between clockwise & anticlockwise direction. So the formula becomes $\frac{(n-1)!}{2}$

EXAMPLES

E4. Six friends are sitting around a circular table to play a board game. How many different seating arrangements are possible if two of the friends, Alice and Bob, insist on sitting next to each other?

(a) 24 (b) 36

(c) 48 (d) 64

Ans: (c)

Explanation:

Here, treat Alice and Bob as a single entity because they must sit next to each other. This reduces the problem to arranging 5 entities (Alice & Bob, and the other 4 friends) in a circle.

Now, we can use circular permutation to arrange these 5 entities. The formula for circular permutation is (n – 1)!, where '*n*' is the number of entities.

In this case, '*n*' is 5.

Number of Arrangements = [(5 – 1)!] × 2! = 4! × 2! = 48.

Here 2! is also multiplied for the arrangement of Alice and Bob (AB, BA).

COUNTING FORMULA FOR COMBINATION

Selecting '*r*' things out of '*n*' distinct things is denoted as:

${}^{n}C_{r}$ or $C(n, r)$ and ${}^{n}C_{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2)...(n-r+1)}{r!}$

For example: Selecting a playing 11 from a team of 15 players is ${}^{15}C_{11.}$

The Following Results must be Remembered

1. Given a set of n points in a plane, with the condition that no three of them are collinear:
   (a) Number of straight lines that can be formed:
   Number of ways by which we can select any two points gives the total number of straight lines = ${}^{n}C_{2}$.
   (b) Number of triangles that can be formed:
   Number of ways by which we can select any three non-collinear points gives total number of triangles = ${}^{n}C_{3}$
   (c) The total number of diagonals in a polygon with n sides:
   Number of diagonals = Total lines – (number of sides of polygon) = ${}^{n}C_{2} - n$
2. Given a set of n points in a plane, with the condition that m points of them are collinear:
   (d) Number of straight lines that can be formed:
   Number of ways by which we can select any two points gives the total number of straight lines = ${}^{n}C_{2} - {}^{m}C_{2} + 1$
   (e) Number of triangles that can be formed:
   Number of ways by which we can select any three non-collinear points gives total number of triangles = ${}^{n}C_{3} - {}^{m}C_{3}$

Note:

1. The number of combinations of 'n' distinct things taken 'r' at a time when 'p' particular things are always included = ${}^{n-p}C_{r-p}$
   Eg. total number of ways of selecting 11 players out of 15 players when Rohit Sharma and Virat Kohli are always included = ${}^{15-2}C_{11-2} = {}^{13}C_{9}$
2. The number of combinations of 'n' distinct things taken 'r' at a time when 'p' particular things are always excluded = ${}^{n-p}C_{r}$
   Eg. total number of ways of selecting 11 players out of 15 players when Hardik Pandya and Ashwin are always excluded = ${}^{15-2}C_{11} = {}^{13}C_{11}$

GAP METHOD

If you want to consider arrangements of some items when no two of some specified items are together.

To arrange 'n' different things in such a way that no two of the 'r' things are together, then we arrange the '*r*' things in gaps created by arranging remaining (*n*–*r*) things.

Suppose you have a group of 3 boys (B1, B2, B3) and 3 girls (G1, G2, G3), and you want to arrange them such that no two boys are next to each other.

Arrangement of girls: First of all, arrange 3 girls for which there is no restriction. This can be done in 3! ways.

Creation of gaps: Arrangement of 3 girls in a line creates 4 gaps. One such arrangement is (_G1_G2_G3_)

Arrangement of boys in gaps: Now select 3 gaps out of 4 (in ${}^{4}C_{3}$ ways) for 3 boys and arrange them in gaps in 3! ways.

Total Arrangements: $3! \cdot {}^{4}C_{3} \cdot 3! = 6 \times 4 \times 6 = 144$ ways.

EXAMPLES

E5. In how many ways 7 men and 7 women can be seated around a round table such that no two women can sit together

(a) $(7!)^2$ (b) $7! \times 6!$

(c) $(6!)^2$ (d) $7!$

Ans: (b)

Explanation:

Firstly 7 men can be seated in a circle in 6! ways. Now no two women are to sit together and as such the 7 women are to be arranged in seven empty seats (gaps) between two consecutive men. So 7 women can be seated in 7 gaps in 7! ways.

Therefore, total number of ways = 6! . 7!

PERMUTATION OF ALIKE OBJECTS

When you want to find the number of permutations of a set of objects, some of which are alike, you can use the concept of permutations with repetition. The formula for permutations of alike objects is:

$$P(n;n_1,n_2,n_3,\ldots,n_k)=\frac{n!}{n_1!n_2!n_3!\ldots\ldots n_k!}$$

where P represents the number of permutations, n is the total number of objects in the sets $n_1, n_2, n_3,\ldots, n_k$ of similar type.

Let's consider an example. Suppose you have the word "BOOKKEEPER," and you want to find the number of permutations of its letters. The letters in "BOOKKEEPER" are repeated, so we'll use the permutation formula with repetition:

- Total letters (n) = 10
- Number of 'O's (n_1) = 2
- Number of 'B's (n_2) = 1
- Number of 'K's (n_3) = 2
- Number of 'E's (n_4) = 3
- Number of 'P's (n_5) = 1
- Number of 'R's (n_6) = 1

$$P(10;2,1,2,3,1,1,)=\frac{10!}{2!1!2!3!1!1!}$$

EXAMPLES

E6. How many words can be formed with the letters of the word MATHEMATICS by rearranging them?

(a) $\frac{11!}{2!2!}$ (b) $\frac{11!}{2!}$

(c) $\frac{11!}{2!2!2!}$ (d) $11!$

Ans: (c)

Explanation:

Since there are 2 M's, 2 A's and 2 T's.

∴ Required number of ways are $\frac{11!}{2!2!2!}$.

DIVISION INTO GROUPS

Dividing a set of elements into distinct groups or subsets, is often called "partitioning" or "division into groups." The number of ways to perform such a division depends on various factors, such as the number of elements, the size of each group, and any additional constraints.

Partitioning into Unequal Groups

The number of ways in which 'n' distinct objects can be split into three groups containing respectively r, s and t objects, where r, s and t are distinct and r + s + t = n, is given by

$$^{n}C_{r}.\ ^{n-r}C_{s}.\ ^{n-r-s}C_{t}=\frac{n!}{r!s!t!}$$

Partitioning into Equal Groups

If 3n things are divided into three equal groups, then the number of ways is given by

$$=\frac{(3n)!}{n!n!n!3!}\quad=\frac{(3n)!}{3!\,(n!)^3}$$

In case of equal sized groups false counting occurs as internal arrangement in groups is not considered. To avoid this false counting we divide by factorial of the number of equal size group's (3! in this case).

In general, if we have to divide 'n' distinct objects into *l* groups containing p objects each, m groups containing q objects each then ways of group formation

$$=\frac{n!}{(p!)^l\times(q!)^m\times l!\times m!}$$ where $lp + mq = n$.

EXAMPLES

E7. The total number of ways in which 5 balls of different colors can be distributed among 3 persons so that each person gets at least one ball is

(a) 75 (b) 150

(c) 210 (d) 243

Ans: (b)

Explanation:

Firstly we should form groups of balls in which they can be distributed among 3 boys. The possible ways in which grouping can be done (giving at least one ball to each boy) is

Case-1: 1, 1, 3

Case-2: 2, 2, 1

Formation of groups for case-1: $\frac{5!}{1!1!3!2!}$

Formation of groups for case-2: $\frac{5!}{2!2!1!2!}$

Distribution among boys can be done in

$= \frac{5!}{1!1!3!2!} \times 3! + \frac{5!}{2!2!1!2!} \times 3! = 150$ ways

3! Is being multiplied to distribute 3 groups formed among 3 boys.

TOTAL NUMBER OF COMBINATIONS

(a) The number of combinations of n different things taking some or all (or atleast one) at a time

$= {}^nC_1 + {}^nC_2 + ...+ {}^nC_n = \mathbf{2^n - 1}$

(b) The number of ways of selecting zero or more things out of 'n' identical things = **(*n* + 1)**

(Selection of '0' thing = 1 way) + (Selection of '1' thing = 1 way) + (Selection of '2' things = 1 way) + ...+ (Selection of 'n' things = 1 way) = **(*n* + 1)** ways

(c) The number of ways to select some or all(or atleast one) out of (p + q + r) things where p are alike of first kind, q are alike of second kind and r are alike of third kind is = **(*p* +1) (*q* +1) (*r* + 1) – 1**

In the end '1' is subtracted from all cases when no item is selected from p,q and r things.

(d) The number of ways to select some or all out of (p + q + t) things where p are alike of first kind, q are alike of second kind and remaining t are different is = **(*p* +1) (*q* +1) $2^t - 1$**

(2^t is multiplied as '*t*' things are different (case (a)).

EXAMPLES

E8. Ishaan plans to organize a dinner party for his friends. In all he has 7 friends. In how many ways can he invite one or more of them to the party?

(a) 49 (b) 63

(c) 127 (d) 128

Ans: (c)

Explanation:

This can be thought as number of combinations of 7 different things taking some or all (or atleast one) at a time

$= {}^7C_1 + {}^7C_2 + {}^7C_3 ..... + {}^7C_7 = 2^7 - 1 = 128 - 1 = 127$ ways

E9. A basket of fruits contains four types of fruits, fruits of the same species being identical. It has 3 Apples, 4 Bananas, 2 Mangoes and 2 Melons. Find the number of ways in which at least one fruit is selected from the basket.

(a) 120 (b) 119

(c) 180 (d) 179

Ans: (d)

Explanation:

Consider:

The number of ways of selecting zero or more things out of '*n*' = **(*n* + 1) ways**

Apples can be selected in (3+1) ways

Bananas can be selected in (4+1) ways

Mangoes can be selected in (2+1) ways and Melons can be selected in (2+1) ways.

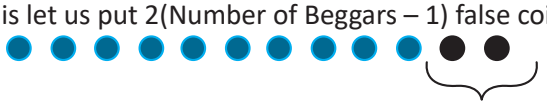
Ways of selecting at least one fruit from the basket

$= (3+1)(4+1)(2+1)(2+1) - 1$

$= 4 \times 5 \times 3 \times 3 - 1 = 180 - 1 = 179$

DISTRIBUTION OF ALIKE OBJECTS (BEGGAR'S METHOD)

Let there be 10 identical coins to be distributed among three beggars A, B and C. To divide these identical coins into 3 parts we require 2 partition walls between them, for this let us put 2(Number of Beggars – 1) false coins.



False Coins

Now we have 10 identical coins of one kind and 2 identical false coins of other kind. Consider this problem as permutation of alike objects. One of the possible arrangements be

In the above arrangement the 2 false coins divide the 10 given identical coins into 3 groups. The first beggar A will receive 2 coins, second beggar B will receive 3 coins and C will receive the remaining 5 coins.

- Total number of such arrangements = $\frac{12!}{10!2!} = {}^{12}C_2 = {}^{10+3-1}C_{3-1}$ (where number of identical coins = 10 and number of beggars = 3).

So, from the above method we can generalize:

> Total number of ways in which 'n' identical things can be distributed among 'r' persons so that each person may get any number of things is = ${}^{n+r-1}C_{r-1}$

- Consider the equation: **x + y + z + w = 10**
  - **Find non-negative integral solutions.**
    Consider x, y, z and w as four beggars and solving this equation is equivalent to distributing 10 identical coins to these 4 beggars when each beggar can receive any number of coins. So, ${}^{10+4-1}C_{4-1} = {}^{13}C_3$ non-negative integral solutions exist for the above equation.
  - **Find the number of natural solutions of the equation.**
    Finding the number of natural solutions of the above equation is equivalent to distributing 10 identical coins to 4 beggars, when each beggar gets at least one coin. So, give one coin to each beggar. Now if each one receives one coin, we are left with 6 coins. Equation becomes $x + y + z + w = 6$, hence, ${}^{6+4-1}C_{4-1} = {}^{9}C_3$ natural solutions exist for the above equation.

EXAMPLES

E10. A shop offers four flavors of ice-cream. A man has to buy 25 ice creams, buying at least four ice-creams of each flavor. In how many ways can the man plan his purchases, if ice-creams of each flavor are identical and available in abundance.

(a) 13C3 (b) 12C3
(c) 25C3 (d) 21C3

Ans: (b)

Explanation:

Let the four flavors be F1, F2, F3, F4. Consider these four flavors as four beggars among which 25 identical ice-creams are to be distributed giving four ice-creams to each beggar.

F1 + F2 + F3 + F4 = 25.

Firstly, give four ice –creams to each beggar. Now we have four beggars and 9 ice-creams, hence, in ${}^{9+4-1}C_{4-1} = {}^{12}C_3$ ways distribution can be done.

THE PRINCIPLE OF INCLUSION AND EXCLUSION (PIE)

The Principle of Inclusion and Exclusion (PIE) is a counting principle that provides a systematic way of counting the number of elements in the union of multiple sets. It is particularly useful when dealing with counting problems involving overlapping sets. It ensures that an object (or set of objects) is not counted twice.

Statement of the Principle

For two sets A and B, the principle can be stated as:

$|A \cup B| = |A| + |B| - |A \cap B|$

This formula accounts for the elements in both sets A and B but avoids double-counting of elements that are in both sets ($|A \cap B|$).

Extension to Three Sets

For three sets A, B, and C, the principle is extended as:

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

This formula adjusts for the overlaps in each pair of sets and also corrects for any elements that may have been subtracted too many times.

In this way the principle can be extended to any number of sets.

EXAMPLES

E11. Number of words that can be made with the letters of the word "DEBATE" if each word neither begins with D nor ends in E, is:

(a) 240 (b) 280
(c) 480 (d) 504

Ans: (d)

Explanation:

First we have to find all the arrangements of the word 'DEBATE' is 6 ! = 720

Using PIE,

Number of arrangement which either starts with D or ends with E is

(Words starting with D) + (Words ending with E) – (Words starting with D and ending with E) = (5! + 5! – 4!) = (120 + 120 – 24)= 216

Hence total number of arrangement which neither starts with D nor ends with E

= (720 – 216) = 504

DERANGEMENT

Derangement of a set is a permutation of its elements in which none of the elements appear in their original position. In other words, a derangement is a permutation with no fixed points.

Imagine you are hosting a dinner party with three guests: Alice, Bob, and Carol. You want to explore the idea of derangements in the context of seating arrangements.

Original Seating Arrangement

Alice sits in seat 1.

Bob sits in seat 2.

Carol sits in seat 3.

Derangements:

Let D_3 = arrangements where no one sits in their original seat.

- Alice sits in seat 2, Bob sits in seat 3, and Carol sits in seat 1.
- Alice sits in seat 3, Bob sits in seat 1, and Carol sits in seat 2.

***D_3* = 2** (arrangements where no one is in their original seat)

In general,

***D_n* = ways in which 'n' things are arranged so that all 'n' things occupy wrong places**

D_n = arrangement without restriction

– ways in which 1 thing is in correct position while (n – 1) things are deranged

– ways in which 2 things are in correct position while (n – 2) things are deranged

.....

– ways in which all n things are in correct position and there is no derangement

Recursive formula:

$$D_n = n! - {}^nC_1 D_{n-1} - {}^nC_2 D_{n-2} - \ldots\ldots\ldots\ldots - {}^nC_n D_0$$

The general formula for derangement is,

$$D_n = n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots\ldots\ldots + (-1)^r \frac{1}{r!}\right)$$

Note:

- $D_0 = 0! = 1$
- $D_1 = 1! - {}^1C_1 D_0 = 0$
- $D_2 = 2! - {}^2C_1 D_1 - {}^2C_2 D_0 = 1$
- $D_3 = 3! - {}^3C_1 D_2 - {}^3C_2 D_1 - {}^3C_3 D_0 = 2$
- $D_4 = 4! - {}^4C_1 D_3 - {}^4C_2 D_2 - {}^4C_3 D_1 - {}^4C_4 D_0 = 9$
- $D_5 = 5! - {}^5C_1 D_4 - {}^5C_2 D_3 - {}^5C_3 D_2 - {}^5C_4 D_1 - {}^5C_5 D_0 = 44$

EXAMPLES

E12. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is

(a) 264 (b) 265

(c) 53 (d) 67

Ans: (c)

Explanation:

Firstly, put card numbered 1 in envelope 2.

Now, card 2 can't be put correctly, so two cases arise.

Case1: If 'card 2' goes in 'envelope 1' then it is derangement of 4 things (cards other than 1 and 2) which can be done in $D_4 = 9$ ways

Case2: If 'card 2' doesn't go in 'envelope 1', it is Derangement of 5 things which can be done in $D_5 = 44$ ways.

Therefore, total ways = 9 + 44 = 53.

RANK OF A WORD

The "rank of a word in a dictionary" typically refers to the position of a word when the words are arranged in lexicographical order. Lexicographical order is the order in which words are listed in a dictionary, based on the alphabetical order of their letters. Here we will not consider the meaning of the word.

Let's consider a question to illustrate the concept:

All the six letters of the name SACHIN are arranged to form different words without repeating any letter in one word. The words so formed are, then arranged as in a dictionary. What will be the position of the word SACHIN in that sequence?

Dictionary order of the alphabets is: A, C, H, I, N, S

The number of words beginning with A (i.e. the number of words in which A comes in first place) is 5!

A						5! Words

Similarly, the number of words beginning with C is 5!, beginning with H is 5! and beginning with I is also 5! and beginning with N is also 5!

C						5! Words
H						5! Words
I						5! Words
N						5! Words

Next in order is S (with which our word begins), after S other letters will also come in alphabetical order.

So, after S, again A will occur then C, then H, then I and then N. So 'SACHIN' happens to be the first word beginning with S.

Therefore, the rank of the word SACHIN is = 5! + 5! + 5! + 5! + 5! + 1(for the word SACHIN itself) = 601.

SUM OF NUMBERS

Let's consider an example to illustrate this.

Example: Suppose you have the set of digits {1, 2, 3}. You want to find the sum of all numbers formed by permuting these digits.

Permutations:

123, 132, 213, 231, 312, 321

Formula for the Sum(S)

The formula to find the sum of all numbers formed by permuting the digits of a set is given by:

$S = (n-1)! \cdot (10^0 + 10^1 + 10^2 + \ldots\ldots + 10^{n-1}) \cdot$ (Sum of numbers), where 'n' is the number of digits in the set.

This term (n-1)! accounts for the total number of permutations of the digits. Second bracket representing powers of 10 represents the place value of each position. The place value of each position multiplied with the digit in the third bracket represents the contribution of each digit in each position.

In our example with the set {1, 2, 3}:

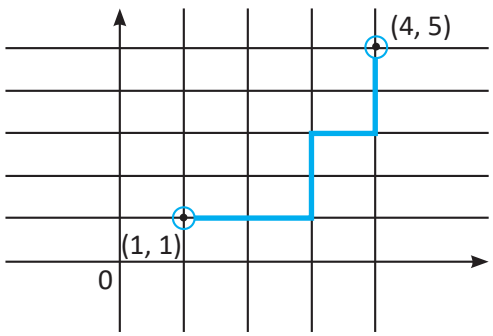
$S = 2! \cdot (10^0 + 10^1 + 10^2) \cdot (1 + 2 + 3) = 2 \times 111 \times 6 = 1332$

GRID PROBLEM

The "grid problem" in permutations typically involves counting the number of paths or arrangements within a grid. Let's explore a simple example to illustrate this concept.

Example: Paths in a Grid

Consider a grid (Cartesian plane). You start from (1,1) and want to reach (4,5), moving only up or to the right. The task is to count the number of different paths you can take.



When considering the possible paths (tracing them out), one of the paths is "Right, Right, Up, Up, Right, Up, Up".

While counting the paths we observe that in each path there are 3 right steps (R, R, R) and 4 up steps (U, U, U, U)

Number of different paths

= Number of ways of arranging 'RRRUUUU' (Permutation of alike objects) $= \frac{7!}{3!4!}$

MAXIMIZING ${}^nC_r$

- ${}^nC_r$ is maximum at:

$$\begin{cases} r = \frac{n}{2}, & \text{if } n \text{ is even} \\ r = \frac{n-1}{2} \text{ or } \frac{n+1}{2}, & \text{if } n \text{ is odd} \end{cases}$$

- ${}^{13}C_r$ is maximum when r = 6 and 7
- ${}^{10}C_r$ is maximum when r = 5

Some Important Identities

(a) ${}^nC_0 = 1 = {}^nC_n$

(b) ${}^nC_1 = n$

(c) ${}^nC_r = {}^nC_{n-r}$

(d) ${}^nC_x = {}^nC_y \rightarrow$ Either $x = y$ or $x + y = n$

(e) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(c) ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n = 2n$

(d) ${}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n = 2n - 1$

QUESTIONS

Q1. How many 3-digit natural numbers (without repetition of digits) are there such that each digit is odd and the number is divisible by 5?

(UPSC CSAT 2022)

(a) 8 (b) 12

(c) 16 (d) 24

Ans: (b)

Explanation:

To be divisible by 5, a number must end in 5 or 0.

Since we are looking for a 3-digit number with all odd digits, the last digit must be 5.

The first two digits must be odd and can be chosen from the set {1, 3, 7, 9}.

For 1st digit (4 choices) and 2nd digit (3 choices).

So, total number of 3-digit numbers with odd digits that end in 5 is: 4 x 3 x 1 = 12

Q2. A flag has to be designed with 4 horizontal stripes using some or all of the colours red, green and yellow. What is the number of different ways in which this can be done so that no two adjacent stripes have the same colour? (UPSC CSAT 2023)

(a) 12 (b) 18

(c) 24 (d) 36

Ans: (c)

Explanation:

Choices for flag stripes are as follows:

First Stripe: 3 choices for the colour of the first stripe (red, green, or yellow).

Second Stripe: Since no two adjacent stripes can have the same colour, there are 2 choices for the colour of the second stripe. It must be different from the colour of the first stripe.

Third Stripe: Again, there are 2 choices for the color of the third stripe, different from the color of the second stripe.

Fourth Stripe: The fourth stripe must have a different color from the third stripe, so there are 2 choices for the color of the fourth stripe.

Total Ways = 3 (choices for the first stripe) × 2 (choices for the second stripe) × 2 (choices for the third stripe) × 2 (choices for the fourth stripe)

Total Ways = 3 x 2 x 2 x 2 = 24 different ways to design the flag with the given conditions.

Q3. A, B and C are three places such that there are three different roads from A to B, four different roads from B to C and three different roads from A to C. In how many different ways can one travel from A to C using these roads? (UPSC CSAT 2022)

(a) 10 (b) 13

(c) 15 (d) 36

Ans: (c)

Explanation:

There are two ways in which one can go from *A* to *C*, directly (3 choices of roads) or Via B.

One can travel from A to B in 3 ways and B to C in 4 ways. So, one can travel A to C via B = 3 × 4 = 12.

Total ways = 3 + 12 = 15.

Q4. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of four parallel lines, is (UPSC CSAT 2019)

(a) 18 (b) 24

(c) 32 (d) 36

Ans: (d)

Explanation:

A parallelogram is formed by choosing two parallel lines from the first set of four parallel lines and two parallel lines from the second set.

∴ Number of parallelograms formed

= (Ways of selecting 2 lines from set I)

× (Ways of selecting 2 lines from set II)

$= {}^4C_2 \times {}^4C_2 = \frac{4!}{2!2!} \times \frac{4!}{2!2!} = 6 \times 6 = 36$

Q5. How many diagonals can be drawn by joining the vertices of an octagon? (UPSC CSAT 2018)

(a) 20 (b) 24

(c) 28 (d) 64

Ans: (a)

Explanation:

Using the formula,

Number of diagonals of n-sided polygon

= Total lines – (number of sides of polygon)

$= {}^nC_2 - n$

Here, $n = 8$ (for Octagon)

$\text{Diagonals} = {}^8C_2 - 8 = \frac{8!}{2!6!} - 8 = 28 - 8 = 20.$

Q6. The digits 1 to 9 are arranged in three rows in such a way that each row contains three digits, and the number formed in the second row is twice the number formed in the first row; and the number formed in the third row is thrice the number formed in the first row. Repetition of digits is not allowed. If only three of the four digits 2, 3, 7 and 9 are allowed to be used in the first row, how many such combinations are possible to be arranged in the three rows? (UPSC CSAT 2022)

(a) 4 (b) 3

(c) 2 (d) 1

Ans: (c)

Explanation:

We can only use three of the four digits – 2, 3, 7, and 9, in the first row.

The first digit in the first row cannot be 7 or 9, as otherwise thrice the number will not be a three-digit number.

So, the first digit in the first row can either be 2, or 3.

The possible cases are: 237, 273, 239, 293, 279, 297, 327, 372, 329, 392, 379, or 397.

On eliminating the numbers whose 3x is not a three-digit number, we are left with: 237, 273, 239, 293, 279, 297, 327, and 329.

On Checking the following numbers:

237 × 2 = 474 (digit repetition, and so eliminated)

273 × 2 = 546; 273 × 3 = 819

239 × 2 = 478; 239 × 3 = 717 (digit repetition, and so eliminated) 293 × 2 = 586; 293 × 3 = 879 (digit repetition, and so eliminated) 279 × 2 = 558 (digit repetition, and so eliminated)

297 × 2 = 594 (digit repetition, and so eliminated)

327 × 2 = 654; 327 × 3 = 981

329× 2 = 658; 329 × 3 = 987 (digit repetition, and so eliminated) So, only two cases are possible.

Q7. In a tournament of Chess having 150 entrants, a player is eliminated whenever he loses a match. It is given that no match results in a tie/draw. How many matches are played in the entire tournament?

(a) 151 (b) 150

(c) 149 (d) 148

Ans: (c)

Explanation:

The tournament starts with 150 players.

After the first round (in which 75 matches are held): 75 players are eliminated, and 75 remain.

After the second round (in which 37 matches are held): 37 players are eliminated, and 38 remain.

After the third round (in which 19 matches are held): 19 players are eliminated, and 19 remain.

After the fourth round (in which 9 matches are held): 9 players are eliminated, and 10 remain.

After the fifth round (in which 5 matches are held): 5 players are eliminated, and 5 remain.

After the sixth round (in which 2 matches are held): 2 players are eliminated, and 3 remain.

After the seventh round (in which 1 match is held): 1 player is eliminated, and 2 remain.

After the eighth round (in which 1 match is held): 1 player is eliminated, and 1 remains.

So, total number of matches

= 75 + 37 + 19 + 9 + 5 + 2 + 1 + 1 = 149.

Q8. Raj has ten pairs of red, nine pairs of white and eight pairs of black shoes in a box. If he randomly picks shoes one by one (without replacement) from the box to get a red pair of shoes to wear, what is the maximum number of attempts he has to make?

(a) 27 (b) 36

(c) 44 (d) 45

Ans: (d)

Explanation:

Raj has ten pairs of red, nine pairs of white and eight pairs of black shoes in a box. So, he has 20 red shoes, 18 white shoes, and 16 black shoes.

To find the maximum number of attempts we need to visualize the worst-case scenario. Let Raj draw all white and black shoes, which amounts to 18 + 16 = 34 shoes.

Now, if he picks any two shoes, they are certainly going to be red.

However, we also need to ensure that the red shoes picked by him make a pair, i.e. there should be one red shoe for left foot and one red shoe for the right foot. For this to happen we need to pick 11 red shoes.

So, maximum number of attempts to get a red pair of shoes = 34 + 11 = 45

Q9. There are four letters and four envelopes and exactly one letter is to be put in exactly one envelope with the correct address. If the letters are randomly inserted into the envelopes, then consider the following statements:

1. **It is possible that exactly one letter goes into an incorrect envelope.**
2. **There are only six ways in which only two letters can go into the correct envelopes.**

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

Ans: (b)

Explanation:

Statement 1: If one letter is placed in an incorrect envelope, then the letter that belonged to that envelope must have also been placed in an incorrect envelope. So, it's not possible that only one letter gets placed in an incorrect envelope. Either no letter will get misplaced, or at least two letters will get misplaced.

So, Statement 1 is incorrect.

Statement 2: We need to place two letters in the correct envelopes and two in incorrect envelopes.

We can choose the two letters to be placed in the correct envelope in ${}^4C_2$ ways, i.e. 6 ways.

The remaining two letters will automatically get chosen to get placed in each other's envelopes (i.e. incorrect envelopes).

So, Statement 2 is correct.

PRACTICE QUESTIONS

1. In a Company there are 3 vacancies. One is for the post of Director and the other two for the post of Deputy Director. For the two vacancies, applications from 6 candidates are received. Amongst the six applicants, only two are eligible for the post of Director while they all are eligible for the post of Deputy Director. The number of possible combinations of selectees is

   (a) 4 (b) 12
   (c) 18 (d) None of the above

2. In a question paper there are seven questions to be attempted. Each question can only be answered as either - True (T) or False (F). Each candidate answered all the questions. No two candidates have given the answers to the seven questions in an identical sequence. For this to happen the maximum number of candidates is:

   (a) 14 (b) 64
   (c) 1028 (d) 128

3. How many different sums can be formed with the denomination Rs. 10, Rs. 20, Rs. 50, Rs. 100, and Rs. 500 taking at least three denominations at a time?

   (a) 16 (b) 17
   (c) 18 (d) 24

4. The total number of arrangements that can be made out of the letters of the word "ATLANTA" without altering the relative positions of vowels and consonants is.

   (a) 24 (b) 64
   (c) 1028 (d) 128

5. A bag contains 10 balls. 5 balls are green, 3 are white and 2 are red. What is the minimum number of balls that must be picked up from the bag (without replacing any of it) to be assured of picking at least one ball of each colour?

   (a) 10 (b) 6
   (c) 9 (d) 8

6. In a drawing competition a flag is to be painted which consists of three parallel stripes. There are 6 different colors to choose from. The flag must be painted in a way that no two stripes should have the same colour. In how many ways can the flag be painted?

   (a) 120 (b) 81
   (c) 66 (d) 36

7. How many 4-digit numbers can be formed using the digits {1, 2, 3, ... 9} such that any digit that appears in such a number appears at least twice?

   (a) 220 (b) 225
   (c) 245 (d) 260

8. If 3 men and 3 women are to be arranged in a row so that the women are not next to each other, how many possible arrangements are there?

   (a) 124 (b) 720
   (c) 144 (d) 240

9. In how many ways can 6 letters be posted in 4 post boxes, if any number of letters can be posted in all of the four post boxes?

   (a) 6^4 (b) 4^6
   (c) ${}^6C_4$ (d) ${}^6P_4$

10. The password of a computer system consists of 8 digits' alphanumeric code. If all the odd positions are occupied by a vowel and even positions are occupied by non zero digits. Repetition of numbers is not allowed but alphabets can repeat. How many such passwords can be formed?

    (a) 1890000 (b) 1790000
    (c) 2090000 (d) 3000000

11. ABCD is a rectangle. Three points are taken each on the side AD and BC, two points each are taken on side AB and CD. None of the points taken is on the corner. How many different triangles can be formed using these points.

    (a) 120 (b) 124
    (c) 118 (d) 116

12. Five coins are tossed simultaneously. In how many of the outcomes will the third coin turn up a head?

    (a) 2^5 (b) 2^4
    (c) 2^6 (d) 2^3

13. **How many odd 4-digit positive integers that are multiples of 5 can be formed without using the digit 4?**
(a) 648 (b) 729
(c) 1296 (d) 916

14. **How many 5 digit even numbers can be formed using 0, 2,3,6,9 if repetition of digits is not allowed?**
(a) 60 (b) 72
(c) 120 (d) 48

15. **A three digit number has to be formed using the digits 3,4,5,8. If the number is divisible by 3, how many such numbers can be formed?**
(a) 10 (b) 12
(c) 18 (d) 20

16. **How many numbers are there between 99 and 1000, having at least one of their digits 5?**
(a) 240 (b) 272
(c) 252 (d) 27

17. **In a cricket championship, there are 36 matches. If each team plays one match with every other team, the number of teams is**
(a) 9 (b) 8
(c) 7 (d) None of these

18. **In how many ways can 10 papers be arranged if the best and the second best paper never come together?**
(a) (10!)/2 (b) (9!)
(c) (9!)×8 (d) (8!)×2

19. **In how many ways can 15 identical white and 10 identical black balls be arranged in a row so that no two black balls are together?**
(a) 150 (b) $^{15}C_{10}$
(c) $^{16}C_{10}$ (d) $^{14}C_{10}$

20. **Three dices each having 6 faces, numbered from 1 to 6 are rolled simultaneously. What is the number of possible outcomes such that at least two dice show the number 4?**
(a) 12 (b) 15
(c) 16 (d) 18

21. **3 men and 1 woman board a train in which 7 seats are vacant, two of these 7 seats are reserved for ladies. A woman may or may not sit on the seat reserved for ladies. In how many different ways can the 7 seats be occupied by these passengers?**
(a) 76 (b) 210
(c) 25 (d) 180

22. **A garland is to be made using 7 flowers of different colors but the flowers of pink and red colors are not to be kept together. In how many different ways can the garland be made?**
(a) 260 (b) 360
(c) 480 (d) 720

23. **A committee constituting 5 men and 5 women is to be formed. There are a total of 10 women and 8 men. Anita is not ready to be a part of the committee if any of her 3 male friends are a part of the committee. In how many possible ways can the committee be formed?**
(a) 7182 (b) 7056
(c) 1200 (d) 1800

24. **Ramesh has 10 pairs of black shoes, 12 pairs of white shoes, and 15 pairs of gray shoes. All these shoes are kept in a big box. He picks up shoes one by one blindfolded. If he wishes to wear black shoes, what is the minimum number of shoes that he must pick to be assured of getting a black pair of shoes?**
(a) 65 (b) 56
(c) 29 (d) 72

25. **In how many ways can a cricketer score a 50 by hitting only 6, 4 and 1 runs, If he compulsorily hits at least 6 sixes?**
(a) 8 (b) 10
(c) 12 (d) 15

ANSWERS

1. (*d*)	2. (*d*)	3. (*a*)	4. (*a*)	5. (*c*)	6. (*a*)	7. (*b*)	8. (*c*)	9. (*b*)	10. (*a*)
11. (*c*)	12. (*b*)	13. (*a*)	14. (*a*)	15. (*b*)	16. (*c*)	17. (*a*)	18. (*c*)	19. (*c*)	20. (*c*)
21. (*d*)	22. (*c*)	23. (*a*)	24. (*a*)	25. (*a*)					

EXPLANATIONS

1. (d)

Explanation:

Number of ways to select Director = ${}^2C_1$.

Number of ways to select Deputy Director = ${}^5C_2$

Total number of ways = ${}^2C_1 \times {}^5C_2$ = 2× 5×4/2

Number of possible combinations of selectees

= 2 × 10 = 20

2. (d)

Explanation:

There are 7 questions and each Question can either be answered in 2 ways.

No of ways = 2 × 2 × 2 × 2 × 2 × 2 × 2 = 2^7 = 128

If we select the 129th candidate then the sequence will start repeating.

3. (a)

Explanation:

Number of denominations = 5

Case-1: Using all the 5 denominations of currency notes.

Number of ways = 1

Case-2: Using 4 denominations of currency notes.

Here we have to choose 4 out of the 5 denominations of currency notes.

Number of ways = ${}^5C_4 = 5!/(4! * 1!) = 5$

Case-3: Using 3 denominations of currency notes.

Here we have to choose 3 out of the 5 denominations of currency notes.

Number of ways of choosing 3 out of 5 denominations = ${}^5C_3 = 5!/(3!.2!) = 10$

Total number of ways = 1+5+10 = 16

4. (a)

Explanation:

The four consonants all different can be arranged in 2nd, 3rd, 5th and 6th (four) places in 4!

= 4 × 3 × 2 × 1 = 24 ways.

Three vowels A, A, A out of which two are alike can be arranged in 1st, 4th and 7th positions in 1 way

A_ _ A_ _A

Hence the total number of ways are 24x1 = 24 ways

5. (c)

Explanation:

As per the question, we have to pick at least one ball of each colour.

Now even if we pick 5 balls, there's a probability that all of them may be green.

Hence, we must pick more than 5 balls for sure.

Now, even if we pick 5 + 3 balls, i.e. 8 balls, there's a probability that all of them may be green and white only.

Therefore we must pick 8+ 1 = 9 balls to be sure that at least one ball of all the three colours is picked up.

6. (a)

Explanation:

6 ways
5 ways
4 ways

Required number of ways = 6 × 5 × 4 =120 ways

7. (b)

Explanation:

4 digit numbers formed using digits {1, 2, 3,..., 9}.

Any digit that appears should appear at least twice.

Some 4-digit numbers that satisfy the condition: 1111, 1122, 1212 etc.,

Some 4-digit numbers that do not satisfy the condition: 1234, 1223

Possibility-1: All 4 digits are the same. Ex: 1111

This can be done in 9 ways.

Possibility-2: 4 digits by two different numbers. Ex: 1122

We need to select 2 digits from 9: This can be done in ${}^9C_2$ ways = 36

Now we need to reorder such digits: $4!/(2!2!)$ =6

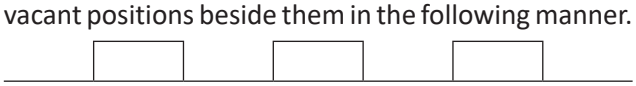
So total 4 digit numbers formed in Possibility-2 are = 36 × 6 = 216

So, Total such numbers= 9 + 216 = 225

8. (c)

Explanation:

Lets first arrange 3 men (in the boxes) and create 4 vacant positions beside them in the following manner.



Out of 4 vacant positions we have to select 3 places for the women.

So the total number of possibilities

= 3! × ${}^4C_3$ × 3! = 6 × 4 × 6 = 144

Note: here 3! Is for arranging 3 men, ${}^4C_3$ is for selecting 3 spaces out of 4 nearby positions for women and finally 3! is for arranging the women.

9. (b)

Explanation:

The first letter can be posted in any of the 4 post boxes. Therefore, we have 4 possibilities for the first letter.

Similarly, the second, the third, the fourth, the fifth and sixth letter can each be posted in any of the 4 post boxes.

Each of the 6 letters has 4 possibilities because we can post any number of letters in all of the boxes.

Therefore, the total number of ways in which the 6 letters can be posted in 4 boxes is $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$

10. (a)

Explanation:

Let us consider the 8 boxes for 8 characters of the code.

5	9	5	8	5	7	5	6

The odd boxes can be filled by all the 5 vowels. As repetition is allowed, all the boxes at odd positions have 5 options.

Total number of ways for arranging alphabets at odd positions are = 5×5×5×5

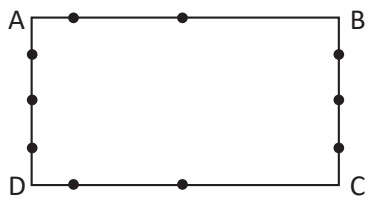
The boxes at the even position are to be filled by non zero digits. And as repetition of digits is not allowed,

Total number of ways for placing digits at even positions are = $9 \times 8 \times 7 \times 6$

Total possibilities $= 5 \times 9 \times 5 \times 8 \times 5 \times 7 \times 5 \times 6 = 1,890,000$

11. (c)

Explanation:



Total number of ways to select 3 points out of 10 = $^{10}C_3$

But there are two cases when a triangle will not be formed i.e when 3 collinear points on side AD are selected or when 3 collinear points on side BC are selected.

Total number of ways to form a triangle out of these points = $^{10}C_3 - 2$

$\Rightarrow 10!/(3!*7!) - 2 = 120 - 2 = 118$

12. (b)

Explanation:

When a coin is tossed once, there are two outcomes. It can turn up a head or a tail.

When 5 coins are tossed simultaneously, the total number of outcomes = 2^5

Out of these, if the third coin has to turn up a head, then the number of possibilities for the third coin is only 1 as the outcome is fixed as head.

Number of outcomes for the remaining 4 coins = ^{4}

$\therefore$ the number of outcomes in which the 3rd coin turns a head = $1 \times 2^4 = 2^4$

13. (a)

Explanation:

Given conditions: 4 digit odd number which is a multiple of 5 and should not contain 4.

Any number ending with 0 is an even number., So the unit digit has to be 5.

Left most digit cannot be 0 and also not 4, So for left most place we have 8 options.

The hundreds and tens places can take values from 0 to 9 except 4. So, 9 options for each of the two digits.

Number of such 4-digit numbers = 8 × 9 × 9 × 1 = 648

14. (a)

Explanation:

Case 1: When 0 is at unit's place

4	3	2	1	1

Fixed position for 0

Number of such possible numbers $= 4 \times 3 \times 2 \times 1 \times 1 = 24$

Case 2: When 0 is not at the unit's place.

3	3	2	1	2

Here, first we have to ensure that only an even digit comes at the unit's place.

Since in this case 0 is not placed at units place. We have two options to fill units digit i.e. 2 & 6.

Number of such possible numbers $= 3 \times 3 \times 2 \times 1 \times 2 = 36$

Total numbers = 24 + 36 = 60.

15. (b)

Explanation:

We have to select the 3 digits in a way such that their sum is divisible by 3.

Case-1: when 8 is not selected as a digit, then the digits are 3, 4, 5

3 + 4 + 5 = 12, hence all the numbers formed by using 3, 4, 5 will be divisible by 3

Total such numbers = 3! = 6

Case-2: when 5 is not selected as a digit, then the digits are 3,4,8

3 + 4 + 8 = 15, hence all the numbers formed by using 3,4,5 will be divisible by 3

Total such numbers = 3! = 6

Case-3: When 3 is not selected as a digit, then the digits are 4,5,8

4 + 5 + 8 = 17, hence all the numbers formed by using 4,5,8 will not be divisible by 3

Case-4: When 4 is not selected as a digit, then the digits are 3,5,8

3 + 5 + 8 = 16, hence all the numbers formed by using 3,5,8 will not be divisible by 3

Thus, total possible 3 digit numbers formed by using 3,4,5,8 , which are divisible by 3 = 6 + 6 = 12

16. (c)

Explanation:

Numbers between 99 and 1000 are all three-digit numbers.

Total number of 3 digit numbers having at least one of their digits as 5 = (Total numbers of three-digit numbers) – (Total number of 3 digit numbers in which 5 does not appear at all)

= (9 × 10 × 10) – (8 × 9 × 9)

= 900 – 648 = 252

17. (a)

Explanation:

Let *n* be the number of teams.

${}^{n}C_2 = 36$

(n(n-1)/2) = 36

$\Rightarrow n(n-1) = 72$

$\Rightarrow n = 9$

18. (c)

Explanation:

Total number of ways to arrange 10 papers = 10!

The cases when the best and second best papers are put together = 9! × 2

(**Note:** here we have considered the best and the second best, together as one unit.)

Number of cases when the best and second best is not put together = $10!-9!*2=9!(10-2)=9!*8$

19. (c)

Explanation:

When 15 identical white balls are placed in a row, It creates 16 spaces beside them.

To place 10 black balls we have to select 10 spaces out of 16 spaces.

Number of ways to select 10 ways out of 16 spaces = ${}^{16}C_{10}$

Here arrangement is not important as the balls are identical.

20. (c)

Explanation:

Case-1: when exactly 2 dice show the number '4'.

Number of possible outcomes=${}^{3}C_2 \times 5 = 15$

Here, ${}^{3}C_2$ is for selecting 2 dice showing the digit '4' and multiplication of 5 is for the number of possible outcomes for 3rd dice.

Case-2: When all the three dices show the number '4'

Number of possible outcomes = 1

Total number of possible outcomes= 15 +1 =16

21. (d)

Explanation:

Case-1: When woman occupies the reserved seat

Number of possible sitting arrangements

$= {}^{2}C_1 {}^{5}C_3 * 3! = 60$

${}^{2}C_1$ is for selecting the seat for the woman.

${}^{5}C_3$ is for selecting the seat for 3 men.

3! Is for arrangement of men

Case-2: When woman doesn't occupy the reserved seat

Number of possible sitting arrangements

$= {}^{5}C_4 \times 4! = 120$

Total number of possible ways of occupying 7 seats = 60 + 120 = 180

22. (c)

Explanation:

Number of ways to arrange the 7 flowers in a circular garland = $(7-1)! = 6!$

Number of cases when red and pink are kept together = $(6-1)!*2! = 5!*2!$

Arrangements when red and pink are not together

$= 6! - (5!*2!) = 5!(6-2) = 480$

23. (a)

Explanation:

Number of women (W) = 10

Number of men (M) = 8

Number of men to be selected= 5

Number of women to be selected= 5

Case-1: When Anita is selected

Number of possible ways to form the committee = ${}^{9}C_{4} \times {}^{5}C_{5}$ =126

Case-2: When Anita is not selected

Number of possible ways to form the committee = ${}^{9}C_{5} \times {}^{8}C_{5}$ =7056

Total number of ways to select the committee = 7056 + 126 = 7182

24. (a)

Explanation:

When he picks 24 shoes, It is possible that all shoes might be of white color.

When he picks 54 shoes , he might have picked 24 white shoes ,30 gray shoes.

When he picks up the 55th shoe, it must be of black color.

On picking up the 56th shoe it must also be a black shoe but it is possible that both the black shoes are of the same foot.

So, in the same manner it is possible to have picked the next 10 shoes of the same foot(from 55th to 64th). But when he picks the 65th shoe, it would complete a pair

25. (a)

Explanation:

Case 1: when he hits exactly 6 sixes

Possible ways to score a fifty:

1. Hits 3 fours
   Runs scored = $6\times6+3\times4+2\times1$
2. Hits 2 fours
   Runs scored = $6\times6+2\times4+6\times1$
3. Hits 1 four
   Runs scored = $6\times6+1\times4+10\times1$
4. Hits 0 fours.
   Runs scored = $6\times6+0\times4+14\times1$

Case-2: when he hits exactly 7 sixes

Possible ways to score a fifty:

1. Hits 2 fours
   Runs scored = $7\times6+2\times4+0\times1$
2. Hits 1 fours
   Runs scored = $7\times6+1\times4+4\times1$
3. Hits 0 fours
   Runs scored = $7\times6+0\times4+8\times1$

Case-3: when he hits exactly 8 sixes

Possible ways to score a fifty

1. Hits 0 four
   Runs scored = $8\times6+0\times4+2\times1$

Number of possible ways to score a fifty= 8

Probability

14

INTRODUCTION

Probability simply means the likelihood or chance of something happening. It helps us answer questions like, "What are the odds of an event occurring?"

Think of probability as a way to measure how likely it is that a specific outcome will happen. We often express it as a number between 0 (impossible) and 1 (certain). For example, if the probability of rain is 0.3, it means there's a 30% chance of rain.

IMPORTANT TERMS RELATED TO PROBABILITY

Random Experiment

A random experiment is a procedure, situation, or event where the outcome is uncertain or unpredictable. In the context of probability and statistics, a random experiment typically involves some form of chance or randomness.

Examples of random experiments include tossing a fair coin, rolling a fair six-sided die, drawing a card from a shuffled deck, or observing the weather on a given day. In all of these cases, the outcomes are uncertain, and the experiment can be repeated to observe different results.

Sample Space

Sample space represents the set of all possible outcomes of a random experiment or a random event. Examples of sample spaces:

- When tossing a fair coin, the sample space is {Heads, Tails}.
- When rolling a six-sided die, the sample space is {1, 2, 3, 4, 5, 6}.
- When selecting a card from a standard deck of 52 playing cards, the sample space is the set of all 52 cards.

Event

An event is a specific outcome or a set of outcomes that can occur within a sample space. An event is typically a subset of the sample space, which means it consists of one or more outcomes from the set of all possible outcomes.

- **Examples of Events**
  - When tossing two fair coins, getting head on both coins is an event.
  - When rolling a six-sided die, getting a prime number or getting an even number are different events.

Complement of an Event

The complement of an event, often denoted as A' (pronounced "A prime" or "not A"), is the set of outcomes in the sample space that are not part of the event A. **An event and its complement are mutually exclusive, which means they cannot happen at the same time.** If an outcome is in A, it cannot be in A', and vice versa.

Suppose you have a six-sided die, and event A is defined as rolling an even number ($A = \{2, 4, 6\}$). The complement of event A (A') would be rolling an odd number ($A' = \{1, 3, 5\}$). If the probability of rolling an even number is $\frac{3}{6}$ (or $\frac{1}{2}$), then the probability of rolling an odd number (the complement) would be $1 - \frac{1}{2} = \frac{1}{2}$

TYPES OF EVENTS

Simple(Elementary) Event

It refers to an event that consists of a single, specific outcome within a sample space.

- **Examples of simple events**
  - When rolling a six-sided fair die, the simple events are represented by each possible outcome: rolling a 1, rolling a 2, rolling a 3, and so on, up to rolling a 6.
  - When tossing a fair coin, the simple events are getting heads and getting tails, as these are the two distinct and elementary outcomes.

Compound Event

These are events that involve more than one outcome or event occurring together. These events can be created by combining simple events, multiple simple events, or a combination of simple and compound events. Some of

the types of Compound Events:

- **Union of Events (A ∪ B):** This represents the event where at least one of the events *A* or *B* occurs. For example, in the context of rolling a die, A might be the event "rolling an even number" (*A* = {2, 4, 6}), and *B* might be the event "rolling a number greater than 4" (*B* = {5, 6}). The union (*A* ∪ *B)* represents the event "rolling an even number or rolling a number greater than 4."
- **Intersection of Events (A ∩ B):** This represents the event where both events A and B occur simultaneously. Using the same example, the intersection A ∩ B represents the event "rolling an even number and rolling a number greater than 4." In this case, the only outcome that satisfies both conditions is rolling a 6.

Equally Likely Events

Equally likely events are those where two or more events have the same probability of occurring. In other words, all the possible outcomes are equally likely to happen, and there is no favoritism or bias toward any specific outcome.

- **Examples of equally likely events:**
  - **Tossing a Fair Coin:** When tossing a fair coin, getting a "head" and getting a "tail" are equally likely events because the coin has two sides with equal probability.
  - **Rolling a Fair Six-Sided Die:** In the case of rolling a fair six-sided die, each number from 1 to 6 is an equally likely event because the die is unbiased, and each face has an equal chance of landing face up.
  - **Drawing a Card from a Well-Shuffled Deck:** In a well-shuffled standard deck of 52 playing cards, drawing any particular card (e.g., the Ace of Spade) is an equally likely event because each card has an equal chance of being drawn.

Mutually Exclusive Events

Mutually exclusive events are events that cannot occur at the same time. If one of these events happens, it precludes the occurrence of the other event within the same trial or experiment. In other words, the outcomes of mutually exclusive events are separate and cannot overlap.

- **Examples of mutually exclusive events**
  - When flipping a fair coin, the events "getting head" and "getting tail" are mutually exclusive because both cannot happen at the same time.
  - In a deck of playing cards, the events "drawing a red card" and "drawing a black card" are mutually exclusive since a card cannot be both red and black.

Exhaustive Events

Exhaustive events are a set of events that collectively cover all possible outcomes of a random experiment. In other words, when you consider exhaustive events together, they account for every possible result that can occur in a given situation.

For Example, In a single roll of a fair six-sided die, the exhaustive events can be defined as "rolling an even number" (E) and "rolling an odd number" (O). Together, these two events are exhaustive, as they cover all possible outcomes.

PROBABILITY

If there are 'n' exhaustive, mutually exclusive and equally likely cases for an event A and 'm' of those are favourable to it, then probability of happening of the event A is defined by the ratio *m*/*n* which is denoted by P(A). Thus

$$P(A) = \frac{m}{n} = \frac{\text{No. of favourable cases to A}}{\text{No. of exhaustive cases to A}}$$

COIN TOSS PROBABILITY

When working with coins in probability, we often consider a fair coin, which is a coin with two equally likely outcomes: heads (H) and tails (T).

Tossing a Single Coin

Total number of possible outcomes = 2

Sample Space = {H, T}

A fair coin has an equal chance of landing heads or tails on any single flip.

- The probability of getting heads (H) on a single coin flip is *P*(H) = 0.5.
- The probability of getting tails (T) on a single coin flip is P(*T*) = 0.5.

Tossing Two Coins

Total number of possible outcomes = $2^2 = 4$

Sample Space = {HH, HT, TH, TT}

- When flipping a fair coin multiple times, each flip is independent of the others. The probability of getting heads or tails remains 0.5 on each flip.

For example: P(2H) = (0.5)(0.5) = 0.25

Tossing Three Coins

Total number of possible outcomes = $2^3 = 8$

Sample Space = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

- When flipping a fair coin multiple times, each flip is independent of the others. The probability of getting heads or tails remains 0.5 on each flip.

For Example: P(1H) = P(HTT) + P(THT) + P(TTH)

= (0.5)(0.5)(0.5) + (0.5)(0.5)(0.5) + (0.5)(0.5)(0.5)

$= \frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$

DICE ROLL PROBABILITY

Standard six-sided dice are commonly used in probability scenarios, and they have six equally likely outcomes: 1, 2, 3, 4, 5, and 6.

Single Die Probability

Total number of possible outcomes = 6

Sample Space = {1, 2, 3, 4, 5, 6}

The probability of rolling a specific number (e.g., 3) on a fair six-sided die is P(3) = $\frac{1}{6}$

The probability of rolling an even number (2, 4, or 6) on a single die is P(even number) = $\frac{3}{6}=\frac{1}{2}$

Rolling Two Dice Probability

Total number of possible outcomes = $6^2 = 36$

Sample Space:

{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

When rolling multiple dice, each die's outcome is independent of the others.

The probability of a specific combination of outcomes is calculated by multiplying the probabilities of each die's outcome. For example, the probability of rolling a 4 on one die and a 2 on another die i.e., (4, 2) is $=\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$

- Probability of getting a doublet (Same number)

  $=\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$

- Sum of Dice Rolls: For example, there are multiple ways to obtain a sum of 7: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), and (6, 1). Each of these combinations has a probability of $\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$.

  Therefore, the probability of rolling a sum of 7 is $=\frac{6}{36}=\frac{1}{6}$.

PROBABILITY OF DRAWING CARDS

A standard deck of 52 playing cards consists of four suits [hearts, diamonds, clubs, and spades), each with 13 ranks numbers from 2 to 10, and face cards (Jack, Queen, King), and an Ace].

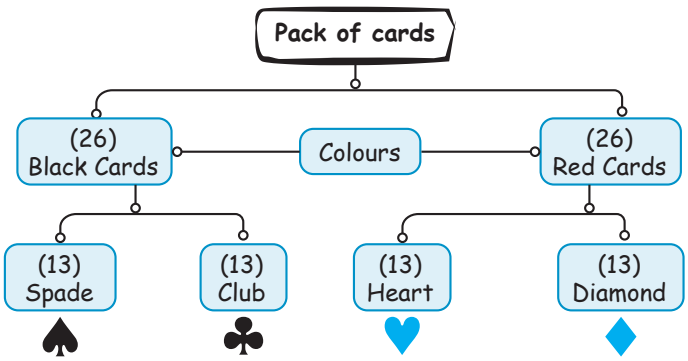


Fig. 14.1: Pack of cards

- Total cards: 52
- Red Cards = Black Cards = 26 each
- Hearts = Diamond = Spade = Clubs = 13 cards each
- Total King = Total Queen = Total Jack = Total Ace = 4 each
- Face Cards : King (4) + Queen (4) + Jack (4) = 12

The probability of drawing a specific card (e.g., the Ace of Spades) from a standard deck is P(Ace of Spades) = $\frac{1}{52}$

The probability of drawing a card of a specific rank (e.g., any Ace) is P(Any Ace) = $\frac{4}{52}=\frac{1}{13}$, since there are four Aces in the deck.

ODDS IN FAVOUR AND ODDS AGAINST AN EVENT

"Odds in favour" refer to a way of expressing the likelihood of an event occurring. These odds indicate how many times the event is expected to happen compared to how many times it is expected not to happen.

If an event A happens in '*m*' number of cases and if total number of exhaustive cases are '*n*' then we can say that-

- The probability of event *A*, *P* (A) = $\frac{m}{n}$
- $P(\overline{A}) = 1 - \frac{m}{n} = \frac{n-m}{n}$
- Odds in favour of A = $\frac{P(A)}{P(\overline{A})} = \frac{m/n}{(n-m)/n} = \frac{m}{n-m}$
- Odds in against of A = $\frac{P(\overline{A})}{P(A)} = \frac{(n-m)/n}{m/n} = \frac{n-m}{m}$

EXAMPLES

E1. Three ships A, B and C sail from England to India. If the ratio of their arriving safely are 2 : 5, 3 : 7 and 6 : 11 respectively then the probability of all the ships for arriving safely is

(a) $\frac{18}{595}$ (b) $\frac{6}{17}$

(c) $\frac{3}{10}$ (d) $\frac{2}{7}$

Ans: (a)

Explanation:

We have the ratio of the ships A, B and C for arriving safely are 2 : 5, 3 : 7 and 6 : 11 respectively.

$\therefore$ The probability of ship A for arriving safely

$=\frac{2}{2+5}=\frac{2}{7}$

Similarly, for $B=\frac{3}{3+7}$

$=\frac{3}{10}$ and for $C=\frac{6}{6+11}=\frac{6}{17}$

Probability of all the ships for arriving safely

$=\frac{2}{7}\times\frac{3}{10}\times\frac{6}{17}=\frac{18}{595}$.

ADDITION THEOREM

- $(A \cup B)$ = **A + B** = A **or** B denotes occurrence of at least one of A or B
- $P(A \cup B) = P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$

  If A & B are mutually exclusive then

  $P(A \cup B) = P(A) + P(B)$
- In case of three events, A, B and C

  $P(A \cup B \cup C) = P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
- If three events A, B and C are pairwise mutually exclusive then,
  - $P(A \cap B) = P(B \cap C) = P(C \cap A) = P(A \cap B \cap C) = 0$
  - $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

INDEPENDENT EVENTS

Independent events are events that are not influenced by the occurrence or non-occurrence of other events. In other words, the outcome of one event has no impact on the probability of the other event(s).

For example: when flipping a fair coin, the outcomes of consecutive coin flips are independent. The probability of getting heads on one flip does not affect the probability of getting heads on the next flip.

In a deck of cards, drawing one card and then drawing another card with replacement (putting back the first card) is treated as independent events. The probability of drawing a specific card on the second draw is not influenced by what was drawn on the first draw.

CONDITIONAL PROBABILITY

Conditional probability deals with the probability of an event occurring given that another event has already occurred. In other words, it quantifies the likelihood of an event happening under a specific condition or constraint. Conditional probability is denoted as P(A/B), where "P(A/B)" represents the probability of event A occurring given that event B has occurred.

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{n(A \cap B)}{n(B)}$$

MULTIPLICATION THEOREM

The Multiplication Theorem is used to calculate the probability of the joint occurrence of two or more events.

$(A \cap B)$ = **A × B** = A **and** B denotes the simultaneous occurrence of A and B.

- $P(A \cap B) = P(A).\ P(B/A) = P(B).\ P(A/B)$

If A and B are two independent events then happening of B will have no effect on A.

So $P(A/B) = P(A)$and similarly $P(B/A) = P(B)$

So, $P(A \cap B) = P(A).\ P(B)$

This rule can be further extended to any number of events, if A, B and C are independent events then

$P(A \cap B \cap C) = P(A).\ P(B).\ P(C)$

EXAMPLES

E2. If $4P(A) = 6P(B) = 10P(A \cap B)$ then $P\left(\frac{B}{A}\right)$ =

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$

(c) $\frac{7}{10}$ (d) $\frac{19}{60}$

Ans: (a)

Explanation:

$P(\frac{B}{A}) = \frac{P(A \cap B)}{P(A)}$

$= \frac{(1/10)}{(1/4)} = \frac{2}{5}$

TOTAL PROBABILITY THEOREM

The Total Probability Theorem, is used to find the probability of an event A by considering all the possible ways or partitions through which event A can occur. This theorem is particularly useful when the event A can happen in different ways or under different conditions.

For a partition of the sample space into mutually exclusive events B_1, B_2, B_3, ..., B_n (i.e., events that do not overlap and together cover the entire sample space), the probability of event A can be expressed as:

$A = (A \cap B_1) \cup (A \cap B_2) \cup (A \cap B_3) \cup ..... \cup (A \cap B_n)$

Now, $P(A \cap B_i) = P(A).\ P(B_i/A) = P(B_i).\ P(A/B_i)$

So, $A = \sum_{i=1}^{n} P(A \cap B_i) = \sum_{i=1}^{n} P(B_i).P(A/B_i)$

Consider the following scenario, a factory produces light bulbs of two different types, Type A and Type B.

Type A bulbs have a 60% chance of being of high quality, while Type B bulbs have a 40% chance of being of high quality. The factory produces Type A bulbs 70% of the time and Type B bulbs 30% of the time.

You want to find the probability of randomly selecting a high-quality bulb. We can use the Total Probability Theorem to find the overall probability of selecting a high-quality bulb (event H), considering the two types of bulbs (Type *A* and Type *B*) as partitions (B_1 and B_2).

- Event B_1: Selecting a Type A bulb
- Event B_2: Selecting a Type *B* bulb.

The Total Probability Theorem states that:

$P(H) = P(H/B_1)\cdot P(B_1) + P(H/B_2)\cdot P(B_2)$

$P(H) = (0.6)\cdot(0.7) + (0.4)\cdot(0.3) = 0.42 + 0.12 = 0.54$

So, the probability of randomly selecting a high-quality bulb from the factory is 0.54, or 54%.

BAYES' THEOREM

Bayes' theorem relates conditional probability and is often called inverse probability.

Let us consider any event 'A' of sample space 'S' .This event would have occurred due to the different causes (or due to the occurrence of any of the events A_1, A_2, ...A_i).

Now, if event A has occurred and we want to determine the probability that it happened due to a specific cause, let's say cause A_i. That means we want to calculate $P(A_i/A)$.

So, $P(A_i/A) = \frac{P(A_i \cap A)}{P(A)} = \frac{P(A_i)\cdot P(A/A_i)}{P(A)}$ where the denominator can be calculated using the total probability theorem. Application of Bayes' theorem can be better understood with an **example:**

Suppose you are a doctor, and a patient comes to you with symptoms that could be indicative of a rare disease, Disease X. The prevalence of Disease X in the general population is 1%, meaning that 1% of the population has the disease. You have a diagnostic test for Disease X, but the test is not perfect. The test has the following characteristics:

Sensitivity: The probability of a positive test result when the patient has the disease is 95%

Specificity: The probability of a negative test result when the patient does not have the disease is 90%

The patient takes the test and receives a positive result. What is the probability that the patient has Disease X?

To find the probability that the patient has Disease X , we can use Bayes' Theorem:

- P(Disease X | Positive Test)

$$= \frac{\text{P(Positive Test | DiseaseX)} \times \text{P(DiseaseX)}}{\text{P(Positive Test)}}$$

The denominator,

- P(Positive Test)

  = P(Positive Test | DiseaseX) × P(DiseaseX)

  + P(Positive Test | NoDiseaseX) × P(No DiseaseX)

  = (0.95) × (0.01) + (0.10) × (0.99) = 0.1085

- P(Disease X | Positive Test)

$$= \frac{(0.95)\cdot(0.01)}{0.1085} = \frac{0.0095}{0.1085} = \frac{95}{1085} = \frac{19}{217}$$

BINOMIAL PROBABILITY

Binomial probability is a specific type of probability distribution used to model the number of successes in a fixed number of independent Bernoulli trials (experiments with two possible outcomes: success and failure). The binomial probability formula is used to calculate the probability of getting a specific number of successes in a given number of trials. Let's explore this concept with an **example:**

Imagine you're flipping a fair coin (one that has an equal chance of landing heads or tails). You want to find the probability of getting exactly 2 heads in 3 coin flips.

In this case, we have the following parameters:

- n(number of trials): We're flipping the coin 3 times.
- r(number of successes): We want to get 2 heads.
- p(probability of success): The probability of getting a head on a single coin flip is 0.5 because it's a fair coin. The probability of failure (getting a tail) is also 0.5.

Binomial Probability Formula:

- $P(r \text{ successes}) = {}^{n}C_{r}\, p^{r}\, (1-p)^{n-r}$

In our coin-flipping example: $n = 3$ (3 coin flips).

$r = 2$ (we want 2 heads).

$p = 0.5$ (probability of getting a head).

(1−p)=0.5 (probability of getting a tail).

So, P(2 Heads) = ${}^{3}C_{2}\,(0.5)^{2}\,(0.5)^{3-2} = 3\times\frac{1}{4}\times\frac{1}{2}=\frac{3}{8}$

QUESTIONS

Q1. A coin is tossed and a dice is rolled. The probability that the coin shows the head and the dice shows 6 is

(a) $\frac{1}{8}$ (b) $\frac{1}{12}$

(c) $\frac{1}{2}$ (d) 1

Ans: (b)

Explanation:

Probability of getting head on coin= $\frac{1}{2}$ and probability of getting 6 on dice= $\frac{1}{6}$

Hence, required probability $=\left(\frac{1}{2}\right)\left(\frac{1}{6}\right)=\frac{1}{12}$

Q2. The probability of getting a total of 5 or 6 in a single throw of 2 dice is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{1}{6}$

Ans: (b)

Explanation:

Total number of ways = 36

Favorable numbers of cases are (1, 4), (2, 3), (3,2), (4, 1), (1, 5), (2, 4), (3, 3), (4, 2), (5, 1) = 9

Hence the required probability $\frac{9}{36}=\frac{1}{4}$

Q3. The probability that a leap year selected randomly will have 53 Sundays is

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$

(c) $\frac{4}{53}$ (d) $\frac{4}{49}$

Ans: (b)

Explanation:

A leap year contains 366 days *i.e.* 52 weeks and 2 days, clearly there are 52 Sundays in 52 weeks.

For the remaining two days, we may have any of the following two days (i) Sunday and Monday, (ii) Monday and Tuesday, (iii) Tuesday and Wednesday, (iv) Wednesday and Thursday, (v) Thursday and Friday, (vi) Friday and Saturday and (vii) Saturday and Sunday.

Now for 53 Sundays, one of the two days must be Sunday (either (i) or (vii)). Hence required probability = $\frac{2}{7}$

Q4. If A and B are any two events, then the probability that exactly one of them occur is

(a) $P(A) + P(B) - P(A \cap B)$ (b) $P(A) + P(B) - 2P(A \cap B)$

(c) $P(A) + P(B) - P(A \cup B)$ (d) $P(A) + P(B) - 2P(A \cup B)$

Ans: (b)

Explanation:

Required probability = A occurs and B does not occur or B occurs and A does not occur

$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$

$= P(A) + P(B) - 2P(A \cap B).$

Q5. The probability of happening at least one of the events A and B is 0.6. If the events A and B happens simultaneously with the probability 0.2, then P($\bar{A}$) + P($\bar{B}$) will be

(a) 0.4 (b) 0.8

(c) 1.2 (d) 1

Ans: (c)

Explanation:

We are given that $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.2$.

We know that if A and B are any two events, then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.6 = 1 - P(\bar{A}) + 1 - P(\bar{B}) - 0.2 \Rightarrow P(\bar{A}) + P(\bar{B}) = 2 - 0.8 = 1.2.$

Note:

Students must take care not to confuse between and/ or events. It is advised to write the equations and their values despite the obvious nature to avoid mistakes.

Q6. The probability of solving a question by three students are $\frac{1}{2},\frac{1}{4},\frac{1}{6}$ respectively. Probability of question is being solved will be

(a) $\frac{33}{48}$ (b) $\frac{35}{48}$

(c) $\frac{31}{48}$ (d) $\frac{37}{48}$

Ans: (a)

Explanation:

This question can also be solved by one student, by two students or by three students.

$P(A)=\frac{1}{2},P(B)=\frac{1}{4},P(C)=\frac{1}{6}$ $P(A\cup B\cup C)=P(A)+P(B)+P(C)$

$-[P(A).P(B)+P(B).P(C)+P(C).P(A)]+[P(A).P(B).P(C)]$

$=\frac{1}{2}+\frac{1}{4}+\frac{1}{6}-\left[\frac{1}{2}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{2}\right]+\left[\frac{1}{2}\times\frac{1}{4}\times\frac{1}{6}\right]=\frac{33}{48}.$

Q7. A bag 'A' contains 2 white and 3 red balls and bag 'B' contains 4 white and 5 red balls. One ball is drawn at random from a randomly chosen bag and is found to be red. The probability that it was drawn from bag 'B' was

(a) $\frac{5}{14}$ (b) $\frac{5}{16}$

(c) $\frac{5}{18}$ (d) $\frac{25}{52}$

Ans: (d)

Explanation:

Let E_1 be the event that the ball is drawn from bag A and E_2 be the event that it is drawn from bag *B* and *E* that the ball is red. We have to find $P(E_2/E)$

Since both the bags are equally likely to be selected, we have $P(E_1) = P(E_2) = (1/2)$.

Also, $P\left(\frac{E}{E_1}\right)=\frac{3}{5}$ and $\left(\frac{E}{E_2}\right)=\frac{5}{9}$

Hence by Bayes' theorem, we have

$$P(E_2/E)=\frac{P(E_2)P(E/E_2)}{P(E_1)P(E/E_2)+P(E_2)P(E/E_2)}$$

$$=\frac{\frac{1}{2}.\frac{5}{9}}{\frac{1}{2}.\frac{3}{5}+\frac{1}{2}.\frac{5}{9}}=\frac{25}{52}.$$

Q8. The probability that a man can hit a target is $\frac{3}{4}$. He tries 5 times. The probability that he will hit the target at least three times is

(a) $\frac{291}{364}$ (b) $\frac{371}{464}$

(c) $\frac{471}{502}$ (d) $\frac{459}{512}$

Ans: (d)

Explanation:

We have $p=\frac{3}{4}$ and therefore $q=\frac{1}{4}$ where p is probability of success and q is probability of failure and $n = 5$

Therefore required probability

$= P(3 \text{ success}) + P(4 \text{ success}) + P(5 \text{ success})$

$$={}^5C_3\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2+{}^5C_4\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)+{}^5C_5\left(\frac{3}{4}\right)^5$$

$$=10\times\frac{27}{1024}+5\times\frac{81}{1024}+1\times\frac{243}{1024}=\frac{918}{1024}=\frac{459}{512}$$

Q9. A bag contains 15 red balls and 20 black balls. Each ball is numbered either 1 or 2 or 3. 20% of the red balls are numbered 1 and 40% of them are numbered 3. Similarly, among the black balls, 45% are numbered 2 and 30% are numbered 3. A boy picks a ball at random. He wins if the ball is red and numbered 3 or if it is black and numbered 1 or 2. What are the chances of his winning?

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(a) $\frac{1}{2}$ (b) $\frac{4}{7}$

(c) $\frac{5}{9}$ (d) $\frac{12}{13}$

Ans: (b)

Explanation:

Total red balls = 15 Total black balls = 20

Probability of picking a random ball is Red = 15/35

Probability of picking Red and Number 3

= 15/35×40/100 - Eq (i)

(As 40% red balls are Number 3)

Probability of picking a random ball is Black = 20/35

(As Black Number 1 balls = 100% - Black Number 2 balls – Black Number 3 balls

= 100% – 45% – 30% = 25%)

Black and Number 1 = 20/35 × 25/100 - Eq (ii)

Black and Number 2 = 20/35 × 45/100 – Eq (iii)

(As 45% black balls are Number 2)

Total probability = Eq (i) + Eq (ii) + Eq (iii) = 4/7

PRACTICE QUESTIONS

1. **Three of the six vertices of a regular hexagon are chosen at random. What is the chance that the triangle with these three vertices does not form an equilateral triangle?**

   (a) $\frac{1}{20}$ (b) $\frac{3}{10}$

   (c) $\frac{1}{10}$ (d) $\frac{9}{10}$

2. **Two boxes A and B contain six oranges and four blue balls; and four orange and six blue balls respectively. One ball is drawn at random from box A and placed in box B. Then one ball is transferred at random from box B to box A. If one ball is now drawn at random from box A, what is the chance that the ball drawn is orange in colour?**

   (a) $\frac{32}{55}$ (b) $\frac{31}{55}$

   (c) $\frac{18}{55}$ (d) $\frac{19}{55}$

3. **In a regular decagon, there are diagonals of distinct sizes. If all the possible diagonals are drawn and one is chosen at random, then what is the chance that it is not the longest one?**

   (a) $\frac{5}{7}$ (b) $\frac{6}{7}$

   (c) $\frac{4}{7}$ (d) $\frac{3}{7}$

4. **Two numbers a and b are chosen at random from the set of first 25 natural numbers. What is the chance that ($a^2 - b^2$) is divisible by 3 is:**

   (a) $\frac{51}{100}$ (b) $\frac{8}{25}$

   (c) $\frac{41}{75}$ (d) None of these

5. **Two letters are taken out at random from the words MATHEMATICS and STATISTICS, one from each. What is the chance that the letters taken out are the same?**

   (a) $\frac{7}{55}$ (b) $\frac{9}{55}$

   (c) $\frac{7}{56}$ (d) None of these

6. ***A* and *B* alternately throw a pair of dice. *A* wins if he throws 5 before B throws 6 and B wins if he throws 6 before A throws 5. Find the chance of A's win, if A begins.**

   (a) $\frac{11}{19}$ (b) $\frac{17}{19}$

   (c) $\frac{16}{19}$ (d) $\frac{9}{19}$

7. **Given that the sum of two non-negative quantities is 100, the chance that their product is not less than $\frac{3}{4}$ their greatest product value is:**

   (a) $\frac{99}{100}$ (b) $\frac{51}{100}$

   (c) $\frac{49}{100}$ (d) None of these

8. **The letters in the word ATACAMA are rearranged to form a new word, with or without meaning. What is the chance that all the A's come together in the new word formed?**

   (a) $\frac{4}{35}$ (b) $\frac{1}{7}$

   (c) $\frac{6}{35}$ (d) None of the above

9. **In a pack of 8 cards, the cards are numbered from 1 to 8. Two cards are drawn at random without replacement. What is the chance of drawing 1 odd numbered card and the other numbered as a multiple of 2?**

   (a) $\frac{1}{2}$ (b) $\frac{3}{7}$

   (c) $\frac{4}{7}$ (d) $\frac{1}{4}$

10. **A three-digit number is formed using the digits from 1, 2, 5, 7 randomly without repetition. What is the chance that this number is divisible by 3?**

    (a) $\frac{7}{9}$ (b) $\frac{1}{9}$

    (c) $\frac{2}{9}$ (d) None of the above

11. **Five persons are to be chosen from a group of 9 people. What is the chance that a certain married couple will be chosen?**

    (a) $\frac{4}{9}$ (b) $\frac{5}{18}$

    (c) $\frac{3}{18}$ (d) None of the above.

12. **If 5 objects are distributed at random among 5 persons, the chances that at least one of them will not get anything is**

    (a) $\frac{5^5}{120}$ (b) $\frac{(5^5-120)}{5^5}$

    (c) $\frac{120}{5^5}$ (d) None of the above

13. The chance that A can solve a set of equations is 5/6 and B can solve it is 4/7. If both of them attempt to solve the equations, then what is the chance that the equations are solved?

(a) $\frac{1}{90}$ (b) $\frac{81}{91}$

(c) $\frac{13}{14}$ (d) None of the above

14. 25 girls, among whom are *A* and *B* sat down at a round table. What are the chances that there are exactly 5 girls between *A* and *B*?

(a) $\frac{20!}{25!}$ (b) $\frac{23!}{24!}$

(c) $\frac{23! \times 2!}{24!}$ (d) $\frac{2!}{25!}$

15. A bag contains 8 red and 7 green balls. Five balls are drawn two times successively. All the five balls are drawn such that the balls are not replaced before the second draw. What is the chance that the first draw gives 5 red balls and second draw gives 5 green balls?

(a) $\frac{2}{1287}$ (b) $\frac{2}{429}$

(c) $\frac{4}{429}$ (d) $\frac{4}{1287}$

16. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be spades. What is the chance that the missing card is a spade?

(a) $\frac{11}{50}$ (b) $\frac{3}{26}$

(c) $\frac{39}{51}$ (d) $\frac{23}{52}$

17. There is a job opening and 15 applications are received out of which 8 are women and 7 are men. What is the chance that at least 1 woman is selected if 2 persons are to be selected in total?

(a) $\frac{56}{105}$ (b) $\frac{4}{5}$

(c) $\frac{4}{15}$ (d) $\frac{5}{13}$

18. A person forgets the middle two digits of a mobile number. He started dialling the numbers randomly. What is the chance that he dials the correct number if the two digits which he forgot are different?

(a) $\frac{44}{45}$ (b) $\frac{1}{90}$

(c) $\frac{1}{45}$ (d) Cannot be determined

19. There are 4 volumes of a medical journal among 50 books arranged on a shelf in a random order. What is the chance that they occur in increasing order from right to left, if the volumes are not necessarily kept side by side?

(a) $\frac{1}{24}$ (b) $\frac{1}{46}$

(c) $\frac{1}{50}$ (d) None of the above

20. Three persons *A*, *B* and *C* are to speak at a function along with 5 other persons. if they all speak in random order, what is the chance A speaks after B and B speaks after C?

(a) ⅔ (b) ⅚

(c) ⅓ (d) ⅙

21. How many times a coin needs to be tossed in order to get at least once a tail with a chance greater than or equal to 0.8?

(a) 2 (b) 1

(c) 3 (d) 5

22. A man takes a step forward with a probability of 0.7 and backward with a probability 0.3. The probability that at the end of 13 steps he is one step away from the starting point is

(a) $^{13}C_7(0.21)^6$ (b) $^{13}C_7(0.21)^7$

(c) $^{13}C_1(0.3)$ (d) $^{13}C_1(0.7)$

23. Consider the first 50 natural numbers. What is the chance that the sum of any two randomly chosen numbers (without replacement) will be odd?

(a) $\frac{24}{49}$ (b) $\frac{1}{2}$

(c) $\frac{25}{98}$ (d) $\frac{25}{49}$

24. The ratio of number of boys and girls in two houses Blue and Red in a school are 3:1 and 2:5 respectively. An individual is selected to be the captain of the house. The chance that this individual is selected from the house Blue is $\frac{2}{3}$. What is the chance that the captain will be a boy?

(a) $\frac{1}{2}$ (b) $\frac{25}{42}$

(c) $\frac{17}{42}$ (d) $\frac{27}{47}$

25. Rohan speaks truth 4 out of 5 times and Sohan 3 out of 4 times. What is the chance that they will contradict each other in any statement?

(a) $\frac{7}{20}$ (b) $\frac{9}{20}$

(c) $\frac{13}{20}$ (d) $\frac{3}{20}$

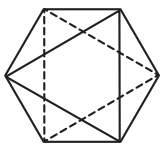
ANSWERS

1. (*d*)	2. (*a*)	3. (*b*)	4. (*c*)	5. (*a*)	6. (*d*)	7. (*b*)	8. (*a*)	9. (*c*)	10. (*d*)
11. (*b*)	12. (*b*)	13. (*c*)	14. (*c*)	15. (*a*)	16. (*a*)	17. (*b*)	18. (*b*)	19. (*a*)	20. (*d*)
21. (*c*)	22. (*a*)	23. (*d*)	24. (*b*)	25. (*a*)					

EXPLANATIONS

1. (d)

Explanation:



There are 6 vertices in a hexagon.

The number of triangles that can be formed = ${}^6C_3 = 20$

But there are 2 triangles out of 20 triangles that are equilateral as shown in the above figure.

Thus, the chances that the triangle formed are not equilateral = $\frac{18}{20} = \frac{9}{10}$

2. (a)

Explanation:

E_{oo} denote that a orange colored ball is transferred from box A to box B and then a orange colored ball is transferred from box B to box A.

E_{ob} denote that a orange color ball is transferred from box A to box B and thereafter a blue colour ball is transferred from box B to box A.

E_{bo} denote that a blue colour ball is transferred from box A to box B and then a orange colour ball is transferred from box B to box A.

E_{bb} denote that a blue colour ball is transferred from box A to box B and then a blue colour ball is transferred from box B to box A.

Chance that event E_{oo} will occur = $\frac{6}{10}\times\frac{5}{11}=\frac{3}{11}$

Chance that event E_{ob} will occur = $\frac{6}{10}\times\frac{6}{11}=\frac{18}{55}$

Chance that event E_{bo} will occur = $\frac{4}{10}\times\frac{4}{11}=\frac{8}{55}$

Chance that event E_{bb} will occur = $\frac{4}{10}\times\frac{7}{11}=\frac{14}{55}$

Chance of drawing a orange coloured ball from box A after the above four events will be $\frac{6}{10},\frac{5}{10},\frac{7}{10},\frac{6}{10}$

Chances that the ball drawn will be orange

$= \frac{6}{10}\times\frac{3}{11}+\frac{5}{10}\times\frac{18}{55}+\frac{7}{10}\times\frac{8}{55}+\frac{6}{10}\times\frac{14}{55} = \frac{32}{55}$

3. (b)

Explanation:

Total number of diagonals in a regular polygon of n side = $\frac{n(n-3)}{2}$

Number of diagonals in a decagon = $\frac{10(10-3)}{2} = 35$

Number of the longest diagonals = number of pairs of opposite points in a decagon = 5

Number of diagonals which are not the longest one's = 35 – 5 = 30

Required probability = $\frac{30}{35} = \frac{6}{7}$

4. (c)

Explanation:

Total number of natural numbers upto 25 which are divisible by 3 = 8

Number of ways 2 natural numbers can be chosen out of 25 = ${}^{25}C_2 = 300$

$a^2–b^2$ is divisible by 3 if and only if either a and b are divisible by 3 or none of a and b is divisible by 3.

Thus, number of ways = ${}^8C_2 + {}^{17}C_2 = 28+ 136 = 164$

Therefore, the chance that a^2-b^2 is divisible by 3 = $\frac{164}{300} = \frac{41}{75}$

5. (a)

Explanation:

Same letters are A, C, I, S and T

Chance of choosing A = $\frac{2}{11}\times\frac{1}{10} = \frac{2}{110}$

Chance of choosing C = $\frac{1}{11}\times\frac{1}{10} = \frac{1}{110}$

Chance of choosing I = $\frac{1}{11}\times\frac{2}{10} = \frac{2}{110}$

Chance of choosing S = $\frac{1}{11}\times\frac{3}{10} = \frac{3}{110}$

Chance of choosing T = $\frac{2}{11}\times\frac{3}{10} = \frac{6}{110}$

Chance that the letters taken out are same

$= \frac{2}{110}+\frac{1}{110}+\frac{2}{110}+\frac{3}{110}+\frac{6}{110} = \frac{7}{55}$

6. (d)

Explanation:

5 can be thrown with a pair of dice in the following number of ways

(1,4), (4,1), (2,3), (3,2)

Chance of throwing a five = $\frac{4}{36} = \frac{1}{9}$

Chance of not throwing a five = $\frac{8}{9}$

6 can be thrown with a pair of dice in the following number of ways

(1,5), (5,1), (2,4), (4,2) (3,3)

Chance of throwing a six = $\frac{5}{36}$

Chance of not throwing a six = $\frac{31}{36}$

A will win the game if he throws five in 1st or 3rd or 5th... chance

Chance of A winning in 1st throw = $\frac{1}{9}$

Chance of A winning in 3rd throw = $(8/9)\times(31/36)\times(1/9)$

Chance of A winning in 5th throw = $(8/9)^2\times(31/36)^2\times(1/9)$

and so on

Chance of A winning the game

$$=\frac{1}{9}+\left(\frac{8}{9}\times\frac{31}{36}\right)\times\frac{1}{9}+\left(\frac{8}{9}\times\frac{31}{36}\right)^2\times\frac{1}{9}\ldots$$

It forms a GP with first term (a) = $\frac{1}{9}$ and common difference(r) = $\left(\frac{8}{9}\times\frac{31}{36}\right)$

Sum of infinite GP = $\frac{a}{(1-r)}$

$$\text{Sum}=\frac{\frac{1}{9}}{1-\left(\frac{8}{9}\times\frac{31}{36}\right)}=\frac{9}{19}$$

7. (b)

Explanation:

Let x and y be the two non-negative integers

Since $x + y = 100$, $(xy)_{max} = 50\times50 = 2500$

(xy will be maximum for $x = y$)

Also, $xy \geq \left(\frac{3}{4}\right)\times2500 = 1875 \Rightarrow \times (100-x) \geq 1875$

$\Rightarrow x^2-100x + 1875 \leq 0 \Rightarrow 25 \leq x \leq 75$

So, favourable number of ways = 75-25+1=51

Total number of ways = 100

Chance as per conditions of the question = $\frac{51}{100}$

8. (a)

Explanation:

Total number of words, with or without meaning, formed from the letters of the word ATACAMA = $\frac{7!}{4!} = \frac{7\times6\times5\times4\times3\times2\times1}{4\times3\times2\times1}$ (Since there are 4 A's, the numerator is divided by 4!) = 210

Total number of words, with or without meaning, if all the A's are taken together =4! = $4\times3\times2\times1=24$

Chance that all the A's come together in the new word formed = $\frac{24}{210}=\frac{4}{35}$

9. (c)

Explanation:

Number of odd numbered cards in the pack = 4; Number of cards which are numbered as multiple of 2 = 4

Chance of odd numbered card being drawn in 1st instance = $\frac{4}{8}$.

Chance of drawing card which is numbered as multiple of 2 in 2nd instance = $\frac{4}{7}$

Chance of drawing card which is numbered as multiple of 2 in 1st instance = $\frac{4}{8}$

Chance of odd numbered card being drawn in 2nd instance = $\frac{4}{7}$

Chance of drawing 1 odd numbered card and the other numbered as a multiple of 2 = $\left(\frac{4}{8}\times\frac{4}{7}\right)+\left(\frac{4}{7}\times\frac{4}{8}\right)=\frac{4}{7}$

10. (d)

Explanation:

Since the sum of any three digits out of the given four is not divisible by 3, thus, no three-digit number so formed will be divisible by 3.

11. (b)

Explanation:

Total number of ways in which 5 people can be chosen out of 9 people = ${}^9C_5 = 126$

Number of ways in which the married couple is chosen = ${}^7C_3 \times 1 = 35$

Thus, the required probability = $\frac{35}{126} = \frac{5}{18}$

12. (b)

Explanation:

Total Number of ways to distribute 5 objects among 5 persons = $5\times5\times5\times5\times5 = 5^5$ ways

Number of ways of distribution in which each one gets only one thing = $5\times4\times3\times2\times1 = 120$

So, the number of distributions in which atleast one of them does not get anything = 5^5–120

Required probability = (5^5–120)/5^5

13. (c)

Explanation:

Chance that A solves the equations = $\frac{5}{6}$; Chance that A does not solve the equations = $\frac{1}{6}$

Chance that B solves the equations = $\frac{4}{7}$; Chance that B does not solve the equations = $\frac{3}{7}$

Chance that the equations are solved = A solves the problem AND B does not solve the problem

OR

A does not solve the problem AND B solves the problem

OR

A solves the problem AND B also solves the problem

$$=\left(\frac{5}{6}\right)*\left(\frac{3}{7}\right)+\left(\frac{1}{6}\right)*\left(\frac{4}{7}\right)+\left(\frac{5}{6}\right)*\left(\frac{4}{7}\right)=\frac{39}{42}=\frac{13}{14}$$

14. (c)

Explanation:

25 girls can be seated around a round table in 24! ways.

So, the total number of cases = 24!

Excluding A and B, out of remaining 23 girls, 5 girls can be selected in $^{23}C_5$ ways which can be arranged in 5×4×3×2×1 i.e., 5! ways.

Remaining 25–(5+2) girls can be arranged in 18! ways.

Also, A and B mutually can be arranged in 2! ways.

Thus, Required number of arrangements

= $^{23}C_5 \times 5! \times 2! \times 18! = 23! \times 2!$

Chance that there are exactly 5 girls between A and B = (23! × 2!)/24!

15. (a)

Explanation:

Let A be the event drawing 5 red balls in first draw and B be the event of drawing 5 green balls in the second draw.

Total number of chances of drawing 5 balls out of 15 balls = $^{15}C_5$

Chances that 5 red balls are drawn in the first draw = $^8C_5$ / $^{15}C_5 = \frac{8}{429}$

Chances that 5 green balls are drawn in the second draw = $^7C_5$ / $^{10}C_5 = \frac{1}{12}$

Thus, required number of chances

$$=\left(\frac{8}{429}\right)*\left(\frac{1}{12}\right)=\frac{2}{1287}$$

16. (a)

Explanation:

Chance that the missing card is diamond = $\frac{13}{52}=\frac{1}{4}$

Chance that the missing card is heart = $\frac{13}{52}=\frac{1}{4}$

Chance that the missing card is spade = $\frac{13}{52}=\frac{1}{4}$

Chance that the missing card is club = $\frac{13}{52}=\frac{1}{4}$

Chances of drawing two spades given that one spade is missing = $^{12}C_2/^{51}C_2 = \frac{22}{425}$

Chances of drawing two spades given that one heart is missing = $^{13}C_2/^{51}C_2 = \frac{26}{425}$

Chances of drawing two spades given that one diamond is missing = $^{13}C_2/^{51}C_2 = \frac{26}{425}$

Chances of drawing two spades given that one club is missing = $^{13}C_2/^{51}C_2 = \frac{26}{425}$

Required number of chances

$$=\frac{\left(\frac{1}{4}\right)\times\left(\frac{22}{425}\right)}{\left(\frac{1}{4}\right)\times\left(\frac{22}{425}\right)+\left(\frac{1}{4}\right)\times\left(\frac{26}{425}\right)+\left(\frac{1}{4}\right)\times\left(\frac{26}{425}\right)+\left(\frac{1}{4}\right)\times\left(\frac{26}{425}\right)}=\frac{11}{50}$$

17. (b)

Explanation:

Total number of ways 2 persons can be selected out of 15 = $^{15}C_2 = 105$

No of ways in which 1 woman can be selected = $^8C_1 \times ^7C_1 = 56$

No of ways in which 2 women can be selected = $^8C_2 = 28$

Thus, total number of ways $=\frac{(56+28)}{105}=\frac{84}{105}=\frac{4}{5}$

18. (b)

Explanation:

Number of ways in which middle 2 digits can be dialled = $^{10}P_2 = 90$

There is only one correct combination of numbers.

Therefore, chance that correct number is dialled = $\frac{1}{90}$

19. (a)

Explanation:

Total number of ways of arranging 50 books on a shelf = 50!

Number of ways 4 places for the four volumes can be chosen = ${}^{50}C_4$

In the remaining 46 places the remaining 46 books can be arranged in 46! ways.

In the 4 places, four volumes of the journal can be arranged in a particular order in only one way.

Required number of ways = ${}^{50}C_4 \times 46! = \frac{50!}{4!}$

Thus, the chance that they occur in increasing order from right to left $= \left(\frac{50!}{4!}\right)50! = \frac{1}{24}$

20. (d)

Explanation:

Total number of ways in which all the 8 persons can speak = 8!

Number of ways in which A, B and C can speak in the given order = ${}^{8}C_3$ = 8!/(5! × 3!) = 56

Number of ways to arrange remaining 5 persons = 5! = 120

Favourable number of ways = 56 × 120

Chance that A speaks after B and B speaks after $C = (56\times120)/8! = \frac{1}{6}$

21. (c)

Explanation:

Chance of getting atleast one tail in n times

$$= 1 - \left(\frac{1}{2}\right)^n$$

Therefore, $1 - (1/2)^n \geq 0.8 \Rightarrow 2^n \geq 5$

For the above inequality to be true, n should be atleast 3.

22. (a)

Explanation:

The man will be one step away from the starting point if (A) either he is one step ahead or (B) one step behind the starting point. Therefore, required probability = $P(A) + P(B)$

The man will be one step ahead at the end of 13 steps if he moves 7 steps forward and 6 steps backward.

The probability of the above event is = ${}^{13}C_7(0.7)^7\,(0.3)^6$

The man will be one step behind at the end of 13 steps if he moves 7 steps backwards and 6 steps forward.

The probability of the above event is

= ${}^{13}C_7(0.3)^7\,(0.7)^6$

Therefore, the required probability

= ${}^{13}C_7(0.7)^7\,(0.3)^6 + {}^{13}C_7(0.3)^7\,(0.7)^6 = {}^{13}C_7(0.21)^6$

23. (d)

Explanation:

There are 25 odd and 25 even natural numbers from 1 to 50.

For the sum of any two natural numbers to be odd, one of them should be even and the other as odd.

Chance of choosing 2 natural numbers such that 1st is even and 2nd is odd $= \left(\frac{25}{50}\right) = \left(\frac{25}{49}\right) = \frac{25}{98}$

Chance of choosing 2 natural numbers such that 1st is odd and 2nd is even $= \left(\frac{25}{50}\right)\times\left(\frac{25}{49}\right) = \frac{25}{98}$

Required number of chances $= \frac{25}{98} + \frac{25}{98} = \frac{25}{49}$

24. (b)

Explanation:

Chance that the individual is selected from house Red = $1 - \frac{2}{3} = \frac{1}{3}$

Chance that the captain is a boy from house blue $= \left(\frac{2}{3}\right)\times\left(\frac{3}{4}\right) = \frac{1}{2}$

Chance that the captain is a boy from house Red $= \left(\frac{1}{3}\right)\times\left(\frac{2}{7}\right) = \frac{2}{21}$

Chances that the captain will be a boy

$$= \frac{1}{2} + \frac{2}{21} = \frac{25}{42}$$

25. (a)

Explanation:

Chance that Rohan speaks truth = $\frac{4}{5}$; Chance that Rohan does not speak truth = $\frac{1}{5}$

Chance that Sohan speaks truth = $\frac{3}{4}$; Chance that Sohan does not speak truth = $\frac{1}{4}$

Chances that they will contradict each other

= Rohan says truth and Sohan lie OR Rohan lie and Sohan says truth

$$= \left(\frac{4}{5}\right)\times\left(\frac{1}{4}\right) + \left(\frac{1}{5}\right)\times\left(\frac{3}{4}\right) = \frac{7}{20}$$

Progressions

15

INTRODUCTION

In this chapter, we'll dive into the concept of progression in mathematics, which refers to arranging numbers in a specific sequence according to certain rules. We'll explore three main types of progressions. First is Arithmetic Progression (AP), where the difference between consecutive numbers is constant. Second, we encounter Geometric Progression (GP), where each number is a fixed multiple of the previous one. Lastly, we'll examine Harmonic Progression (HP), characterized by the reciprocal values of an arithmetic sequence. Understanding these progressions will help unlock patterns in numbers and provide solutions to various mathematical problems.

ARITHMETIC PROGRESSIONS (AP)

An arithmetic progression, often called an "AP", is a sequence of numbers where each number is found by adding the same amount to the previous one. This consistent amount we add is called the "common difference."

Let's understand this with a real world example. Let's say you have a savings account, and you deposit $100 every month. Your starting balance is $1,000.

- In the first month, your balance is $1,000 + $100 = $1,100.
- In the second month, your balance is $1,100 + $100 = $1,200.
- In the third month, your balance is $1,200 + $100 = $1,300.
- And so on. Here the common difference is $100.

Generally, the first term of an AP is denoted by 'a' and common difference by 'd'.

The common difference is found by subtracting any term of the series from its next term. That is, common difference of an *AP*, $\boldsymbol{d = T_n - T_{n-1}}$

NTH TERM OF AN AP

An *AP* with first term '*a*' and common difference 'd' can be written as $a, a+d, a+2d, a+3d, \ldots\ldots$

Clearly, the coefficient of d in any term is one less than the position of the term in that series. Thus, n^{th} term of an AP is given by $\mathbf{T_n = a + (n - 1)\, d}$

In the above example, for the fourth month (fifth term in series n = 5) in the savings account example the balance:

$T_5 = 1000 + (5 - 1)\ 100 = \$1400$

Balance at the end of 100th month ($n = 101$);

$T_{101} = 1000 + (101 - 1)\ 100 = \$11000$

To illustrate, let's determine the 20th term of the AP: 4, 8, 12, 16, ...

Using the standard formula, we assign values a = 4, $n = 20$, and $d = 4$ in the formula and compute it as

$T_{20} = 4 + (20 - 1) \times 4 = 4 + 19 \times 4 = 80$. However, if we interpret the formula as "To find the nth term, add (n – 1) times the common difference to the first term," we can arrive at the answer much more swiftly.

To find the 20th term of this AP, simply add 19 (which is one less than 20) times the common difference to the first term, which is 4. Thus, $T_{20} = 76 + 4 = 80$

SUM OF FIRST N TERMS OF AN AP

Sum of first n terms means sum from first term to *n*th term.

If '*a*' is first term and '*d*' is common difference, then sum of the first n terms of AP is $\mathbf{S_n = \frac{n}{2}[2a + (n-1)d]}$

If last term of an A.P. containing *n* terms be T_n, then

- $T_n = a + (n - 1)d$
- $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + \{a + (n-1)d\}] = \frac{n}{2}[a + T_n]$

Consider the AP; 4, 8, 12, 16, 20, 24.

Average of these six terms = $\frac{4+8+12+16+20+24}{6} = 14$

Observations: 14 is also the average of the first (4) and the last term (24) of the A.P.

In fact, it is also the average of 8 and 20 (which correspond to the 2nd and 5th terms of the AP).

Further, 14 is also the average of the 3rd (12) and 4th (16) terms of the A.P.

Note: In an AP, the average of terms equidistant from beginning and end is same and is equal to average of an AP. These pairs are known as "corresponding terms" in an AP.

Consider, $S_n = \frac{n}{2}[a + T_n]$

= Number of terms × (Average of first and last term)

= Number of terms × (Average of any pair of corresponding terms)

Properties of Arithmetic Progression

- The common difference of an AP can be zero, positive or negative.
- If a, b, c are in $AP \Rightarrow 2b = a + c$
- If an AP is reversed then the resulting sequence is also an AP.
- If a constant is added or subtracted from each term of an AP, then the resulting sequence will also be in AP with same common difference
- If each term of a given AP is multiplied or divided by a non-zero constant k, then the resulting sequence will also be in AP, with common difference d×k or d/k respectively, (where d = common difference of given AP)
- If corresponding terms of two *APs* are added or subtracted then the resulting sequence is also an AP.
- Three numbers in AP should be taken as a – d, a, a + d. This reduces one variable 'd' if sum is given. Similarly four numbers in A.P. can be taken as $a - 3d, a - d, a + d, a + 3d$ and so on.
- For any sequence whose sum of first r terms is S_r, r^{th} term, $t_r = S_r - S_{r-1}$.

ARITHMETIC MEAN (MEAN OR AVERAGE) (AM)

If three terms are in AP then the middle term is called the AM between the other two, so if a, b, c are in AP, b is AM of a & c.

AM for any n numbers $a_1, a_2, ..., a_n$ is AM

$= \frac{a_1 + a_2 + a_3 ..... + a_n}{n}$

N-ARITHMETIC MEANS BETWEEN TWO NUMBERS

The n numbers A_1, A_2.....An are said to be AM's between the numbers a and b if a, A_1, A_2,.....A_n, b are in AP. Clearly in AP: a, A_1, A_2,.....A_n, b and there are n+2 terms.

So, $b = a + (n + 2 - 1)d \Rightarrow d = \frac{b-a}{n+1}$.

Other terms in AP can be found using the formula for n^{th} term.

$A_1 = a + \frac{b-a}{n+1}, A_2 = a + \frac{2(b-a)}{n+1}, ..., A_n = a + \frac{n(b-a)}{n+1}$

TYPES OF QUESTIONS

There are mainly three type of question on Arithmetic Progressions

TYPE 1: QUESTIONS BASED ON FINDING n^{th} TERM OF AN AP

QUESTIONS

Q1. Find the number of terms in the sequence 14, 17, 20, 23, ..., 92.

(a) 25 (b) 26

(c) 27 (d) 28

Ans: (c)

Explanation:

Let a be the first term and d be the common difference

$a = 14, d = 3$ so, $92 = 14 + (n - 1)3 \Rightarrow n = 27$

Q2. Find 28th term of the AP 3, 6, 9, 12,....., 102.

(a) 84 (b) 78

(c) 81 (d) 87

Ans: (a)

Explanation:

First term, $a = 3$; Common difference, $d = 3$

$T_n = a + (n - 1)d$

$\Rightarrow T_{28} = 3 + (28 - 1)\ 3$

$\Rightarrow T_{28} = 84$

Q3. Find 20th term from the end of the AP 4, 11, 18, 25, 32,....., 214.

(a) 84 (b) 78

(c) 81 (d) 87

Ans: (c)

Explanation:

Let total number of terms in the series be n. So n^{th} term of series is 214

Where, First term, $a = 4$

Common difference $d = 7$

$T_n = a + (n-1)\,d$

$\Rightarrow 214 = 4 + (n-1)\,7 \quad \Rightarrow 214 = 4 + 7n - 7 \Rightarrow n = 31$

Therefore, there are 31 terms in the given series. Then the 20th term from the end is the 12th term from the beginning.

$T_n = a + (n-1)d \quad \Rightarrow T_{12} = 4 + (12-1)\,7$

$\Rightarrow T_{12} = 81$

Q4. If 5 times of 5th term of an AP is equal to 12 times of 12th term, then 17th term of the AP is -

(a) 4 (b) 0

(c) 7 (d) 5

Ans: (b)

Explanation:

Let a be the first term and d be the common difference

5th term of the $AP = a + (5-1)\,d = a + 4d$

12th term of the AP $= a + (12-1)\,d = a + 11d$

According to the question,

$5 \times [a + 4d] = 12 \times [a + 11d]$

$\Rightarrow 5a + 20d = 12a + 132d$

$\Rightarrow 7a + 112d = 0 \quad \Rightarrow a + 16d = 0$

$\Rightarrow a + (17-1)\,d = 0$

$\Rightarrow$ 17th term of AP $= T_{17} = 0$

TYPE 2: QUESTIONS BASED ON FINDING SUM OF n TERMS OF AN AP

QUESTIONS

Q5. Find sum of all odd integers from 5 to 1005.

(a) 253005 (b) 105505

(c) 1010 (d) None of these

Ans: (a)

Explanation:

This will clearly form an AP with first term a = 5, common difference = 2 and last term = 1005.

Let there be n terms (odd numbers) from 5 to 1005.

$T_n = a + (n-1)\,d$

$\Rightarrow 1005 = 5 + (n-1)2$

$\Rightarrow 1005 = 5 + 2n - 2$

$\Rightarrow 1005 = 3 + 2n$

$\Rightarrow 2n = 1002$

$\Rightarrow n = 501$

Therefore sum of 501 terms is

$S = \frac{n}{2}\,[a + T_n] \Rightarrow S = \frac{501}{2}\,[5 + 1005]$

$\Rightarrow S = 253005$

Q6. Consider an AP whose 3rd term is 24 and 7th term is 36, then sum of its first 17 terms is-

(a) 714 (b) 715

(c) 716 (d) None of these

Ans: (a)

Explanation:

Let a be the first term and d be the common difference

$T_3 = a + 2d = 24;$

$T_7 = a + 6d = 36$

Solving these equations we get, $d = 3$ and $a = 18$

$s_{17} = \frac{17}{2}\,[2 \times 18 + 16 \times 3] = 714$

Q7. The sum of 24 terms of the following series $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \ldots$ is

(a) 300 (b) $300\sqrt{2}$

(c) $200\sqrt{2}$ (d) None of these

Ans: (b)

Explanation:

We have

$= 1\sqrt{2} + 2\sqrt{2} + 3\sqrt{2} + 4\sqrt{2} + \ldots\ldots$

$\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \ldots\ldots$

$\Rightarrow \sqrt{2}$ [1+2+3+4+.....upto 24 terms]

$\Rightarrow \sqrt{2}\ [\frac{n}{2}\,[2a + (n-1)\,d]]$

$\Rightarrow \sqrt{2}\left[\frac{24}{2}\left[2(1) + (24-1)(1)\right]\right]$

$\Rightarrow \sqrt{2}\,[12(2 + 23)] = 300\sqrt{2}$

Q8. In an AP the expression for sum of n terms is $2n^2 + 11n$. The term number which is equal 77 is-

(a) 17 (b) 18

(c) 19 (d) None of these

Ans: (b)

Explanation:

$S_n = 2n^2 + 11n$

$T_n = S_n - S_{n-1}$

$= 2n^2 + 11n - [2\,(n-1)^2 + 11\,(n-1)] = 77$

$\Rightarrow 4n + 9 = 77$

$\Rightarrow n = 17$

TYPE 3: QUESTIONS BASED ON ARITHMETIC MEAN

QUESTIONS

Q9. The four arithmetic means between 3 and 23 are

(a) 5, 9, 11, 13 (b) 7, 11, 15, 19

(c) 5, 11, 15, 22 (d) 7, 15, 19, 21

Ans: (b)

Explanation:

Let four arithmetic means are A_1, A_2, A_3 and A_4

So 3, A_1, A_2, A_3, A_4, 23 are in AP.

First term $a = 3$,

and 6th term of AP is 23,

So, $T_6 = 23 = a + 5d$

$\Rightarrow d = 4$

Thus $A_1 = 3+4 = 7, A_2 = 7+4 = 11$

$A_3 = 11+4 = 15, A_4 = 15+4 = 19$

Q10. Three numbers are in A.P. whose sum is 33 and product is 792, then the smallest number from these numbers is

(a) 4 (b) 8

(c) 11 (d) 14

Ans: (a)

Explanation:

Suppose that three numbers are $a–d$, a, a + d, therefore,

$a – d + a + a + d = 33$ or $a = 11$

Now according to question

$(a – d)a(a + d) = 792$

$11(121– d^2) = 792 \Rightarrow d = 7$

Then the required numbers are 4, 11, 18.

Hence the smallest number is 4.

GEOMETRIC PROGRESSIONS (GP)

A geometric progression (GP), also known as a geometric sequence, is a sequence of numbers in which each term after the first is found by multiplying the previous term by a fixed, non-zero number called the common ratio. In other words, each term is obtained by multiplying the previous term by the same constant factor.

The general form of a geometric progression is: $a, ar, ar_2, ar_3, \ldots$

Here: 'a' is the first term (the starting value), 'r' is the common ratio.

Let's understand this with an example. Suppose you have a bank account, and you deposit \$100 as an initial deposit. The bank offers an annual interest rate of 5%, and your money is compounded annually.

In this scenario, your account balance can be represented as a geometric progression.

1st year: \$100 (your initial deposit)

2nd year: \$100 × 1.05 = \$105 (your initial deposit multiplied by the common ratio of 1.05)

3rd year: \$105 × 1.05 = \$110.25

4th year: \$110.25 × 1.05 = \$115.76

and so on. Here, the first term 'a' is \$100, and the common ratio 'r' is 1.05 (because the balance increases by 5% each year). This creates a geometric progression where each term is obtained by multiplying the previous term by 1.05.

Common ratio can be found out by dividing any term (except first term) by its just previous term, thus common ratio, $\mathbf{r = T_n / T_{n-1}}$

nth TERM OF A GP

If we examine the series a, ar, ar^2, ar^3, ar^4,... we notice that in any term the index of r is always less by one than the number of terms in the series.

A GP with the first term 'a' and common ratio 'r' can be written a, ar, ar^2, ar^3,.....

Clearly the power of r in any term is one less than the position of the term in that series.

Thus, nth term of a GP is given by $\mathbf{T_n = a\, r^{n-1}}$

Example: Suppose you have a company that manufactures a popular gadget, and you start with an initial production of 100 units. You have a consistent doubling of production each month to meet increasing demand. You want to find the estimated production after 5 months.

Here: 'a' (the first term) is 100 (the initial production), 'r' (the common ratio) is 2, as the production is doubling each month. 'n' is 5 because you want to find the production after 5 months.

Using the formula $T_n = a\, r^{n-1}$

$T_5 = 100 \times 2^{(5-1)} = 100 \times 2^4 = 100 \times 16 = 1600$.

SUM OF n TERMS OF A GP:

The sum of the first 'n' terms of a geometric progression (GP), often denoted as S_n is:

- $S_n = \dfrac{a(r^n - 1)}{r - 1}, r \neq 1$
- $S_n = na, r = 1$

If $r = 1$ then it is a constant GP, so its sum can be directly computed as (number of terms) × (first term).

Note: Sum of an infinite GP:

For example: 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, ...∞. The sum of infinite GP is possible if it is converging (in our example the terms are reducing and are tending to zero). GPs are converging if $-1 < r < 1$.

Then the sum of the infinite G.P

$= a + ar + ar^2 + ...\infty = \frac{a}{1-r}$, $(-1 < r < 1)$

For 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, ...∞.

Sum to infinite terms $S_\infty = \frac{1}{1-\frac{1}{2}} = 2$

Properties of a Geometric Progression

- If a, b, c are in GP $\Rightarrow b^2 = ac$
- If each term of a GP is multiplied or divided or raised to power by the same non-zero number, then the resulting sequence is also a GP.
- If $a_1, a_2, a_3, ...$ and $b_1, b_2, b_3, ...$ are two GPs with common ratio r_1 and r_2 respectively, then the sequence $a_1b_1, a_2b_2, a_3b_3, ...$ is also a G.P. with common ratio $r_1 r_2$ i.e. if corresponding terms of two GPs are multiplied then the resulting sequence is also a GP and its common ratio is the product of common ratios of the two GPs.
- Product of terms of a GP equidistant from beginning and end is equal i.e. if $a_1, a_2, a_3, a_4, ... a_{n-1}, a_n$ are in GP, then $a_1a_n = a_2a_{n-1} = a_3a_{n-2} = ...$
- Three numbers in GP should be taken as $\frac{a}{r}, a, ar$.
- This reduces one variable 'r' if the product is given.

  Similarly four numbers in GP can be taken as $\frac{a}{r^3}$, $\frac{a}{r}, a, ar, ar^3$ and so on.

GEOMETRIC MEAN (GM)

If three terms are in GP then the middle term is called the GM between the other two, so if a, b, c are in GP, b is GM of a & c. If a and c are both positive, then $b = \sqrt{ac}$ and if a and c are both negative, then $b = -\sqrt{ac}$.

GM for any n positive numbers $a_1, a_2, ..., a_n$ is

GM = $(a_1 \times a_2 \times a_3 \times ..... \times a_n)^{\frac{1}{n}}$

N-GEOMETRIC MEANS BETWEEN TWO NUMBERS

The n numbers $G_1, G_2, ...G_n$ are said to be GM's between the numbers a and b if a, $G_1, G_2, ...G_n, b$ are in *GP*. Clearly in GP a, $G_1, G_2, ...G_n, b$ there are $n + 2$ terms.

So, $b = a(r)^{(n+2-1)} \Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$. Other terms in GP can be found using the formula for nth term.

$$G_1 = a \times \left(\frac{b}{a}\right)^{\frac{1}{n+1}}, G_2 = a \times \left(\frac{b}{a}\right)^{\frac{2}{n+1}} ...., G_n = a \times \left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

TYPES OF QUESTIONS

There are mainly three types of question on Geometric Progression

TYPE 1: QUESTIONS BASED ON FINDING NTH TERM OF A GP

QUESTIONS

Q11. If the 10th term of a geometric progression is 9 and 4th term is 4, then its 7th term is

(a) 6 (b) 36

(c) $\frac{4}{9}$ (d) $\frac{9}{4}$

Ans: (a)

Explanation:

Accordingly,

$\Rightarrow T_{10} = ar^{10-1}$

$\Rightarrow 9 = ar^9$ ...(i)

Similarly $T_4 = ar^{4-1}$

$\Rightarrow 4 = ^{ar3}$ ...(ii)

From eq. 1 and 2

$\Rightarrow r^3 = \frac{3}{2}$ and $a = \frac{8}{3}$

So for 7th term $T_7 = ar^6$

$\Rightarrow T_7 = (8/3)(3/2)^2 \Rightarrow T_7 = 6$

Trick 7th term is equidistant from 10th and 4th so it will be $\sqrt{9 \times 4} = 6$.

Q12. The 6th term of GP is 32 and its 8th term is 128. Then the common ratio of GP is-

(a) –1 (b) 2

(c) 4 (d) –4

Ans: (b)

Explanation:

Accordingly, $T_n = ar^{n-1}$

$\Rightarrow T_6 = ar^5 = 32$ ...(i)

Similarly $T_8 = ar^{8-1}$

$\Rightarrow 128 = ar^7$ ...(ii)

Dividing eq. (ii) from eq. (i)

$\Rightarrow r^2 = 4 \quad \Rightarrow r = 2$

Q13. Fifth term of a G.P. is 2, then the product of its 9 terms is

(a) 256 (b) 512

(c) 1024 (d) Nones of these

Ans: (b)

Explanation:

Let the 9 terms of a G.P. is

$\frac{a}{r^4}, \frac{a}{r^3}, \frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2, ar^3, ar^4$

Given, fifth term $a = 2$

Hence product of 9 terms is $a^9 = 2^9 = 512$.

TYPE 2: QUESTIONS BASED ON SUM OF N TERM OF A GP

QUESTIONS

Q14. If the sum of n terms of a G.P. is 255 and n^{th} terms is 128 and common ratio is 2, then first term will be

(a) 1 (b) 3

(c) 7 (d) None of these

Ans: (a)

Explanation:

Given that $\frac{a(r^n - 1)}{r - 1} = 255$ $(r > 1)$ ...(i)

$ar^{n-1} = 128$ ...(ii)

and common ratio $r = 2$ ...(iii)

From (i), (ii) and (iii)

we get, $a(2)^{n-1} = 128$ ...(iv)

and $\frac{a(2^n - 1)}{2 - 1} = 255$ ...(v)

Dividing (v) by (iv) we get

$\frac{2^n - 1}{2^{n-1}} = \frac{255}{128}$ or $2 - 2^{-n+1} = \frac{255}{128}$

$\Rightarrow 2^{-n} = 2^{-8}$ or $n = 8$

Putting $n = 8$ in equation (iv),

we have $a(2)^7 = 128$ or $a = 1$

Q15. If the sum of an infinite G.P. be 9 and the sum of first two terms be 5, then the common ratio is

(a) $\frac{1}{3}$ (b) $\frac{3}{2}$

(c) $\frac{3}{4}$ (d) $\frac{2}{3}$

Ans: (d)

Explanation:

Infinite series is $a, ar, ar^2, \ldots\ldots\infty$

Sum $S = \frac{a}{1 - r} = 9$ or $a = 9(r - 1)$ ...(i)

and $a + ar = 5$ or $a(r + 1) = 5$

or $9(r - 1)(r + 1) = 5$ [By equation (i)]

$1 - r^2 = \frac{5}{9}$ Therefore, $r = \pm\frac{2}{3}$

TYPE 3: QUESTIONS BASED ON GEOMETRIC MEAN

QUESTIONS

Q16. If three geometric means be inserted between 2 and 32, then the third geometric mean will be

(a) 8 (b) 4

(c) 16 (d) 12

Ans: (c)

Explanation:

$2, g_1, g_2, g_3, 32$

where

$a = 2, ar = g_1, ar^2 = g_2, ar^3 = g_3$ and $ar^4 = 32$

Now $2 \times r^4 = 32$ or, $r^4 = 16$ or $r = 2$.

Then third geometric mean a $r^3 = 2 \times 2^3 = 16$.

HARMONIC PROGRESSIONS (HP)

A harmonic progression (HP), is a sequence of numbers in which the reciprocal of each term forms an arithmetic progression. In other words, if you have a harmonic progression, the reciprocals of the terms will follow an arithmetic sequence.

The sequence $a_1, a_2, a_3 \ldots\ldots a_n \ldots\ldots (a_i \neq 0)$ is said to be an HP if the sequence $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots\ldots \frac{1}{a_n}$ is an AP.

n^{th} TERM OF HP

General HP can be considered as $\frac{1}{a}, \frac{1}{a + d}, \ldots$ where a and d are first term and common difference of corresponding AP.

Thus, nth term of the HP, $T_n = \frac{1}{a+(n-1)d}$

Note: There is no formula for the sum of n terms of an HP. Questions in HP are generally solved by reciprocating the terms, and making use of the properties of the corresponding AP.

HARMONIC MEAN (HM)

If three terms are in HP then the middle term is called the HM between the other two, so if a, b, c are in HP, b is HM of a & c.

H.M. between a & c, then $b = \frac{2ac}{a+c}$

If $a_1, a_2, \ldots\ldots a_n$ are 'n' non-zero numbers then HM. 'H' of these numbers is given by

$$\frac{1}{H} = \frac{1}{n}\left[\frac{1}{a_1} + \frac{1}{a_2} \ldots\ldots + \frac{1}{a_n}\right]$$

QUESTIONS

Q17. The fifth term of the H.P. $2, 2\frac{1}{2}, 3\frac{1}{3}\ldots\ldots$ will be

(a) $5\frac{1}{5}$ (b) $3\frac{1}{5}$

(c) $\frac{1}{10}$ (d) 10

Ans: (d)

Explanation:

Series, $2, 2\frac{1}{2}, 3\frac{1}{3}\ldots\ldots$ are in H.P.

Therefore $\frac{1}{2}, \frac{2}{5}, \frac{3}{10}\ldots\ldots$ will be in A.P.

Now first term $a = \frac{1}{2}$ and common difference $d = -\frac{1}{10}$

So, 5th term of the A.P. $= \frac{1}{2} + (5-1)\left(-\frac{1}{10}\right) = \frac{1}{10}$

Hence 5th term in H.P. is 10.

Q18. The sixth H.M. between 3 and $\frac{6}{13}$ is

(a) $\frac{63}{120}$ (b) $\frac{63}{12}$

(c) $\frac{126}{105}$ (d) $\frac{120}{63}$

Ans: (a)

Explanation: $x_n = \frac{(n+1)ab}{na+b}$

Sixth H.M. $x_n = \frac{7\times 3\times (6/13)}{6\times 3+(6/13)} = \frac{63}{120}$

RELATIONSHIP BETWEEN AM, GM AND HM

If AM,GM,HM are Arithmetic Mean, Geometric Mean and Harmonic Mean between any two numbers, then

- **AM ≥ GM ≥ HM** (equality holds when all terms are equal)
- **$(GM)^2$ = (AM) x (HM)** (i.e., AM, GM, HM are in GP)

QUESTIONS

Q19. If A_1, A_2 are the two A.M.'s between two numbers a and b and G_1, G_2 be two G.M.'s between same two numbers, then $\frac{A_1 + A_2}{G_1 G_2}$

(a) $\frac{a+b}{ab}$ (b) $\frac{a+b}{2ab}$

(c) $\frac{2ab}{a+b}$ (d) $\frac{ab}{a+b}$

Ans: (a)

Explanation:

Given that a, A_1, A_2, b are in A.P.

Therefore, $A_1 = \frac{a+A_2}{2}$, $A_2 = \frac{A_1+b}{2}$

$\Rightarrow A_1 + A_2 = \frac{1}{2}(a + b + A_1 + A_2)$

$\Rightarrow \frac{1}{2}(A_1 + A_2) = \frac{1}{2}(a+b)$

or $A_1 + A_2 = a + b$ ...(i)

and a, G_1, G_2, b are in G.P.

Therefore $G_1^2 = aG_2$, $G_2^2 = bG_1$ ...(ii)

$G_1^2 G_2^2 = ab\,G_1 G_2$ or $G_1 G_2 = ab$

Hence $\frac{A_1 + A_2}{G_1 G_2} = \frac{a+b}{ab}$

Q20. If the A.M. and H.M. of two numbers is 27 and 12 respectively, then G.M. of the two numbers will be

(a) 9 (b) 18

(c) 24 (d) 36

Ans: (b)

Explanation:

Let two numbers a and b.

We have $G = \sqrt{AH} \Rightarrow$ G.M= $\sqrt{27\times12} = 18$

Q21. Consider the following addition problem: 3P + 4P + PP + PP = RQ2; where *P*, *Q* and *R* are different digits. What is the arithmetic mean of all such possible sums? **(UPSC CSAT 2021)**

(a) 102 (b) 120

(c) 202 (d) 220

Ans: (c)

Explanation:

3P + 4P + PP + PP = RQ2

Or (30 + P) + (40 + P) + (10 x P + P) + (10 x P + P)

= (100 x R) + (10 x Q) + 2

Or 24P + 70 = 100R + 10 Q + 2

Or 20P + 70 + 4P = 100R + 10 Q + 2

The unit digit of the resultant is 2.

It will be obtained when 4 is multiplied by P.

So, P must be 3, or 8.

If P = 3, then: 24P + 70 = 24 × 3 + 70 = 72 + 70 = 142

If P = 8, then: 24P + 70 = 24 × 8 + 70 = 192 + 70 = 262

Arithmetic sum of 142 and 262 = (142 + 262)/2 = 202.

Q22. A biology class at high school predicted that a local population of animals will double in size every 12 years. The population at the beginning of the year 2021 was estimated to be 50 animals. If P represents the population after n years, then which one of the following equations represents the model of the class for the population? **(UPSC CSAT 2021)**

(a) P = 12 + 50n

(b) P = 50 + 12n

(c) $P = 50\ (2)^{12n}$

(d) $P = 50\ (2)^{n/12}$

Ans: (c)

Explanation:

The population is doubling every 12 years.

So, this is a case of Geometric Progression.

So, $r = 2$

Initial population = 50

Being a case of GP, the population should be in the form of ar^{n-1} or $50(2)^t$

Thus, option A and B are eliminated since they are not represented in this particular manner.

From C and D (putting a value of $n = 12$ for doubling of population), option C gives the correct answer.

Some Special Series

- Sum of first '*n*' natural numbers = 1 + 2 + 3 .....+ $n = \frac{n(n+1)}{2}$
- Sum of first n odd natural numbers = n^2
- Sum of first n even natural numbers = $n(n + 1)$
- Sum of squares of first '*n*' natural numbers = $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$
- Sum of cubes of first '*n*' natural numbers = $1^3 + 2^3 + 3^3 + \ldots\ldots\ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

PRACTICE QUESTIONS

1. **Find the four consecutive numbers in an increasing arithmetic progression (AP) such that their sum is 180 and the product of the first and last numbers is 200 less than the product of the middle two numbers.**

   (a) 15, 35, 55, 75 (b) 35, 45, 65, 85

   (c) 30, 40, 50, 60 (d) 20, 40, 50, 70

2. **Vidya saved a total of ₹ 3,54,000 over a twelve-year period. Her annual savings increased by ₹ 3,300 each year. Find the amount she saved in the first year.**

   (a) ₹ 11350 (b) ₹ 11305

   (c) ₹ 11530 (d) ₹ 11035

3. **Find the sum of the first 252 terms in the following series: 3 + 6 + 6 + 12 + 9 + 18 + 12 + 24 + 15 + 30 + ...**

   (a) 72909 (b) 72999

   (c) 72099 (d) 72009

4. **If the sum of the first 15 terms of an AP is 1050 and the common difference is 5. Find the 20th term of the given sequence.**

   (a) 103 (b) 301

   (c) 130 (d) None of these

5. **The sum of an infinite GP series is 7. And the sum of terms at the even places is 3. Find the first term of the series.**

   (a) $\frac{4}{3}$ (b) $\frac{10}{7}$

   (c) $\frac{7}{4}$ (d) $\frac{7}{3}$

6. **The terms a, b, c are in a G.P. If x and y are the arithmetic mean of a, b and b, c respectively, then find the value of $\left(\frac{a}{x}+\frac{c}{y}\right)$.**

   (a) 2 (b) -4

   (c) 4 (d) 6

7. The Harmonic mean of the roots of the equation $(1+\sqrt{2})x^2-(1+\sqrt{2})x+2+2\sqrt{2}=0$ is:

   (a) 5 (b) 4

   (c) 3 (d) 2

8. **The sum of the first 10 terms of the series $\frac{7}{2^2 5^2}+\frac{13}{5^2 8^2}+\frac{19}{8^2 11^2}+\ldots$ is $\frac{m}{n}$ where m, n are coprime natural numbers, then the value of $(m+n)$ is:**

   (a) 1019 (b) 1091

   (c) 1109 (d) 9101

9. **Calculate the sum of the series: $\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\frac{1}{5^4}+\frac{1}{5^5}\ldots$**

   (a) $\frac{3}{4}$ (b) $\frac{4}{3}$

   (c) $\frac{3}{5}$ (d) $\frac{1}{4}$

10. **Find the sum of infinite series.**

    $1+\frac{2}{3}+\frac{3}{3^2}+\frac{4}{3^3}+\frac{5}{3^4}+\ldots\infty$

    (a) $\frac{4}{9}$ (b) $\frac{5}{7}$

    (c) $\frac{9}{4}$ (d) $\frac{9}{7}$

11. **If a_1, a_2, a_3, ... are in harmonic progression with $a_1 = 5$ and $a_{10} = 15$. Then, the least positive integer n for which $a_n < 0$, is:**

    (a) 14 (b) 15

    (c) 16 (d) 17

12. **If a, b, and c are non-zero numbers in H.P., then $\frac{1}{a}-bc, \frac{1}{b}-ca, \frac{1}{c}-ab$ are in:**

    (a) A.P. (b) G.P.

    (c) H.P. (d) G.P. & H.P. both

13. **In a geometric progression, the sum of the first and the last terms is 50 and the product of the second and the last second terms is 625. Find the first term of the series:**

    (a) 50 (b) 25

    (c) 15 (d) 5

14. **Vipin is offered Rs. 15 for a job on the first day, and his payment triples daily. If he continues working for ten days, how much will he earn by the end of the tenth day?**

    (a) ₹ 4, 24, 860 (b) ₹ 4, 42, 860

    (c) ₹ 4, 42, 880 (d) ₹ 4, 42, 660

15. If 20 is divided into four parts which are in AP such that the ratio of the product of the first and fourth to the product of the second and third is 5 : 6. What is the value of four parts?

(a) 7/41, 7/ 51, 7/ 61, 7/ 71

(b) 41/7, 51/ 7, 61/ 7, 71/ 7

(c) 43/7, 53/ 7, 63/ 7, 73/ 7

(d) 41/5, 51/ 7, 61/ 9, 71/ 11

16. A shopkeeper sells pens in an arithmetic progression manner. On the first day, he sells 4 pens, 7 pens on the second day, and so on. Find the number of pens he will sell on the 10th day.

(a) 30 (b) 31

(c) 32 (d) 33

ANSWERS

1. (*c*)	2. (*a*)	3. (*d*)	4. (*c*)	5. (*c*)	6. (*a*)	7. (*b*)	8. (*c*)	9. (*d*)	10. (*c*)
11. (*b*)	12. (*a*)	13. (*b*)	14. (*b*)	15. (*b*)	16. (*b*)				

EXPLANATIONS

1. (c)

Explanation:

Let the four terms be $(a-3d)$, $(a-d)$, $(a+d)$ & $(a+3d)$

$(a-3d)+(a-d)+(a+d)+(a+3d)=180$

$4a=180 \Rightarrow a=\frac{180}{4}=45$

Also, $(a-3d)(a+3d)=(a-d)(a+d)-200$

$a^2-9d^2=a^2-d^2-200 \quad 8d^2=200 \quad \Rightarrow d^2=25$

$a = 45$, $d = 5$ (d can't be '–5' as it is an increasing AP)

So, the four numbers are 30, 40, 50, 60

2. (a)

Explanation:

Let the savings in the first year be ₹ P.

Then, the given sequence is

P + (P + 3300) + (P + 6600) +...

It is an AP with a = P, d = (P + 3300) – P = 3300,

$n = 12$, $S_n = 354000$

$S_n=\frac{n}{2}[2a+(n-1)d]$

$354000=\frac{12}{2}[2P+11\times 3300]$

$354000 = 6\,[2P + 36300]$

$2P + 36300 = 59000$

$2P = 59000 - 36300 = 22700$

$\Rightarrow P =$ ₹ 11350

So, Vidya saved ₹ 11,350 in the first year .

3. (d)

Explanation:

The given series is the sum of 2 AP series

1st AP is (3 + 6 + 9 + 12 + 15 + ...)

2nd AP is (6 + 12 + 18 + 24 + 30 + ...)

Consider the 1st AP

First term (a) = 3; common ratio

(d) = 3 & number of terms(n) = 126

$\therefore$ Sum of series 1 (S_1) = (n/2)[2a + $(n-1)d$]

$S_1 = (126/2)\,[2\times 3 + (126-1)\times 3\,] = 63\,[6 + 125\times 3]$

$= 63 \times 381 = 24003$

Consider the 2nd AP

First term (a) = 6; common ratio (d) = 6 & number of terms(n) = 126

$S_2 = (126/2)[2\times 6 + 125\times 6] = 63\times 762 = 48006$

$\therefore$ Sum of the given series = $S_1 + S_2 = 24003 + 48006$

$= 72009$.

4. (c)

Explanation:

Sum of the first 15 terms (S15)= 1050 ; Common ratio (d) = 5

$S_n = (n/2)\times[2a+(n-1)d]$

$1050 = (15/2)\times[2a + (15-1)(5)] = (15/2)\times[2a + 70]$

$1050 \times (2/15) = 2a + 70$

$140 - 70 = 2a \Rightarrow a = 35$

$T_{20} = a + (20-1)d = 35 + (20-1)5$

$T20 = 35 + 95 \Rightarrow T20 = 130$

5. (c)

Explanation:

Let's assume the given GP is

$a + ar + ar^2 + ar^3 + ar^4 + ar^5 + ar^6$.....

First term = a, common ratio = r

Sum of an infinite GP ($S\infty$) = $\frac{a}{1-r} = 7$ (given) ...(i)

The terms at the even places also forms a GP i.e., $ar + ar^3 + ar^5 + ar^7$.....

First term = ar, common ratio = r^2

Sum of the series = $\frac{ar}{1-r^2} = 3$ (given) ...(ii)

Divide equation (i) by (ii)

$\frac{(1+r)}{r} = \frac{7}{3} \Rightarrow 4r = 3$

$r = \frac{3}{4}$ then, $a = \frac{7}{4}$

6. (a)

Explanation:

In the given G.P.

Let $b = ar$, $c = ar^2$

x is the arithmetic mean of a and b

Thus, $x = \frac{a+b}{2} = \frac{a(1+r)}{2}$

y is the arithmetic mean of b and c

$y = \frac{b+c}{2} = \frac{ar(1+r)}{2}$

$$\left(\frac{a}{x} + \frac{c}{y}\right) = \left\{\frac{2a}{a(1+r)} + \frac{2ar^2}{ar(1+r)}\right\} = \left\{\frac{2}{1+r} + \frac{2r}{1+r}\right\} = 2$$

7. (b)

Explanation: $(1+\sqrt{2})x^2 - (1+\sqrt{2})x + 2 + 2\sqrt{2} = 0$

Let "a" and "b" be the roots of the equation.

Sum of roots ($a + b$) = $\frac{(1+\sqrt{2})}{1+\sqrt{2}} = 1$

Product of roots (ab) = $\frac{(2+2\sqrt{2})}{(1+\sqrt{2})} = 2$

Harmonic mean = $\frac{2ab}{(a+b)} = \frac{2*2}{1} = 4$

8. (c)

Explanation: $S = \frac{7}{2^2 5^2} + \frac{13}{5^2 8^2} + \frac{19}{8^2 11^2} + ...$

$3S = \frac{21}{2^2 5^2} + \frac{39}{5^2 8^2} + \frac{57}{8^2 11^2} + ...$

$$3S = \sum_{i=1}^{10} \frac{(3r+2)^2 - (3r-1)^2}{(3r+2)^2 (3r-2)^2}$$

$$3S = \sum_{l=1}^{10} \left(\frac{1}{(3r-1)^2} - \frac{1}{(3r+2)^2}\right)$$

$3S = \frac{1}{2^2} - \frac{1}{2^{10}}$

$3S = \frac{2^8 - 1}{2^{10}}$

$S = \frac{85}{1024} = \frac{m}{n}$

Value of $m + n = 85 + 1024 = 1109$

9. (d)

Explanation:

$\frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \frac{1}{5^4} + \frac{1}{5^5}$.....

It is a geometric progression (GP) where,

$a = \frac{1}{5}$, $r = \frac{1}{5}$

The formula for the sum(S) of an infinite geometric series is:

$S = \frac{a}{(1-r)}$

S = (1/5) / (1 – 1/5)

Simplify the denominator:

S = (1/5) / (4/5)

S = ¼

10. (c)

Explanation:

$S_\infty = 1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \frac{5}{3^4} + ...\infty$ ...(i)

$\frac{1}{3}S_\infty = \frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \frac{5}{3^4} + ...\infty$ ...(ii)

Subtract (ii) from (i)

$\frac{2}{3}S_\infty = 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + ...\infty$

Right hand side is a GP with First term (a) = 1 and common ratio(r) = 1/3

$\frac{2}{3}S_\infty = \frac{1}{1-\frac{1}{3}}$

$$\frac{2}{3}S_\infty = \frac{3}{2}$$

$$S_\infty = \frac{9}{4}$$

11. (b)

Explanation:

$\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots$ in A.P.

$\frac{1}{a_{10}} = \frac{1}{a_1} + 9d \quad \left(\frac{1}{15} - \frac{1}{5}\right) = 9d$

$$d = \frac{-2}{135}$$

So, $a_n < 0$

$$\frac{1}{a_n} < 0$$

$$\frac{1}{a_1} + (n-1)d < 0 \quad \frac{1}{5} + (n-1)\left(\frac{-2}{15 \times 9}\right) < 0$$

$27 - 2(n-1) < 0$

$2(n-1) > 27$

$\Rightarrow n > 14.5$

Thus, the least positive integer n for which $a_n < 0$ is 15

12. (a)

Explanation:

a, b, c are non-zero numbers is H.P.

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. ...(i)

If each number of A.P. is multiplied by the same constant then the resulting sequence is also an A.P.

Multiplying equation (i) by (1-abc)

Hence, $\frac{1-abc}{a}, \frac{1-abc}{b}, \frac{1-abc}{c}$ are in A.P.

$$\Rightarrow \frac{1}{a} - bc, \frac{1}{b} - ca, \frac{1}{c} - ab \rightarrow A.P.$$

13. (b)

Explanation:

Let 'a' be the first term and 'r' be the common ratio of the GP.

$a + ar^{n-1} = 50$ ...(i)

Also, $ar \times ar^{n-2} = 625$

$\Rightarrow a^2 r^{n-1} = 625$ ...(ii)

From Eq. (ii)

$ar^{n-1} = 625/a$

On substitution of this in Eq.(i)

$a + 625/a = 50$

$a^2 - 50a + 625 = 0$

$(a - 25)(a - 25) = 0 \Rightarrow a = 25$

14. (b)

Explanation:

15, 45, 135, 405, ...

This is a GP with $a = 15$, $r = 3$, $n = 10$

$S_n = a(r^n - 1)/(r - 1)$

$\Rightarrow S = 15(3^{10} - 1)/(3 - 1)$

$= 15(59049 - 1)/2 = 15(29524)$

= ₹ 4, 42, 860

15. (b)

Explanation:

Let the four parts be $(a - 3d)$, $(a - d)$, $(a + d)$, $(a + 3d)$.

From question,

$(a - 3d) + (a - d) + (a + d) + (a + 3d) = 20$

$4a = 20 \Rightarrow a = 5$

Also, $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{5}{6}$

$$\Rightarrow \frac{a^2 - 9d^2}{a^2 - d^2} = \frac{5}{6}$$

$$\Rightarrow \frac{5^2 - 9d^2}{5^2 - d^2} = \frac{5}{6}$$

$25 = 49d^2$

$$\Rightarrow d = \frac{5}{7}, -\frac{5}{7}$$

$\therefore a = 8$, $d = \frac{5}{7}$ (as sign of d will not affect A.P.)

So, the four parts are $\frac{41}{7}, \frac{51}{7}, \frac{61}{7}, \frac{71}{7}$.

16. (b)

Explanation:

First term (a) = 4 and common difference (d) = 7 – 4 = 3

The n^{th} term of an arithmetic progression is given by:

$T_n = a + (n-1)d$

For the 10^{th} day ($n = 10$):

$T_{10} = 4 + (10 - 1)(3) = 31$

So, the shopkeeper will sell 31 pens on the 10th day.

Statistics

16

INTRODUCTION

Measures of central tendency are statistical values used to describe the center/middle value(s) of a dataset. These values can be taken as representative values of the complete data set. Mean, Median and Mode are three most common measures of central tendency.

ARITHMETIC MEAN (AVERAGE)

This is the number you get when you add up all the numbers in a group and then divide by how many numbers there are.

If $x_1, x_2,.....x_n$ be n observations, then their arithmetic mean is given by $\bar{x} = \frac{x_1 + x_2 + ... + x_n}{n}$

Suppose you have a dataset representing the daily temperatures in degrees Celsius for a week, as follows: 22, 24, 21, 23, 25, 19, 20.

To find the mean (average) of this dataset, you add up all the values and then divide by the number of values. Here's how you calculate it:

(i) Add up all the values: 22 + 24 + 21 + 23 + 25 + 19 + 20 = 154.

(ii) Divide the sum (154) by the number of values (7): 154/7 = 22

This Mean represents the arithmetic average temperature for the week.

WEIGHTED AVERAGE

A weighted average pays more attention to some things than others. It's a way of finding an average that considers how important each item is. You do this by multiplying each item by how important it is (its "weight"), adding all those results together, and then dividing by the sum total of all weights.

If $w_1, w_2, w_3,.....w_n$ are the weights assigned to the values $x_1, x_2, x_3,.....,x_n$ respectively, then the **weighted average (A.M.)** is defined as: $\frac{w_1x_1 + w_2x_2 + ... + w_nx_n}{w_1 + w_2 + ... + w_n}$

Let's understand this with an example: Suppose in a bank 5 employees receive a pay of 100000, 3 employees receive a pay of 90000, 6 employees receive a pay of 80000, 4 employees receive a pay of 70000, and 2 employees receive a pay of 6000. Then the average pay of employees

$= \frac{5(100000) + 3(90000) + 6(80000) + 4(70000) + 2(6000)}{5 + 3 + 6 + 4 + 2}$

= INR 77,100.

This concept can also be used to calculate the combined average of different groups (series) of different sizes. If $\bar{x}_1, \bar{x}_2, ..., \bar{x}_k$ are the respective means of 'k' different series of sizes $n_1, n_2,....,n_k$ respectively then the mean of the combined series is given by

$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + ...n_k\bar{x}_k}{n_1 + n_2 + ... + n_k}$

PROPERTIES OF AVERAGE

(a) Zero-Sum Property: It means that in a dataset, when you calculate the average (mean), the total difference between numbers below the average and numbers above the average always equals zero.

(b) If you add (or subtract) a constant value 'k' to each of the values in a dataset, the arithmetic mean of that dataset will also increase (or decrease) by the same constant 'k'.

(c) If you multiply (or divide) every value in a dataset by a constant 'k', then the arithmetic mean (average) of the dataset will also be multiplied (or divided) by the same constant 'k'.

QUESTIONS

Q1. The mean of 5 numbers is 18. If one number is excluded, their mean becomes 16. Then the excluded number is

(a) 18 (b) 25

(c) 26 (d) 30

Ans: (c)

Explanation:

Sum of total number = 18 × 5 = 90

After one number is excluded, the sum of remaining numbers = 16 × 4 = 64

Then, the excluded number is 90 – 64 = 26

Q2. The mean weight per student in a group of seven students is 55 kg If the individual weights of 6 students are 52, 58, 55, 53, 56 and 54; then weights of the seventh student is

(a) 55kg (b) 60kg

(c) 57kg (d) 50kg

Ans: (c)

Explanation:

The total weight of 7 students is

= 55 × 7 = 385 kg

Sum of weight of 6 students

= 52 + 58 + 55 + 53 + 56 + 54 = 328 kg

∴ Weight of seventh student = 385 – 328 = 57kg.

Q3. The average weight of students in a class of 35 students is 40 kg. If the weight of the teacher be included, the average rises by $\frac{1}{2}$ kg; the weight of the teacher is

(a) 40.5 kg (b) 50 kg

(c) 41 kg (d) 58 kg

Ans: (d)

Explanation:

Let the weight of the teacher is *w kg*,

Total weight of students = 35 × 40 and total numbers of persons = 35 + 1

So, Average, $40 + \frac{1}{2} = \frac{35\times40+w}{35+1}$

$\Rightarrow 36 \times 40 + 36\times\frac{1}{2} = 35 \times 40 + w$

$\Rightarrow w = 58$

∴ Weight of the teacher = 58 *kg*.

Q4. Mean of 100 items is 49. It was discovered that three items which should have been 60, 70, 80 were wrongly read as 40, 20, 50 respectively. The correct mean is

(a) 48 (b) $82\frac{1}{2}$

(c) 50 (d) 80

Ans: (c)

Explanation:

Sum of 100 items = 49 × 100 = 4900

Sum of items added= 60 + 70 + 80 = 210

Sum of items replaced= 40 + 20 + 50 = 110

New sum= 4900 + 210 – 110= 5000

∴ Correct mean= $\frac{5000}{100}$ = 50.

Q5. If the mean of the distribution is 2.6, then the value of *y* is

Variate x	1	2	3	4	5
Frequency f of x	4	5	y	1	2

(a) 24 (b) 13

(c) 8 (d) 3

Ans: (c)

Explanation:

We know that, Mean $= \frac{\sum_{i=1}^{k} x_i f_i}{\Sigma f_i}$

i.e., $2.6 = \frac{1\times4+2\times5+3\times y+4\times1+5\times2}{4+5+y+1+2}$

or 31.2 + 2.6y= 28 + 3y or 0.4y= 3.2

Therefore, *y* = 8

MEDIAN

This is the number that is exactly in the middle of a group of numbers when you arrange them in order from smallest to largest (or largest to smallest). It's the value from which half of the numbers are bigger, and half are smaller. It's the middle-most value of an arranged data.

To calculate the median of 'n' number of observations

(i) Arrange the observations in order, either from smallest to largest or largest to smallest.

(ii) If n is odd then, median is the middle value.

i.e., Median (M) = Value of $\left(\frac{n+1}{2}\right)^{th}$ observation

(iii) If n is even then, the median is the average of the two middle values.

I.e., Median (M)

$= \text{Value of } \frac{\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n}{2}+1\right)^{th} \text{observation}}{2}$

Example:

Imagine you have the ages of a group of people attending a family reunion. The ages (in years) are as follows: 25, 42, 30, 44, 52, 50, 65

To find the median, first, arrange the ages in ascending order: 25, 30, 42, 44, 50, 52, 65

Now, because there are seven values, the median will be the middle value. In this case, the middle value(4^{th} term) is 44.

Further if one more person aged 70 joins the reunion then the data (arranged) becomes 25, 30, 42, 44, 50, 52, 65, 70

Now the median will be the average of 44(4th term) and 50(5th term)) i.e. (44 + 50)/2 = 47.

QUESTIONS

Q6. The median of 10, 14, 11, 9, 8, 12, 6 is

(a) 10 (b) 12

(c) 14 (d) 11

Ans: (a)

Explanation:

Arrange the items in ascending order *i.e.*, 6, 8, 9, 10, 11, 12, 14.

Since n is odd then, Median $=\left(\frac{n+1}{2}\right)^{th}$ term

$\therefore$ Median $=\left(\frac{7+1}{2}\right)^{th}$ term = 4^{th} term = 10.

Q7. If a variable takes the discrete values

$$\alpha-4, \alpha-\frac{7}{2}, \alpha-\frac{5}{2}, \alpha-3, \alpha-2, \alpha+\frac{1}{2}, \alpha-\frac{1}{2}, \alpha+5 \ (\alpha>0)$$

then the median is

(a) $\alpha-\frac{5}{4}$ (b) $\alpha-\frac{1}{2}$

(c) $\alpha-2$ (d) $\alpha+\frac{5}{4}$

Ans: (a)

Explanation:

Arrange the data as

$$\alpha-\frac{7}{2}, \alpha-3, \alpha-\frac{5}{2}, \alpha-2, \alpha-\frac{1}{2}, \alpha+\frac{1}{2}, \alpha+4, \alpha+5$$

Median= $\frac{1}{2}$[value of 4^{th} item + value of 5^{th} item]

$$\therefore \text{ Median } = \frac{\alpha-2+\alpha-\frac{1}{2}}{2} = \frac{2\alpha-\frac{5}{2}}{2} = \alpha-\frac{5}{4}.$$

Q8. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observation of the set is increased by 2, then the median of the new set

(a) Is increased by 2

(b) Is decreased by 2

(c) Is two times the original median

(d) Remains same as that of original set

Ans: (d)

Explanation: Since n=9, then median term $=\left(\frac{9+1}{2}\right)^{th}$ = 5^{th} term.

Now, the last four observations have increased by 2.

$\because$ The median is 5th observation, which is remaining unchanged.

$\therefore$ There will be no change in median.

MODE

This is the number that appears most frequently in a group of numbers. It's like the most popular or common value in the set.

Imagine you're running a survey to determine the most popular ice cream flavor among your friends. You ask ten of your friends what their favorite ice cream flavor is, and here are their responses: Chocolate, Vanilla, Chocolate, Strawberry, Vanilla, Chocolate, Chocolate, Strawberry, Vanilla, Chocolate.

In this case, the mode represents the ice cream flavor that appears most frequently in your survey responses. After counting the responses, you find that "Chocolate" is the favorite flavor among your friends as it appears four times, So, the mode in this example is "Chocolate."

QUESTIONS

Q9. A set of numbers consists of three 4's, five 5's, six 6's, eight 8's and seven 10's. The mode of this set of numbers is

(a) 6 (b) 7

(c) 8 (d) 10

Ans: (c)

Explanation: Mode of the data is 8 as it is repeated maximum number of times.

Q10. If The mode of the following items is 0, 1, 6, 7, 2, 3, 7, 6, 6, 2, 6, 0, 5, 6, 0

(a) 0 (b) 5

(c) 6 (d) 2

Ans: (c)

Explanation: Since 6 occurs most times (*i.e.*, 5 times) than any other 15 observations.

$\therefore$ Mode = 6.

Note:

Relation between Mean, Median and Mode:

The following empirical relationship exists between the Mean, Median and Mode:

- Mode = 3 Median – 2 Mean

QUESTIONS

Q11. In a dataset of exam scores for a class the values of mode and mean are 60 and 66 respectively. Then which of the following can be value of median

(a) 60 (b) 64

(c) 66 (d) 68

Ans: (b)

Explanation:

Mode = 3 Median – 2 mean

$\therefore$ Median = $\frac{1}{3}$ (mode + 2 mean)

$=\frac{1}{3}(60+2\times66)=64$

MEASURES OF DISPERSION

Measures of dispersion in statistics are used to quantify the extent to which individual data points (observations) in a dataset differ from the central tendency (like the mean or median). They provide insights into the spread, variability, or scatter of the data.

VARIANCE

Variance is a measure of the spread or dispersion of a set of data points (observations). It quantifies the degree to which each data point in a dataset differs from the mean (average) of the dataset. A higher variance indicates that the data points are more spread out from the mean, while a lower variance suggests that the data points are closer to the mean.

It is denoted by var (x) or σ^2

Mathematically, variance is calculated by:

1. **Calculating the mean:** Add up all the data points and divide by the total number of data points to find the average.
2. **Find the differences from the mean for each data point:** Subtract the mean from each data point to determine how much each value deviates from the mean.
3. **Square these differences:** This step ensures that negative differences don't cancel positive ones.
4. **Calculate the average of these squared differences:** Sum up the squared differences and divide by the total number of data points. This gives the variance.

Variance of Individual observations

If $x_1, x_2, \ldots, x_n$ are n values of a variable x, then by definition

$$\text{var}(x) = \sigma^2 = \frac{1}{n}\left[\sum_{i=1}^{n}(x_i - \bar{x})^2\right]$$

On simplification, var $(x) = \sigma^2 = \left[\frac{1}{n}\sum_{i=1}^{n}x_i^2\right] - \left[\bar{x}^2\right]$

= (Mean of Squares) – (Square of Mean)

Properties of Variance

- Zero Variance for Constant Data: If all data points in a set are the same (constant), then the variance of that data set is zero. This is because there is no variability or spread around the mean.
- If you add (or subtract) a constant value "k" to each of the values in a dataset, the variance of that dataset will remain unchanged.
- If you multiply every value in a dataset by a constant "k", then the variance of the dataset will be multiplied by " k^2 ".

STANDARD DEVIATION

The positive square root of the variance is known as standard deviation i.e.

Standard Deviation = σ = + (Var)$^{1/2}$

Standard deviation is more interpretable than variance because it is in the same units as the original data. In contrast, variance is in squared units, which can make it harder to relate to the original data.

S.D. is calculated by taking the positive square root of variance.

QUESTIONS

Q12. The variance of the first n natural numbers is

(a) $\frac{n^2-1}{12}$ (b) $\frac{n^2-1}{6}$

(c) $\frac{n^2+1}{6}$ (d) $\frac{n^2+1}{12}$

Ans: (a)

Explanation:

Variance = σ^2

$$=\frac{1}{n}\Sigma x^2-\left(\frac{\Sigma x}{n}\right)^2, \left(\ \bar{x}=\frac{\Sigma x}{n}\right)$$

i.e. Var(x) = (Mean of Squares) – (Square of Mean)

Sum of squares of first n natural numbers

$$=\frac{n(n+1)(2n+1)}{6}$$

Sum of first n natural numbers = $\frac{n(n+1)}{2}$

Hence,

$$\text{Variance } =\frac{n(n+1)(2n+1)}{6n}-\left(\frac{n(n+1)}{2n}\right)^2=\frac{n^2-1}{12}.$$

Q13. The standard deviation of 25 numbers is 40. If each of the numbers is increased by 5, then the new standard deviation will be

(a) 40 (b) 45

(c) $40+\frac{21}{25}$ (d) None

Ans: (a)

Explanation:

If each item of a data is increased or decreased by the same constant, the standard deviation of the data remains unchanged.

Q14. The mean of five observations is 4 and their variance is 5.2. If three of these observations are 1, 2 and 6, then the other two are

(a) 2 and 9 (b) 3 and 8

(c) 4 and 7 (d) 5 and 6

Ans: (c)

Explanation:

Let the two unknown observations be x and y, then

$$\text{Mean} = 4 = \frac{1+2+6+x+y}{5}$$

$$\Rightarrow x + y = 11 \quad \ldots(i)$$

and variance = 5. 2

Var(x) = (Mean of Squares) – (Square of Mean)

$$\Rightarrow \frac{1^2+2^2+6^2+x^2+y^2}{5}-(\text{mean})^2=5.2$$

$$\Rightarrow 41 + x^2 + y^2 = 5[5.2 + 4^2] = 106$$

or $x^2 + y^2 = 65$ ...(ii)

Solving (i) and (ii) for x and y, we get

$x = 4$ and $y = 7$

or $x = 7$ and $y = 4$

Q15. In an experiment with 15 observations on x, the following results were available $\Sigma x^2 = 2830$, $\Sigma x = 170$. One observation that was 20 was found to be wrong and was replaced by the correct value 30. Then the corrected variance is

(a) 78.00 (b) 188.66

(c) 177.33 (d) 8.33

Ans: (a)

Explanation:

$\Sigma x = 170$, $\Sigma x^2 = 2830$,

Increase in $\Sigma x = 10$, then $\Sigma x' = 170 + 10 = 180$

Increase in $\Sigma x^2 = 900 – 400 = 500$,

then $\Sigma x'^2 = 2830 + 500 = 3330$

$$\therefore \text{Variance } =\frac{1}{n}\Sigma x'^2-\left(\frac{\Sigma x'}{n}\right)^2$$

$$=\frac{3330}{15}-\left(\frac{180}{15}\right)^2=222-144=78.$$

PRACTICE QUESTIONS

1. **'A', 'B' and 'C' are three positive integers such that sum of mean of 'B' and 'C', and 'A' is 12 while the sum of mean of 'A' and 'B', and 'C' is 13. Find the least possible value of mean of 'A' and 'C'.**
   (a) 5 (b) 1
   (c) 3 (d) 2

2. **Find the sum of mode and median of 16, 18, 19, 25, 25, 30, 32 and 34.**
   (a) 25 (b) 50
   (c) 30 (d) 60

3. **The following are the points obtained by a sports team in a series of games.**
   16, 1, 6, 26, 14, 4, 13, 7, 9, 23, 47, 9, 7, 6, 17, 27
   Find the median of the scores obtained by the team.
   (a) 11 (b) 14
   (c) 13 (d) 12

4. **What is the difference between the median and mode of the list given below?**
   6, 2, 3, 4, 3, 5, 2, 7, 4, 2, 6
   (a) 1 (b) 2
   (c) 3 (d) 5

5. **Calculate the mean marks of the students based on the data given in table below:**

Marks	0-10	10-20	20-30	30-40	40-50
No. of students	3	8	12	7	2

   (a) 19 (b) 24
   (c) 30 (d) 32

6. **The Cleveland YMCA program picks up a group of children from a local elementary school and brings them over to their after-school program. The table below shows the ages of children and number of children of that age.**
   Find the average age of the children.

Age	No. of children
5	2
6	3
7	3
8	4
9	2
10	1

   (a) 7.27 (b) 6.87
   (c) 7.55 (d) 6.23

7. **A teacher gave the same test to two classes. In the class with 26 students, the mean score was 86. In the class with 37 students, the mean score was 78. What was the mean score for all students of two classes?**
   (a) 76.5 (b) 81.3
   (c) 85.5 (d) 87.6

8. **There is an old lady who has so many children. She has triplets, who are 13; Billy is 11; Tina is 8; the twins are 5; and Little Lizzy is 2. What is the median of her children's ages?**
   (a) 11 (b) 8
   (c) 9.5 (d) 11.5

9. **Mickey and Vickey go up the hill every day fetching pails of water. Each weekday (Monday through Friday) they together fetch 35 pails. On weekends, they give themselves a little rest, only fetching 26 pails on Saturday and 12 on Sunday. What is the mode of the number of pails of water they fetch each day?**
   (a) 26 (b) 12
   (c) 35 (d) 42

10. **Consider a large population with a mean of 160 and a standard deviation of 25. A random Sample of size 64 is taken from this population. What is the standard deviation of the sample Mean?**
    (a) 2.800 (b) 3.125
    (c) 3.541 (d) 2.450

11. **Find the mean for the following data:**

Class Interval	0-10	10–20	20–30	30–40	40–50	50–60
Frequency	27	10	7	5	4	2

    (a) 16.818 (b) 15.551
    (c) 13.536 (d) 12.485

12. **If the difference between the mode and the median of a given data is 30 (Mode > Median), then what is the difference between the median and the mean?**
    (a) 18 (b) 20
    (c) 15 (d) 12

13. If the ratio of the mode to the median of a given data is 3 : 4, find the ratio of the mean to the median.

(a) $\frac{4}{7}$ (b) $\frac{5}{8}$

(c) $\frac{7}{8}$ (d) $\frac{9}{8}$

14. If the ratio of the mean to the median of a given data is 2 : 3, find the ratio of the mode to the mean.

(a) $\frac{5}{2}$ (b) $\frac{2}{3}$

(c) $\frac{3}{2}$ (d) $\frac{1}{4}$

15. The distribution table of hourly wages and number of labourers getting a particular amount of wage is given below. Find the mean, median, mode of this dataset.

Wages (Rs)	50	60	70	80	90
Number of labourers	4	8	9	6	5

(a) 70 Rs, 70 Rs, 70 Rs

(b) 70 Rs, 80 Rs, 70 Rs

(c) 80 Rs, 80 Rs, 70 Rs

(d) 70 Rs, 70 Rs, 90 Rs

ANSWERS

1. (*d*) 2. (*b*) 3. (*a*) 4. (*b*) 5. (*b*) 6. (*a*) 7. (*b*) 8. (*c*) 9. (*c*) 10. (*b*)
11. (*a*) 12. (*c*) 13. (*d*) 14. (*a*) 15. (*a*)

EXPLANATIONS

1. (d)

Explanation:

According to information given in the question:

$A+\left(\frac{B+C}{2}\right)=12$

Or, $2A + B + C = 24$ ...(i)

And, $\left(\frac{A+B}{2}\right)+C=13$

Or, $A + B + 2C = 26$ ...(ii)

On subtracting equation (i) from equation (ii), we have;

$C - A = 2$

Or, $C = 2 + A$

Minimum value of 'A' = 1

So, minimum value of 'C' = 2 + 1 = 3

So, least possible value of mean of A and $C=\frac{3+1}{2}=2$

2. (b)

Explanation:

Mode = most frequent number occurring in data set = 25

Median = middle value of a data set, when arranged in ascending order

$=\frac{25+25}{2}=\frac{50}{2}=25$ Number of digits is even)

Sum of mode and median = 25 + 25 = 50

3. (a)

Explanation:

On writing the digits of given data in ascending order:

1, 4, 6, 6, 7, 7, 9, 9, 13, 14, 16, 17, 23, 26, 27, 47

Total number of terms (n) = 16 (it is even)

$\text{Median}=\left[\frac{n}{2}^{th}\ \text{term}+\left(\frac{n}{2}+1\right)^{th}\ \text{term}\right]\div 2$

$=\frac{8^{th}\,\text{term}+9^{th}\,\text{term}}{2}$

$=\frac{9+13}{2}$

= 11

4. (b)

Explanation:

Arranging the data in ascending order, we get

2, 2, 2, 3, 3, 4, 4, 5, 6, 6, 7

Since data set has odd number of values,

$\text{Median}=\left(\frac{n+1}{2}\right)^{th}\text{term}=4$

Mode = 2

Difference between median and mode = 2

5. (b)

Explanation:

Marks (X)	No. of students (f_i)	Mid Value (m_i)	$f_i \times m_i$
0–10	3	5	15
10–20	8	15	120
20–30	12	25	300
30–40	7	35	245
40–50	2	45	90
	Σf_i = N = 32		$\Sigma (f_i \times m_i)$ = 770

$$\text{Mean} = \frac{\Sigma f_i m_i}{\Sigma fi}$$

$$= \frac{770}{32} = 24$$

6. (a)

Explanation:

Age (m_i)	Frequency (f_i)	$f_i \times m_i$
5	2	10
6	3	18
7	3	21
8	4	32
9	2	18
10	1	10
	Σf_i = 15	$\Sigma (f_i \times m_i)$ = 109

$$\text{Mean} = \frac{\Sigma fimi}{\Sigma fi}$$

$$= \frac{109}{15} = 7.27$$

7. (b)

Explanation:

Class 1: $\frac{\text{Total score of 26 students}}{26} = 86$

Total score of 26 students = 26 × 86 = 2236

Class 2: $\frac{\text{Total score of 37 students}}{37} = 78$

Total score of 37 students = 37 x 78 = 2886

Therefore, mean score of all students $= \frac{2236+2886}{26+37}$

$$= \frac{5122}{63} = 81.3$$

8. (c)

Explanation:

It is given that,

There are triplets, who are 13 years old;

Billy is 11 years old;

Tina is 8 years old;

The twins are 5 years old;

and Little Lizzy is 2 years old.

Let's arrange these ages in the ascending or descending order as:

2, 5, 5, 8, 11, 13, 13, 13

So, Median = $\frac{\text{fourth term} + \text{fifth term}}{2}$

$$= \frac{8+11}{2} = 9.5$$

9. (c)

Explanation:

By the definition of mode:

"The value that has higher frequency in a given set of values."

So, according to the question,

Each weekday (Monday through Friday) they fetch 35 pails between them.

On weekends only fetching 26 pails on Saturday and only 12 on Sunday.

Dataset here will be: 35, 35, 35, 35, 35, 26, 12 for Monday to Sunday.

So, here 35 appears 5 times, which is higher than frequency of 26 and 12; they appear only once.

Hence, 35 is the mode.

10. (b)

Explanation:

Formula which can be used here is:

SD of sample mean = $\frac{\text{Std deviation}}{\sqrt{n}}$

n = number of elements in sample

$$= \frac{25}{\sqrt{64}}$$

$$= \frac{25}{8}$$

= 3.125

11. (a)

Explanation:

Class Interval	Frequency (f_i)	Mid Value (m_i)	$f_i m_i$
0–10	27	5	135
10–20	10	15	150
20–30	7	25	175
30–40	5	35	175
40–50	4	45	180
50–60	2	55	110
	Σf_i = N = 55		$\Sigma f_i m_i$ = 925

Mean = $\frac{\Sigma fimi}{\Sigma fi} = \frac{925}{55}$ = 16.818

12. (c)

Explanation:

Mode – Median = 30

We know,

Mode = 3(median) – 2(mean)

so, 3(median) – 2(mean) – (median) = 30

2(median) – 2(mean) = 30

2 (Median – mean) = 30

So, Median – mean = $\frac{30}{2}$ = 15

13. (d)

Explanation:

Given that:

Mode = (3/4) × median

We know that:

Mode = 3(median) – 2(mean)

So, 3(median) – 2(mean) = (3/4) × median

12(median) – 8(mean) = 3(median)

$\frac{\text{mean}}{\text{median}} = \frac{9}{8}$

14. (a)

Explanation:

Given:

Mean = (2/3) × median or

Median = (3/2) × mean

We know that:

Mode = 3(median) – 2(mean)

So, Mode = (3 × (3/2) × Mean) – 2(Mean)

Mode = (9/2) Mean – 2(Mean)

Mode = (5/2) × Mean

Hence, $\frac{\text{mode}}{\text{mean}} = \frac{5}{2}$

15. (a)

Explanation:

N = 32

Mean $= \frac{(50\times4)+(60\times8)+(70\times9)+(80\times6)+(90\times5)}{4+8+9+6+5}$

Mean = $\frac{2240}{32}$

Mean = 70 Rs

If we see frequencies then, Rs 70 is having highest frequency of 9 laborers

So, by the definition of mode, we can get

Mode = 70 Rs

Now, according to; Mode = 3(Median) – 2(Mean)

70 = 3(Median) – 2(Mean)

3(Median) = 70 + (2 × 70)

Median = 70 Rs

Hence,

Mean = 70 Rs, Median = 70 Rs; Mode = 70 Rs

Plane Geometry

17

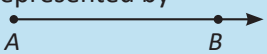
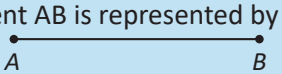
INTRODUCTION

Plane Geometry is a vital branch of mathematics that focuses on the properties and relations of points, lines, angles, and shapes in a two-dimensional space. It lays the foundation for understanding more complex geometric concepts and has practical applications in fields such as art, engineering, and architecture. This chapter will guide you through the essentials of plane geometry, exploring various shapes and their properties, and developing your ability to solve geometric problems.

Point

A point is an exact location or position in space and is typically represented as a dot. Points have no length, width, or dimension.

Line, Ray and Line Segment

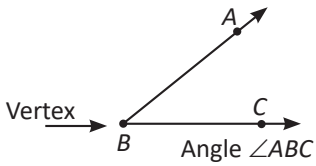
Line
❑ A line $\overleftrightarrow{AB}$ is a represented by
❑ A line has no end point.
❑ It can be extended indefinitely in both directions.
Ray
❑ A ray $\overrightarrow{AB}$ is represented by 
❑ A ray has one end point (its origin point).
❑ A ray can be extended infinitely in one direction from its origin.
Line segment
❑ A line segment AB is represented by 
❑ A line segment has two end points.
❑ A line segment has a definite length.

Plane

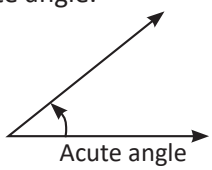
In geometry, a plane is a fundamental two-dimensional, flat surface that extends infinitely in all directions. It has length and width but no thickness. It can be thought of as an infinitely thin sheet.

TYPES OF ANGLES

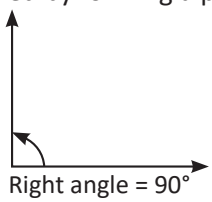
An angle is the union of two rays with a common endpoint, known as the vertex. They are generally measured in degrees.



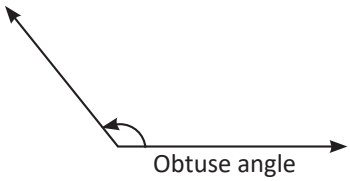
Acute angle: An angle whose measure is between 0° and 90° is called an acute angle.



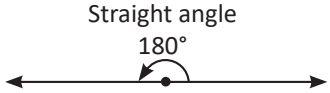
Right Angle: Angles that measure exactly 90 degrees. They are characterized by forming a perfect "L" shape.



Obtuse Angle: An angle whose measure is between 90° and 180° is called an Obtuse angle.



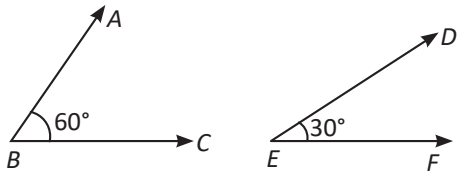
Straight angle: An angle measuring 180° is called a straight angle.



Reflex angle: An angle whose measure is between 180° and 360° is called a reflex angle.

Complementary Angles: Two angles are said to be complementary if the **sum of their measures is 90°**, and each angle is said to be a complement of each other.

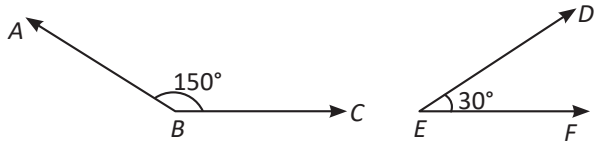
For example: (45°, 45°), (10°, 80°), (20°, 70°), (30°, 60°).



Angles $\angle ABC$ and $\angle DEF$ are complementary angles

Supplementary Angles: Two angles are said to be supplementary if the **sum of their measures is 180°**, and each angle is said to be a supplement of each other.

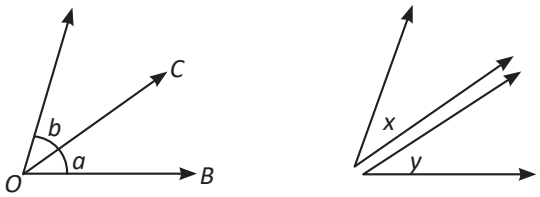
For example: (10°, 170°), (20°, 160°), (30°, 150°), (40°, 140°), (50°, 130°) etc. are all pairs of supplementary angles.



Angles *ABC* and *DEF* are supplementary angles

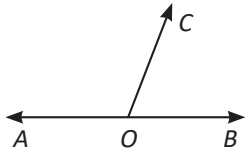
Adjacent Angles: They are angles that are next to each other. The characteristics of adjacent angles are:

- Common Vertex: Adjacent angles have the same vertex or endpoint.
- Common Side: Adjacent angles also share a common side, which is one of the sides of each of the two angles.
- No Overlap: The two angles do not overlap or intersect. They are distinct angles that are side by side.



a and *b* are adjacent angles *x* and *y* are not adjacent angles

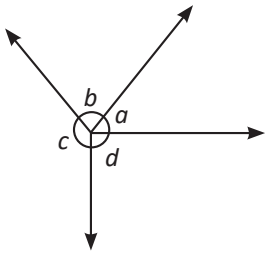
Linear Pair of Angles: Two adjacent angles are said to form a linear pair if their non-common arms are two opposite rays. The most important characteristic of a linear pair is that the two angles are supplementary. In other words, the sum of the measures of the two angles in a linear pair is always 180 degrees.



$\angle AOB = 180°$ (a straight line)

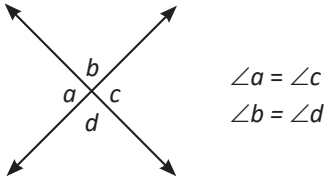
$\angle AOC$ and $\angle COB$ form a linear pair.

Angles at a Point: Sum of the angles around a point is 360°.



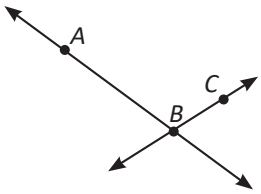
$\angle a + \angle b + \angle c + \angle d = 360°$

Vertically opposite angles: Vertically opposite angles are pairs of angles formed by two intersecting lines opposite to each other. They are always equal (or congruent).



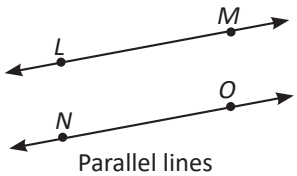
TYPES OF LINES

Intersecting Lines: Two lines are said to be intersecting if they cross or meet at a common point. This point of intersection is called the "intersection point."



Intersecting lines

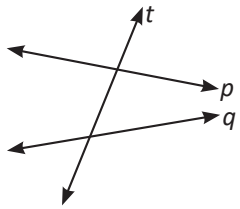
Parallel Lines: Two lines are said to be parallel if they exist in the same plane and remain equidistant from each other throughout their entire length. In other words, no matter how far you extend parallel lines, they will never intersect or cross each other.



Parallel lines

Transversal Line: A transversal line is a straight line that intersects two or more other lines in a plane at distinct points.

In figure, t is a transversal to lines *p* and *q*.

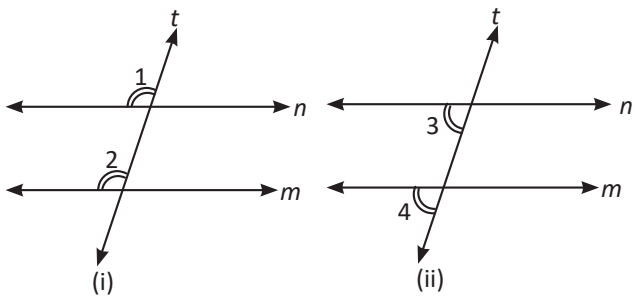


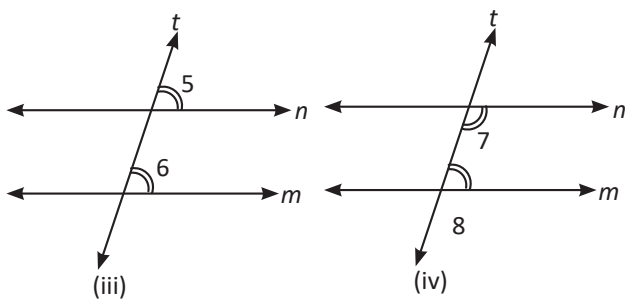
When a transversal line intersects a pair of pair lines, it creates several distinct angles and angle relationships, which are commonly used in geometry.

Corresponding Angles: Corresponding angles are pairs of angles that occupy the same relative position at the intersection of a transversal line with two parallel lines. Corresponding angles can be found in pairs on either side of the transversal line and on the same side of the two parallel lines. They are equal in measure.

Following pair of angles are corresponding:

($\angle 1 = \angle 2$), ($\angle 3 = \angle 4$), ($\angle 5 = \angle 6$), ($\angle 7 = \angle 8$)

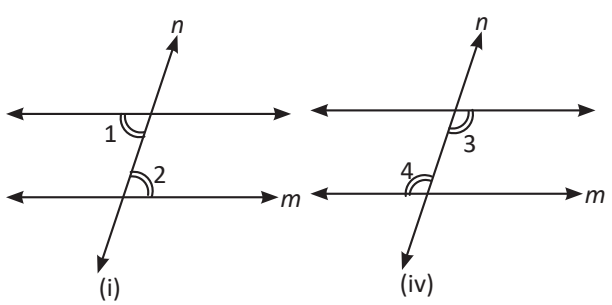




Alternate Interior Angles: These angles are located on opposite sides of the transversal and are positioned between the two parallel lines. They are also equal in measure.

Following pair of angles are alternate interior:

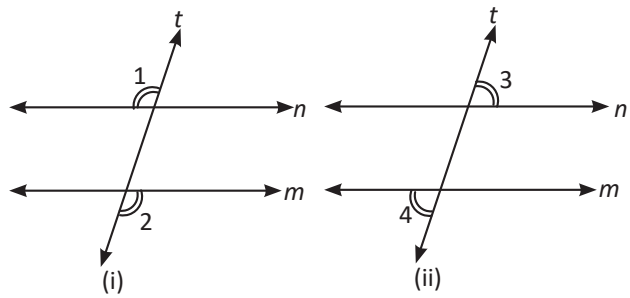
($\angle 1 = \angle 2$) and ($\angle 3 = \angle 4$)



Alternate Exterior Angles: These angles are located on opposite sides of the transversal and are positioned outside the two parallel lines. They are also equal in measure.

Following pair of angles are alternate exterior:

($\angle 1 = \angle 2$) and ($\angle 3 = \angle 4$)

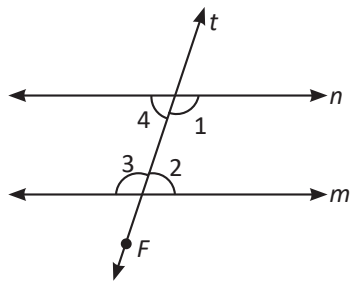


Co-interior angles: Co-interior angles, also known as consecutive interior angles or same-side interior angles are located on the same side of the transversal and are positioned between the two parallel lines.

Co-interior angles are supplementary.

Following pair of angles are Co-interior angles:

($\angle 1 + \angle 2 = 180°$) and ($\angle 3 + \angle 4 = 180°$)

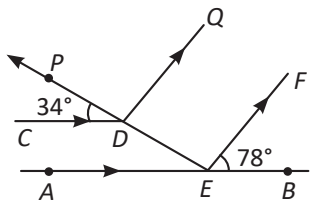


Note:

- If a transversal intersects two lines, making a pair of equal corresponding angles, then the lines are parallel.
- If a transversal intersects two lines in such a way that a pair of alternate interior angles is equal, then the two lines are parallel.
- If a transversal intersects two lines in such a way that a pair of consecutive interior angles is supplementary, then the two lines are parallel.
- If two lines are parallel to the same line, they will be parallel to each other.

EXAMPLES

E1. Consider the following diagram where *AB*||*CD* and *DQ*||*EF*. Then sum of ∠*DEF* and ∠*PDQ* is



(a) 136° (b) 146°

(c) 156° (d) 112°

Ans: (a)

Explanation:

We have, *AB* || *CD* and *PE* is a transversal.

∠*PCD* = ∠*DEA* [Corresponding angles]

∴ ∠*DEA* = 34°

∴ ∠*DEA* + ∠*DEF* + 78° = 180° [Angles on a line]

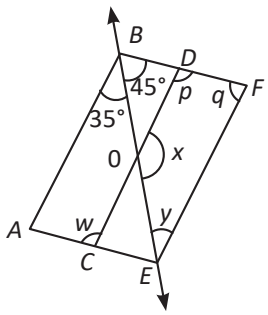
34 + ∠*DEF* + 78 = 180° ∠*DEF* = 180 – 112 = 68°

Now, *DQ*||*EF* and *PE* is a transversal.

∠*PDQ* = ∠*DEF* [Corresponding angles]

∠*PDQ* + ∠*DEF* = 68° + 68° = 136°

E2. Consider the following diagram, *AB*||*CD*||*EF* and *AE*||*BF*. Find the value of ∠*x* + ∠*y* + ∠*p* + ∠*q* + ∠*w*.



(a) 400 (b) 440

(c) 460 (d) 500

Ans: (b)

Explanation:

We have, *AB*||*CD* and *BF* is a transversal.

∠*ABD* = ∠*CDF* [Corresponding angles]

∴ 35° + 45° = ∠*CDF*

or ∠*p* = 80°

Now,

CD||*EF* and *DF* is a transversal.

∴ ∠*p* + ∠*q* = 180° [co-interior angles]

∴ ∠*q* = 180° – 80° = 100°

Also,

AB||*CD* and *BE* is a transversal.

∴ ∠*ABO* = ∠*BOD* [alternate interior angles]

∴ ∠*BOD* = 35°

∠*x* + ∠*BOD* = 180°[Linear pair]

∴ ∠*x* = 180° – 35° = 145°

∠*ABE* = ∠*y* [alternate interior angles]

∴ ∠*y* = 35°.

AE || BF and CD is a transversal.

∠*w* = ∠*p* = 80° [Alternate interior angles]

Hence, ∠*p* = 80°, ∠*q* = 100°, ∠*x* = 145°, ∠*y* = 35° and ∠*w* = 80°

So, Sum of all these angles

= 80° + 100° + 145° + 35° + 80° = 440°

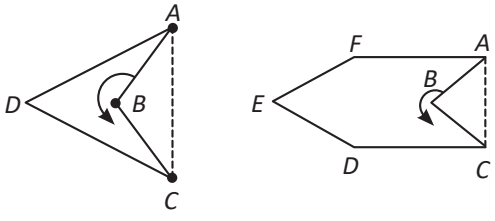
POLYGON

A simple closed figure made up of only line segments is called a polygon. The points where the sides of the polygon meet are called vertices (singular: vertex). Each vertex represents a corner or angle of the polygon. If the number of sides is 'n' then the number of angles is also 'n'. (n is natural no. ≥ 3)

TYPES OF POLYGONS

Convex Polygon: A convex polygon is a type of polygon in which all of its interior angles are less than 180 degrees. In other words, in a convex polygon, none of the angles "bow out" or are greater than a straight angle (180 degrees). Here, all the diagonals of polygon lies inside the polygon.

Concave Polygon: A concave polygon is a type of polygon in which at least one of its interior angles is greater than 180 degrees. In other words, in a concave polygon, at least one angle "bows out" or is wider than a straight angle (180 degrees). Here, at least one of the diagonals of the polygon lies outside the polygon.



TYPES OF CONVEX POLYGON

The various types of convex polygons, classified based on the number of sides are:

Triangle: A polygon with three sides and three angles. Triangles come in different types, including equilateral (all sides and angles are congruent), isosceles (at least two sides and angles are congruent), and scalene (all sides and angles are different).

Quadrilateral: A polygon with four sides and four angles. Common types of quadrilaterals include squares, rectangles, parallelograms, rhombuses, and trapezoids, each with specific properties.

Pentagon: A polygon with five sides and five angles. A regular pentagon has all sides and angles congruent.

Hexagon: A polygon with six sides and six angles. A regular hexagon has all sides and angles congruent.

Heptagon: A polygon with seven sides and seven angles. A regular heptagon has all sides and angles congruent.

Octagon: A polygon with eight sides and eight angles. A regular octagon has all sides and angles congruent. and so on.

Note

- The sum of all interior angles of a n-sided polygon = $(n-2) \times 180°$
- The sum of all exterior angles of a n-sided polygon = $360°$
- Number of Diagonals in a n-sided polygon = $\frac{n(n-3)}{2}$
- Measure of each interior angle of n-sided regular polygon = $\frac{(n-2)180°}{n}$
- Measure of each exterior angle of n-sided regular polygon = $\frac{360°}{n}$

EXAMPLES

E3. In a polygon if sum of its interior angles is 900°, then its number of diagonals is

(a) 14 (b) 20

(c) 9 (d) 27

Ans: (a)

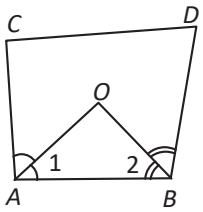
Explanation:

Sum of all interior angles = (n – 2) 180°

$900 = (n-2)\,180° \Rightarrow n-2 = \frac{900}{180} = 5 \Rightarrow n = 7$

Diagonals = $\frac{n(n-3)}{2} = \frac{7(7-3)}{2} = \frac{7(4)}{2} = 14$

E4. In the given figure, *ABCD* is a quadrilateral such that internal angle bisectors of $\angle A$ and $\angle B$ meet at O. Then



(a) $\angle AOB = \frac{1}{2}(\angle C - \angle D)$

(b) $\angle AOB = (\angle C + \angle D)$

(c) $\angle AOB = \frac{1}{2}(\angle C + \angle D)$

(d) None of these

Ans: (c)

Explanation:

In ΔAOB

$\angle AOB + \angle 1 + \angle 2 = 180°$

and $\angle A + \angle B + \angle C + \angle D = 360°$

$\angle AOB = 180° - (\angle 1 + \angle 2)$

$\angle AOB = 180° - (\frac{1}{2}\angle A + \frac{1}{2}\angle B)$

Since, $[\angle 1 = \frac{1}{2}\angle A$ and $\angle 2 = \frac{1}{2}\angle B]$

$\angle AOB = 180° - \frac{1}{2}(\angle A + \angle B)$

$\angle AOB = 180° - \frac{1}{2}(360° - \angle C - \angle D)$

$\angle AOB = 180° - 180° + \frac{1}{2}(\angle C + \angle D)$

$\angle AOB = \frac{1}{2}(\angle C + \angle D)$

TRIANGLE

A geometrical figure formed by joining three non-collinear points by three line segments is called a triangle. It is denoted by the symbol 'Δ'.

Triangles can be classified into different types based on various characteristics, including:

(a) By Side Lengths:

Equilateral Triangle: All three sides are of equal length, and all three angles are equal (60 degrees each).

Isosceles Triangle: Two sides are of equal length, and two angles are equal.

Scalene Triangle: All sides and angles are of different lengths and measures.

(b) By Angles:

Acute Triangle: All angles are acute (less than 90 degrees).

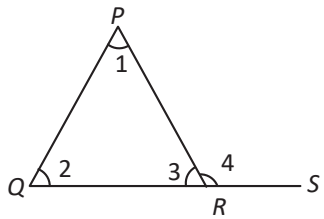
Obtuse Triangle: One angle is obtuse (greater than 90 degrees).

Right Triangle: One angle is a right angle (exactly 90 degrees).

BASIC PROPERTIES AND SOME IMPORTANT THEOREMS OF TRIANGLES

(i) Angle Sum Property of a Triangle: The sum of the interior angles of a triangle is always 180 degrees.

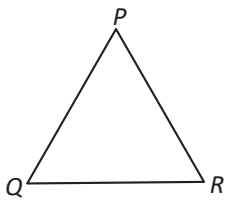
(ii) Exterior Angle Property of a Triangle: If a side of a triangle is produced, the exterior angle so formed is equal to the sum of two interior opposite angles.



$\angle 4 = \angle 1 + \angle 2$

(iii) Triangle Inequality: The sum of any two sides of a triangle is greater than the third side.

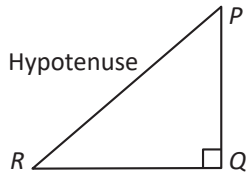
PQ + QR > PR and PR + QR > PQ and PQ + PR > QR



Also, the difference of lengths of any two sides of a triangle is less than the third side.

(iv) The side opposite to the measure of the largest angle is the longest side and vice-versa.

(v) Pythagoras Theorem: It states that in a right angled triangle, the square of the length of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the lengths of the other two sides.

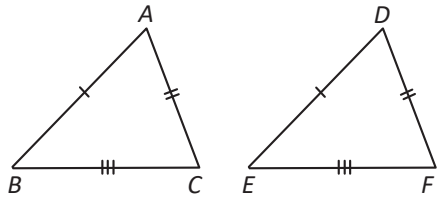


In ΔPQR, $\angle Q = 90°$, we have

$PR^2 = PQ^2 + RQ^2$

CONGRUENCY OF TRIANGLES

The congruency of triangles is a fundamental concept in geometry that refers to two or more triangles having the same shape and size. When two triangles are congruent, it means that their corresponding sides are equal in length, and their corresponding angles are equal in measure.

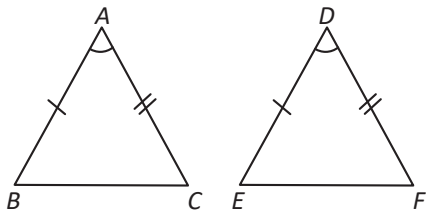


ΔABC and ΔDEF have the same size and shape. They are congruent.

This is denoted as $\Delta ABC \cong \Delta DEF$.

Criteria for Congruence of Triangles

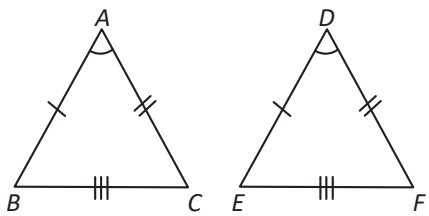
Side-Angle-Side (SAS) Congruence: Two triangles are congruent if they have two sides and the included angle between those sides equal.



In the two triangles, *ABC* and *DEF*

- *Side AB = Side DE*, Side *AC* = Side *DF*
- Included $\angle A$ = included $\angle D$

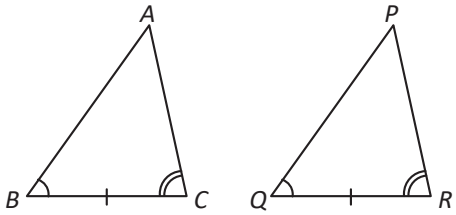
Side-Side-Side (SSS) Congruence: Two triangles are congruent if all three sides of one triangle are equal in length to the corresponding sides of the other triangle.



In the two triangles *ABC* and *DEF*

- Side *AB* = Side *DE*
- Side *BC* = Side *EF*
- Side *AC* = Side *DF*

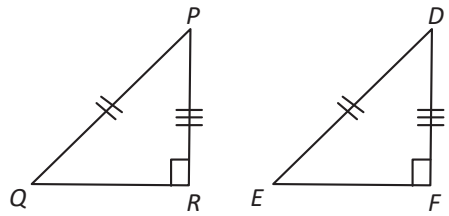
Angle-Side-Angle (ASA) Congruence: Two triangles are congruent if they have two angles and the included side equal.



In the two triangles *ABC* and *PQR*,

- $\angle B = \angle Q$
- $\angle C = \angle R$
- Included side *BC* = Included side *QR*

Right Angle-Hypotenuse-Side (RHS) Congruence (for right triangles): In right triangles, if the hypotenuse and one side of one right triangle are equal in length to the corresponding parts of another right triangle, The triangles are congruent.

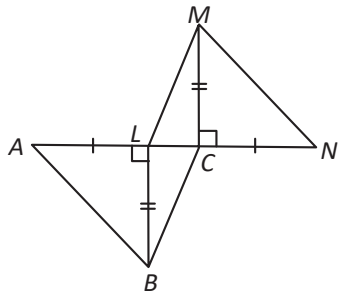


In the two right triangles PQR and *DEF*,

- $\angle R = \angle F = 90°$
- Hypotenuse *PQ* = Hypotenuse *DE*
- Side *PR* = Side *DF*.

EXAMPLES

E5. In the figure below, the following information is given. *BL* ⊥ *AC*, *MC* ⊥ *LN*, *AL* = *CN* and *BL* = *CM*, then which of the following is correct?



(a) $\Delta ALB \cong \Delta NCM$ (b) $\Delta ABC \cong \Delta NML$

(c) $\Delta BLC \cong \Delta MCL$ (d) All are correct

Ans: (d)

Explanation:

In ΔALB and ΔNCM

$AL = CN$ [given]; $BL = CM$ [given]

$\angle ALB = \angle NCM$ [right angles]

So, by SAS Congruency rule $\Delta ALB \cong \Delta NCM$.

In ΔABC and ΔNML

$AB = NM$ [corresponding parts of congruent triangles (CPCT)]

$\angle CAB = \angle LNM$ [CPCT]

$AL = CN \Rightarrow AL + LC = CN + LC \Rightarrow AC = NL$

So, by SAS Congruency rule $\Delta ABC \cong \Delta NML$

In ΔBLC and ΔMCL,

$BL = CM$ [given] $LC = CL$ [common side]

$\angle BLC = \angle MCL$ [right angles]

So, by SAS Congruency rule $\Delta BLC \cong \Delta MCL$

SIMILARITY OF TRIANGLES

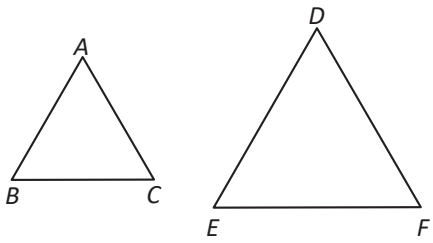
Similarity of triangles, involves triangles that have the same shape but may differ in size. When two or more triangles are similar, it means that their corresponding angles are equal in measure, and their corresponding sides are in proportion. In other words, similar triangles are identical in shape, but they may be scaled versions of each other.

Criteria for Similarity of Triangles

Two triangles ABC and DEF are similar, if

(i) $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$ and

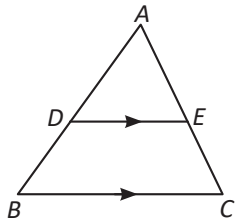
(ii) $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$



- **Angle-Angle (AA) Similarity:** Two triangles are similar if they have two corresponding angles that are equal. In this case, the third angle will also be equal because the sum of the angles in a triangle is always 180 degrees.
- **Side-Angle-Side (SAS) Similarity**: Two triangles are similar if they have one pair of corresponding angles that are equal and the ratio of the lengths of the sides containing those angles is equal.
- **Side-Side-Side (SSS) Similarity:** Two triangles are similar if the ratios of the lengths of their corresponding sides are equal. This means that all three pairs of corresponding sides are in proportion.

Basic Proportionality Theorem or Thales Theorem

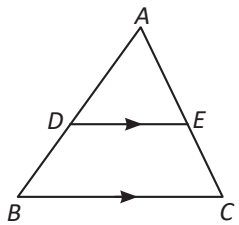
If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the other two sides in the same ratio. Its converse is also true.



- $\frac{AD}{DB} = \frac{AE}{EC}$
- $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$
- $\frac{AB}{DB} = \frac{AC}{EC}$

Mid-Point Theorem

The line segment drawn from the midpoint of any side to the midpoint of any other side of a triangle is parallel to the third side and is half of the length of the third side of the triangle.

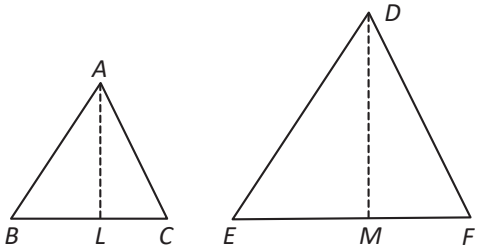


D and *E* are mid-points of *AB* and *AC* respectively then,

- $DE \| BC$
- $DE = \frac{1}{2} BC$

Other Characteristics of Similar triangles

1. If two triangles are similar, then
   (i) The ratio of the corresponding sides is the same as the ratio of the corresponding median.
   (ii) The ratio of the corresponding sides is the same as the ratio of the corresponding angle bisector segments.
   (iii) The ratio of the corresponding sides is the same as the ratio of the corresponding altitudes.
2. The ratio of the areas of two similar triangles are equal to the ratio of the squares of any two corresponding sides.



- $\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

3. The ratio of the areas of two similar triangles is equal to the ratio of
   (i) The squares of the corresponding altitudes.
   (ii) The squares of the corresponding medians.
   (iii) The squares of the corresponding angle bisector segments.

EXAMPLES

E6. Consider the triangle *ABC*, points *D* and *E* are taken on sides *AB* and *AC* respectively such that *DE*||*BC*. If $\frac{AD}{DB} = \frac{2}{3}$, then $\frac{BC}{DE}$ is

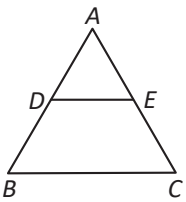
(a) $\frac{5}{2}$ (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

Ans: (a)

Explanation:

In ΔABC, we have



$DE \| BC \Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$

Thus, in triangles *ABC* and *ADE*, we have

$\frac{AB}{AD} = \frac{AC}{AE}$ and, $\angle A = \angle A$

Therefore, by SAS-criterion of similarity, we have

$\Delta ABC \sim \Delta ADE$

$\Rightarrow \frac{AB}{AD} = \frac{BC}{DE}$ ...(i)

It is given that

$\frac{AD}{DB} = \frac{2}{3} \quad \frac{AD}{DB} = \frac{2}{3} \Rightarrow \frac{DB}{AD} = \frac{3}{2}$

$\Rightarrow \frac{DB}{AD} + 1 = \frac{3}{2} + 1$

$\Rightarrow \frac{DB + AD}{AD} = \frac{5}{2}$

$\Rightarrow \frac{AB}{AD} = \frac{5}{2}$ ...(ii)

From (i) and (ii), we get

$\frac{BC}{DE} = \frac{5}{2}$

E7. Consider the triangle *ABC*, with *C* being the right angled vertex. Points *P* and *Q* are taken on sides *AC* and *BC* respectively dividing the sides in the ratio 1:2, then which of the following is correct.

(a) $9\,AQ^2 = 9\,AC^2 + 4\,BC^2$

(b) $9\,BP^2 = 9\,BC^2 + 4\,AC^2$

(c) $9\,(AQ^2 + BP^2) = 13\,AB^2$

(d) All of the above

Ans: (d)

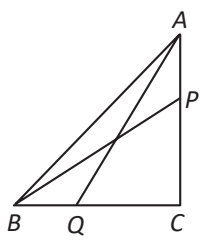
Explanation:

P divides *AC* in the ratio 1 : 2. Therefore,

$$CP = \frac{2}{3}\,AC \qquad ...(i)$$

Also, Q divides *BC* in the ratio 1 : 2.

$$\therefore \quad QC = \frac{2}{3}\,BC \qquad ...(ii)$$



(a) Applying Pythagoras theorem in right-angled triangle ACQ, we have

$AQ^2 = QC^2 + AC^2$

$\Rightarrow AQ^2 = \frac{4}{9}\,BC^2 + AC^2$ [Using (ii)]

$\Rightarrow 9\,AQ^2 = 4\,BC^2 + 9\,AC^2 \qquad ...(iii)$

(b) Applying Pythagoras theorem in right- angled triangle *BCP*, we have

$BP^2 = BC^2 + CP^2$

$\Rightarrow BP^2 = BC^2 + \frac{4}{9}AC^2$ [Using (i)]

$\Rightarrow 9\,BP^2 = 9\,BC^2 + 4\,AC^2 \qquad ...(iv)$

Adding (iii) and (iv), we get

$9\,(AQ^2 + BP^2) = 13\,(BC^2 + AC^2)$

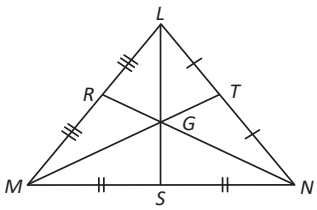
$\Rightarrow 9\,(AQ^2 + BP^2) = 13\,AB^2$

[Since, $BC^2 = AC^2 + AB^2$]

MEDIAN OF A TRIANGLE

A median of a triangle is a line segment that connects a vertex of the triangle to the midpoint of the opposite side.

For example, consider ΔLMN. Let *S* be the midpoint of MN, then *LS* is the line segment joining vertex *L* to the midpoint of its opposite side. So, *LS* is the median. In the figure below, MT and NR are also medians.



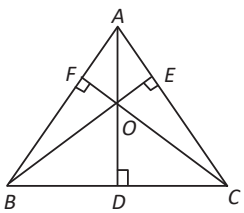
Note :

- A triangle has three medians.
- **The three medians of a triangle intersect at a single point known as the centroid (G).** The centroid is considered the center of gravity of the triangle and has several important properties.
- The centroid of the triangle always lies inside of the triangle.
- The centroid of a triangle divides each one of the medians in the ratio 2 : 1. The segment from the vertex to the centroid is twice as long as the segment from the centroid to the opposite side's midpoint.
- The medians of an equilateral triangle are equal in length.
- The centroid of a triangle is also the point at which each median bisects the triangle's area. This means that each half of the triangle, created by a median, has the same area.

ALTITUDE OF A TRIANGLE

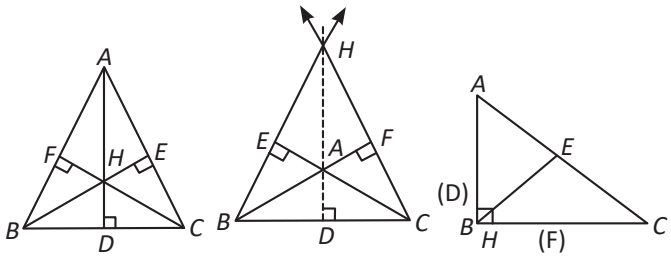
An altitude of a triangle is a perpendicular line segment drawn from a vertex of the triangle to the side opposite to that vertex. It is also known as the height of the triangle relative to that specific vertex.

In the figure below, *AD*, *BE* and *CF* are altitudes.



Note

- A triangle has three altitudes.
- The length of the altitude represents the shortest distance from the vertex to the opposite side.
- **The three altitudes of a triangle intersect at a single point known as the orthocenter (H).** The orthocentre is not necessarily located inside the triangle; it can be inside (acute angled triangle), outside (obtuse angled triangle), or on the triangle (right angled triangle). Consider orthocentre (H) in the following diagrams.



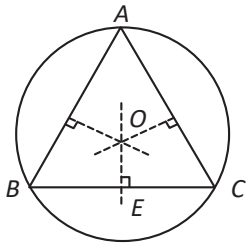
(i) (ii) (iii)

PERPENDICULAR BISECTOR OF A TRIANGLE

The perpendicular bisector of a side of a triangle is a line or line segment that is both perpendicular to that side and passes through its midpoint. In other words, it is a line that divides the side of the triangle into two equal segments while forming a 90-degree angle (perpendicular) with the side.

The three perpendicular bisectors of a triangle's sides intersect at a single point known as the circumcentre (O). The circumcentre is the center of the circumcircle, which is the circle that passes through all three vertices of the triangle. The circumcentre is equidistant from all three vertices of the triangle.

Then, $OA = OB = OC$



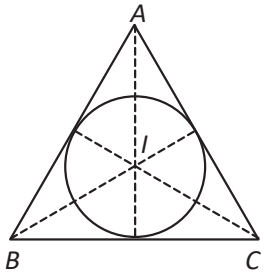
The Circumcentre is not necessarily located inside the triangle; it can be inside (acute angled triangle), outside (obtuse angled triangle), or on the triangle (right angled triangle). If a triangle is right angled, then its circumcentre is the midpoint of the hypotenuse.

ANGLE BISECTOR OF A TRIANGLE

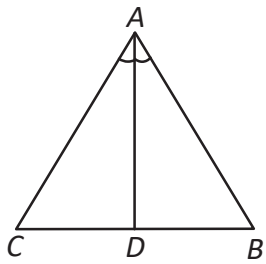
The angle bisector of a triangle is a line or line segment that divides one of the triangle's angles into two equal parts.

The angle bisectors of a triangle intersect at a single point known as the incentre (I). The incentre is the centre of the incircle, which is the largest circle that can be inscribed inside the triangle.

The incenter is equidistant from all three sides of the triangle. It is the point where the angle bisectors intersect and is the centre of the incircle.



Angle Bisector Theorem: The Angle Bisector Theorem states that the angle bisector of an angle in a triangle divides the opposite side into two segments that are proportional to the adjacent sides. If AD is the angle bisector of angle A, then, $\frac{BD}{DC} = \frac{AB}{AC}$

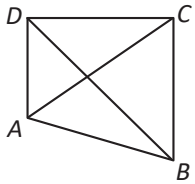


Note

- If the triangle is equilateral, then centroid, incentre, orthocentre, circumcentre coincide.
- Orthocentre, centroid and circumcentre are always collinear and the centroid divides the line joining orthocentre and circumcentre in the ratio 2 : 1.
- In an isosceles triangle centroid, orthocentre, incentre, circumcentre lies on the same line.

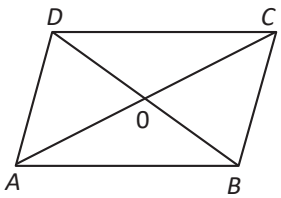
QUADRILATERALS

A quadrilateral is a four-sided polygon. Thus, it has four vertices, four sides, four angles and two diagonals. The Sum of angles of quadrilateral is $\angle A + \angle B + \angle C + \angle D = 360°$



PARALLELOGRAM

A parallelogram (||^{gm}) is a quadrilateral with both pairs of opposite sides equal and parallel.

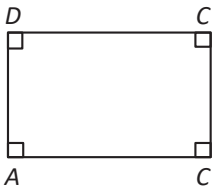


Properties

- Opposite sides are equal i.e. $AB = CD$, $AD = BC$
- Opposite sides are parallel i.e. $AB \| CD$, $AD \| BC$
- Opposite angles are equal i.e. $\angle A = \angle C$; $\angle B = \angle D$
- Sum of adjacent angles is 180° i.e. $\angle A + \angle B = 180°$ or $\angle A + \angle D = 180°$ and $\angle C + \angle D = 180°$ and $\angle B + \angle C = 180°$
- Length of both diagonals are different.
- Diagonals bisect each other at the same point i.e $OA = OC$ and $OB = OD$.
- Diagonal divides ||^{gm} into two congruent triangles.

RECTANGLE

A rectangle is a parallelogram (||^{gm}) with all angles as right angles (90°).

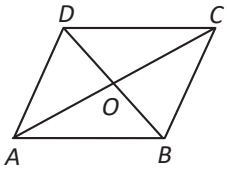


Properties

- A rectangle has all the properties of a parallelogram.
- All angles are 90°.
- Diagonals are equal in length.
- Length of diagonal = $\sqrt{l^2 + b^2}$, where l is the length, and b is the breadth of the rectangle.

RHOMBUS

A rhombus is a parallelogram (||^{gm}) with all sides equal in length.

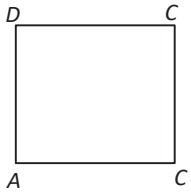


Properties

- A rhombus has all the properties of a parallelogram.
- All sides are equal.
- Diagonals bisect each other at a right angle. i.e. $AO = OC$, $OB = OD$ and $BD \perp AC$

SQUARE

A square is a rectangle with all sides equal or a square is a rhombus with all angles equal (each 90°).

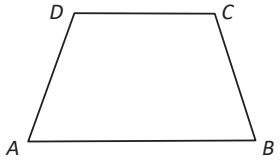


Properties:

- A square has all properties of a parallelogram, rectangle and rhombus as well.
- Diagonals are perpendicular bisectors of each other and are also equal in length.
- Length of diagonal = $\sqrt{a^2 + a^2} = \sqrt{2}a$, where 'a' is a side of the square.

TRAPEZIUM

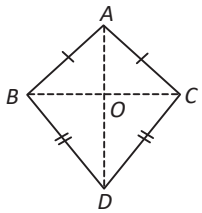
A trapezium, also known as a trapezoid, is a quadrilateral (a four-sided polygon) with one pair of parallel sides.



KITE

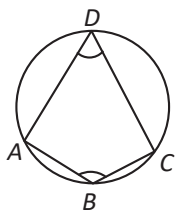
In a quadrilateral, if two pairs of adjacent sides are equal, then it is called a kite.

In the adjoining figure, AB = AC and BD = DC. The longer diagonal bisects the shorter one. i.e. BO = OC [AO ≠ OD]



CYCLIC QUADRILATERAL

A cyclic quadrilateral, also known as a concyclic quadrilateral, is a four-sided polygon in which all four vertices lie on the circumference of a single circle. In a cyclic quadrilateral, the sum of the measures of the opposite angle is always 180 degrees. In the figure below, $\angle A + \angle C = 180°$ or $\angle B + \angle D = 180°$



EXAMPLES

E8. If the mid-points of the sides of any quadrilateral are joined to form another quadrilateral, then the resultant quadrilateral is a

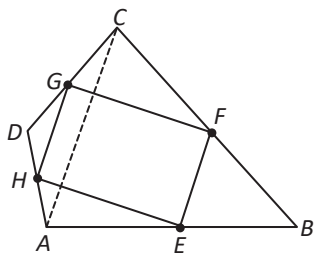
(a) Parallelogram (b) Rectangle

(c) Rhombus (d) Square

Ans: (a)

Explanation:

Let points *E*, F, *G* and *H* are the mid-points of the sides *AB*, *BC*, *CD* and *DA* respectively, of the uadrilateral *ABCD*. Also, join the diagonal AC of the quadrilateral *ABCD*.



Now, in ΔABC, we have *E* and *F* mid-points of the sides *BA* and *BC*.

$EF||AC$ and $EF = \frac{1}{2}$ AC [mid-point theorem] ....

Similarly, from ΔADC, we have

$GH||AC$ and $GH = \frac{1}{2} AC$

So, $EF||GH$ and $EF = GH$

Therefore, quadrilateral *EFGH* is a parallelogram.

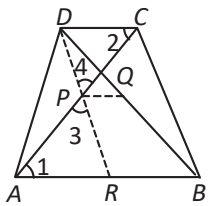
E9. Let *P* and *Q* be the mid-points of the diagonals *AC* and *BD* respectively of trapezium *ABCD*. Then which of the following is not correct?

(a) $PQ||AB$ (b) $PQ = \frac{1}{2}(AB - DC)$

(c) $\Delta APR \cong \Delta DPC$ (d) $PQ = \frac{1}{2}$ (AB + DC)

Ans: (d)

Explanation:



Join *DP* and produce *DP* to meet *AB* in *R*.

Since $AB||DC$ and transversal *AC* cuts them at *A* and *C* respectively.

$\angle 1 = \angle 2$ [$\therefore$ Alternate angles are equal]

Now, in ΔAPR and ΔDPC, we have

$\angle 1 = \angle 2$

$AP = CP$ [Since, P is the midpoint of *AC*]

$\angle 3 = \angle 4$ [Vertically opposite angles]

So, by *ASA* criterion of congruence

$\Delta APR \cong \Delta CPD$

$\Rightarrow$ $AR = DC$ and $PR = DP$ [*CPCT*]

In ΔDRB, *P* and *Q* are the mid-points of sides *DR* and *DB* respectively.

$\therefore$ $PQ||RB$

$\Rightarrow$ $PQ||AB$ [Since, *RB* is a part of *AB*]

$\Rightarrow$ $PQ||AB$ and *DC* [Since, $AB||DC$]

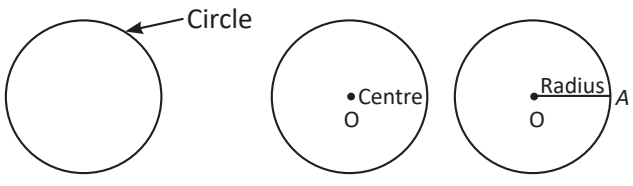
Again, *P* and *Q* are the mid-points of sides *DR* and *DB* respectively in ΔDRB.

$\therefore$ $PQ = \frac{1}{2} RB \Rightarrow PQ = \frac{1}{2}(AB - AR)$

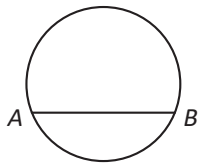
$\Rightarrow$ $PQ = \frac{1}{2}(AB - DC)$[Since, $AR = DC$]

CIRCLES

A circle is defined as the set of all points that are equidistant (at an equal distance) from a single central point, known as the centre. The constant distance from the centre to any point on the circle is called the radius.



Chord: A chord is a line segment with both endpoints on the circle. In the figure, AB is a chord of the circle.



Diameter: The diameter is a chord that passes through the center of the circle and has endpoints on the circle. The diameter is always twice the length of the radius. A diameter divides a circle into two semi-circles. Diameter = 2 × Radius.

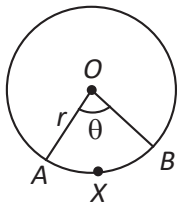
Circumference: The circumference of a circle is the distance around the outer edge of the circle.

It is calculated using the formula:

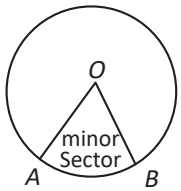
Circumference = $2\pi r$, where "r" is the radius.

Arc: An arc is a portion of the circle's boundary. The length of an arc is proportional to the measure of the central angle that it subtends.

Sector: A sector is the region of the circle enclosed by two radii and the arc between them.



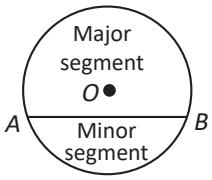
- *AXB* is an arc of the circle.
- *AOB* is a sector of the circle.
- The arc makes an angle at the centre of the circle. This is also known as the central angle.
- The length of the arc of the sector of the circle $(l) = \frac{\theta}{360} \times 2\pi r$ (where r is the radius of the circle)
- The perimeter of the sector of the circle = $l + 2r$
- The area of a sector of a circle = $\frac{\theta}{360} \times \pi r^2$



The sector which is larger than the semicircular region is called the major sector and the region less than the semicircular region is called the minor sector.

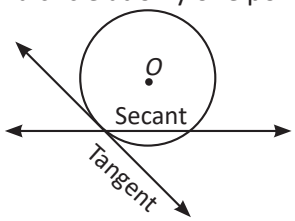
Segments of Circle: A chord divides the circle into two parts. Each part is a segment of the circle.

- Smaller region is known as the minor segment.
- Larger region is known as the major segment.

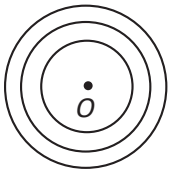


Secant: A secant is a straight line that intersects the circle at two distinct points.

Tangent: A tangent is a straight line which touches the circumference of a circle at only one point.



Concentric Circles: Circles having the same centre and different radius are said to be concentric circles.

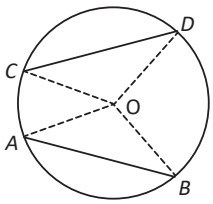


Concentric Circles

BASIC THEOREMS & RESULTS OF CIRCLES

Theorem 1

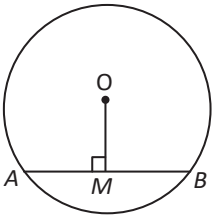
If two chords of a circle are of equal length, they will also subtend equal central angles at the centre of the circle. Its converse is also true.



- Chord AB = chord CD
- $\angle AOB = \angle COD$

Theorem 2

The perpendicular from the centre of a circle to a chord bisects the chord. Its converse is also true.



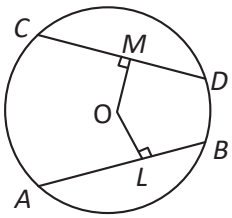
- If M is the mid point of $AB \Rightarrow OM \perp AB$
- If $OM \perp AB$ then $AM = MB$

Theorem 3

There is one and only one circle passing through three non collinear points.

Theorem 4

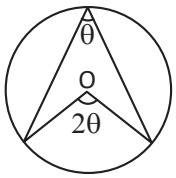
Two equal chords of a circle are equidistant from the centre. Its converse is also true.



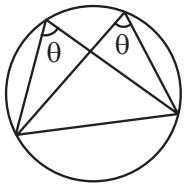
- If $AB = CD \Rightarrow OL = OM$
- If $OL = OM \Rightarrow AB = CD$

Theorem 5

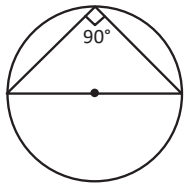
The angle subtended by an arc at the centre is double the angle subtended by it at any point of alternate segment of the circle.



- The angles subtended by a chord in the same segment of a circle are equal.

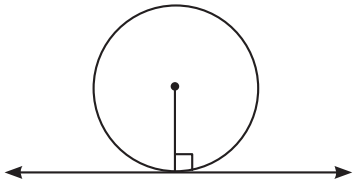


- The angle subtended by a diameter in a semi circle is a right angle.



Theorem 6

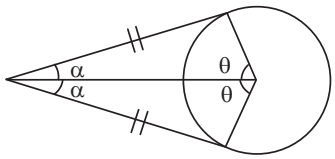
The tangent line at any point of a cirlce is perpendicular to the radius through the point of contact.



Theorem 7

If two tangents are drawn from an external point then:

- They are equal in length.
- They subtend equal angles at the centre.
- They are equally inclined to the line segment joining the centre to that point.

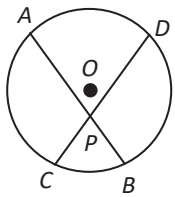


Theorem 8

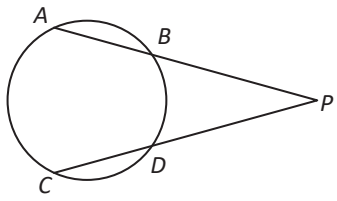
Power of a Point theorem

When two lines intersect a circle then there arises three possibilities:

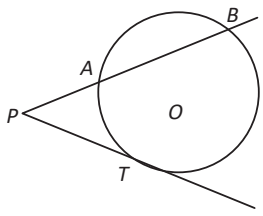
- The two lines are chords of the circle and intersect inside the circle. The product of the segments of one chord is equal to the product of the segments of the other chord.
  - $PA \times PB = PC \times PD$



- The two lines are secants of the circle and intersect outside the circle. The product of the segments of one secant is equal to the product of the segments of the other secant.
  - $PA \times PB = PC \times PD$

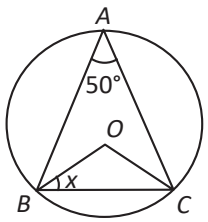


- One of the lines is tangent to the circle while the other is a secant.
  - $PA \times PB = PT^2$



EXAMPLES

E10. In the following diagram find the value of unknown angle *x*.



(a) 40 (b) 50°

(c) 80° (d) 100°

Ans: (a)

Explanation:

$\angle BOC = 2\angle BAC$ [angle at the centre is double the angle subtended in alternate segment]

$\angle BOC = 2 \times 50° = 100°$

In ΔBOC, $BO = OC$ (Radii of the same circle)

$\Rightarrow \angle OBC = \angle OCB = x$

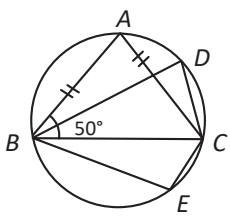
$\therefore x + \angle BOC + x = 180°$ [Sum of $\angle s$ of a Δ]

$\Rightarrow 2x = 180° - \angle BOC$

$\Rightarrow 2x = 180° - 100°$

$\Rightarrow 2x = 80° \Rightarrow x = 40°$.

E11. In the following figure ΔABC is an isosceles triangle with $AB = AC$. Let $\angle ABC = 50°$, then $\angle BDC$ =



(a) 50° (b) 60°

(c) 70° (d) 80°

Ans: (d)

Explanation:

In ΔABC, we have $AB = AC$

$\Rightarrow \angle ACB = \angle ABC$

$\Rightarrow \angle ACB = 50°$ [Since, $\angle ABC = 50°$]

$\therefore \angle BAC = 180° - (\angle ABC + \angle ACB)$

$\Rightarrow \angle BAC = 180° - (50° + 50°) = 80°$

Since $\angle BAC$ and $\angle BDC$

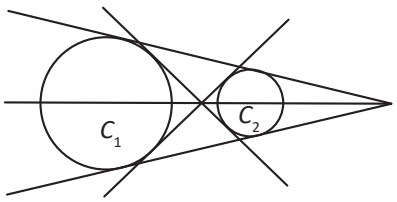
are angles in the same segment.

$\therefore \angle BDC = \angle BAC \Rightarrow \angle BDC = 80°$

COMMON TANGENTS OF TWO CIRCLES

When two circles are positioned in proximity to each other, they can have common tangents. The number and types of common tangents between two circles depend on their relative sizes and positions. Here are the possibilities:

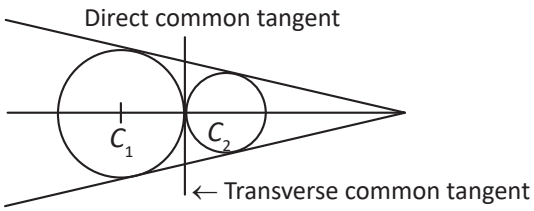
Four Common Tangents: When two circles are non-intersecting (placed apart), i.e. one circle is outside the other and they don't intersect, then four common tangents can be drawn. Two tangents are such that centres of circles lie on the same side of tangents are direct common tangents (DCTs). Two tangents are such that centres of circles lie on either side of the common tangent are transverse common tangents (TCTs).



The distance between the centres is greater than the sum of their radii.

- $C_1C_2 > r_1 + r_2$

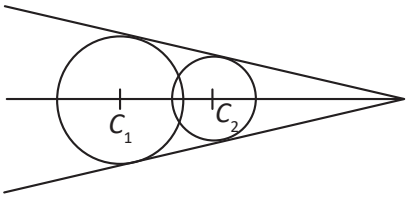
Three Common Tangents: If the circles touch each other externally, there can be three common tangents. Two of these tangents are DCTs and one is TCT.



The distance between the centres is equal to the sum of their radii.

- $C_1C_2 = r_1 + r_2$

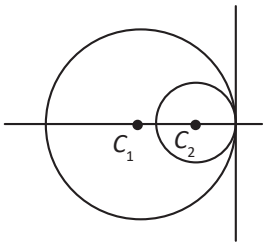
Two Common Tangents: If the circles intersect each other at two distinct points, there can be two common tangents. Both of these tangents are DCTs.



The distance between the centres is less than the sum of their radii and greater than difference of radii.

- $|r_1 - r_2| < C_1C_2 < r_1 + r_2$

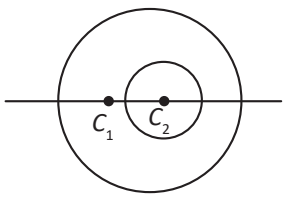
One Common Tangent: If the circles touch each other internally, there can be only one common tangent. It is a TCT.



The distance between the centre is equal to the difference of their radii.

- $C_1C_2 = |r_1 - r_2|$

Zero Common Tangent: When one circle lies completely inside the other, then no common tangent is possible.



The distance between centre is less than the difference of their radii.

- $C_1C_2 < |r_1 - r_2|$

Note: If C_1C_2 is the distance between centres of two circles and r_1 and r_2 are their radii, then:

- Length of direct common tangent $= \sqrt{(C_1C_2)^2 - (r_1 - r_2)^2}$
- Length of transverse common tangent $= \sqrt{(C_1C_2)^2 - (r_1 + r_2)^2}$

EXAMPLES

E12. Consider two non- intersecting circles, the length of whose transverse common tangent is 7 cm and length of direct common tangent is 11 cm. Then the value of the product of radii of the two circles is

(a) 18 (b) 20

(c) 16 (d) 12

Ans: (a)

Explanation:

Length of $TCT = \sqrt{(C_1C_2)^2 - (r_1 + r_2)^2} = 7$ ...(i)

Length of $DCT = \sqrt{(C_1C_2)^2 - (r_1 - r_2)^2} = 11$ ...(ii)

Squaring both (i) and (ii)

$(C_1C_2)^2 - (r_1 + r_2)^2 = 49$

$\Rightarrow (C_1C_2)^2 - (r_1^2 + r_2^2 + 2\,r_1r_2) = 49$ ...(iii)

and $(C_1C_2)^2 - (r_1 - r_2)^2 = 121$

$\Rightarrow (C_1C_2)^2 - (r_1^2 + r_2^2 - 2\,r_1r_2) = 121$ ...(iv)

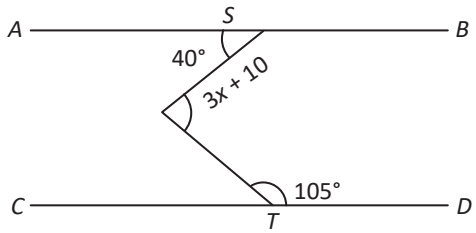
Subtracting (iv) from (iii)

We get $4r_1r_2 = 121 - 49 = 72$

$\Rightarrow r_1r_2 = 18$

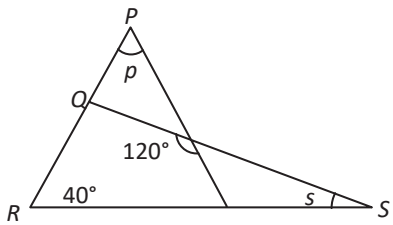
PRACTICE QUESTIONS

1. In the given figure below, *AB*||*CD*. Find the value of *x*.



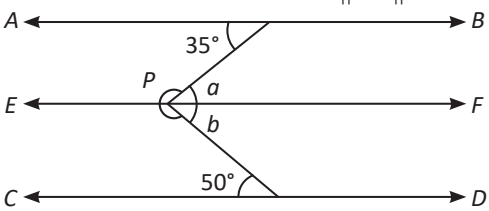
(a) 25° (b) 30°
(c) 35° (d) 37°

2. Find the Value of $\angle p + \angle s$.



(a) 60° (b) 50°
(c) 80° (d) 55°

3. Find the value of $\angle P$. When *AB* || EF || CD



(a) 275° (b) 225°
(c) 85° (d) 135°

4. In a Triangle *ABC*, $\angle B = \angle C = 50°$. Line *BA* is produced up to *D* such that *BA* = *AD*. Find the value of $\angle BCD$.

(a) 60° (b) 30°
(c) 40° (d) 90°

5. In triangle *PQR* a point T is on QR such that *QT* = *PT* = *PR*, and side *QP* is extended up to S such that $\angle RPS = 96°$. Then find the value of $\angle PTR$

(a) 52° (b) 40°
(c) 48° (d) 64°

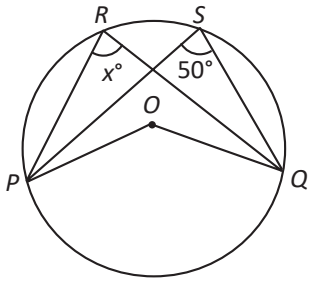
6. In a triangle *PQR*, *A*, *B* and *C* are points on *PQ*, *QR*, *PR* respectively. If *AQ* = *QB*, *CR* = *BR*, and $\angle QPR = 60°$ then find the value of $\angle ABC$.

(a) 60° (b) 50°
(c) 80° (d) 55°

7. $\Delta ABC \sim \Delta PQR$. Their perimeters are 42 cm and 70 cm. If a side of ΔABC = 7.5 cm find the corresponding side of ΔPQR.

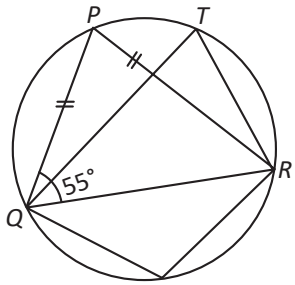
(a) 15 cm (b) 12.5 cm
(c) 7.5 cm (d) 3.75 cm

8. In the given figure, O is the center of a circle, then the value of x will be?



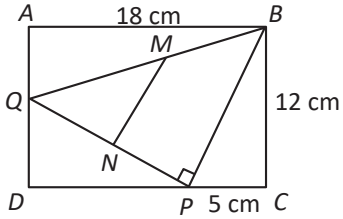
(a) 40° (b) 50°
(c) 130° (d) 100°

9. In the given figure, *PQ* = *PR* and $\angle PQR = 55°$, then find the value of $\angle QTR$.



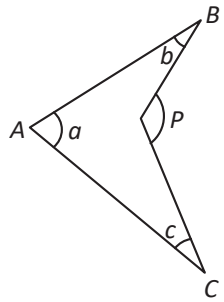
(a) 45° (b) 50°
(c) 70° (d) 55°

10. *ABCD* is a rectangle, *M* and *N* are the mid points of line *QB* and *QP* respectively. Find the length of *MN*.



(a) 7.2 cm (b) 6.5 cm
(c) 5.8 cm (d) 7.5 cm

11. Find the value of $\angle P$.



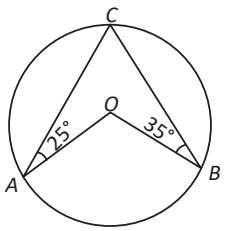
(a) $a + b - c$ (b) $a + c - b$
(c) $a + c$ (d) $a + b + c$

12. ***ABCD*** **is a trapezium, where** ***AB*** **||** ***CD*****,** ***AB*** **= 7cm,** ***BC*** **= 15cm,** ***CD*** **= 24cm and** ***AD*** **= 8cm.** ***DA*** **and** ***CB*** **are extended which meet at point O. find the value of ∠O.**

(a) 45° (b) 60°

(c) 75° (d) 90°

13. In the figure ***∠OAC*** **= 25° and** ***∠OBC*** **= 35° then** ***∠AOB*** **is equal to:**



(a) 110°

(b) 150°

(c) 90°

(d) 120°

14. A boy is standing on the 9 m long pole whose length of shadow is 12 m. If the length of his shadow is 1.6 m, what is the height of the man?

(a) 1 m

(b) 1.2 m

(c) 0.9 m

(d) 0.8 m

ANSWERS

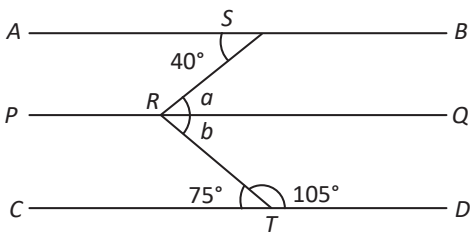
1. (*c*) 2. (*c*) 3. (*a*) 4. (*d*) 5. (*d*) 6. (*a*) 7. (*b*) 8. (*b*) 9. (*c*) 10. (*b*)
11. (*d*) 12. (*d*) 13. (*d*) 14. (*b*)

EXPLANATIONS

1. (c)

Explanation:

Consider an Imaginary line *PQ* parallel to line *AB* & *CD*.



Angle *RTC* = 180 – 105 = 75°

a = ASR = 40° (Alternate Angles)

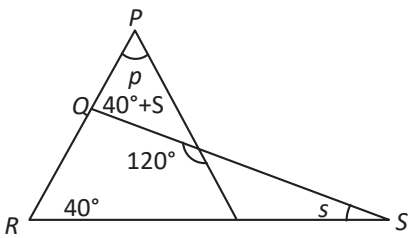
b = *RTC* = 75° (Alternate Angles)

3*x* + 10 = *a* + *b* = 40° + 75° = 115°

3*x* + 10 = 115 ⇒ 3*x* = 105 ⇒ *x* = 105/3 = 35°

2. (c)

Explanation:



Considering triangle QRS

∠PQS = 40° + *s* (∵ An exterior angle of a triangle is equal to the sum of its two opposite non – adjacent interior angles.)

p + 40° + *s* = 120° (∵ An exterior angle of a triangle is equal to the sum of its two opposite non – adjacent interior angles.)

p + *s* = 120° – 40° = 80°

3. (a)

Explanation:

∠a = 35° (∵ Alternate angles)

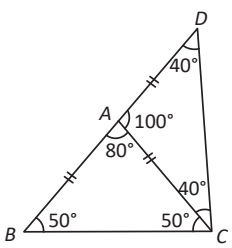
∠b = 50° (∵ Alternate angles)

P = 360° – (*∠a* + *∠b*)

P = 360° – 35° – 50° = 275°

4. (d)

Explanation



In triangle *ABC*

∠B = *∠C* = 50° *∠A* = 180° – 50° – 50° = 80°

Extending line *BA* till *D*

∠DAC = 180° – 80° = 100

$BA = AD$

$BA = AC$ (because $\angle B = \angle C$)

Therefore $DA = AC$

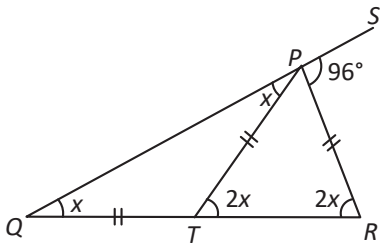
$\angle ADC + \angle ACD = 180° - 100° = 80°$

$\angle ADC = \angle ACD = 80^0/2 = 40°$ (because $AC = AD$)

$\angle BCD = 40° + 50° = 90°$

5. (d)

Explanation:



Let $\angle QPT = x$

Therefore $\angle PQT = x$ ($\because PT = QT$)

$\angle PTR = 2x$ ($\because$ An exterior angle of a triangle is equal to the sum of its two opposite non – adjacent interior angles.)

$\angle PTR = \angle PRT = 2x$ ($\because PT = PR$)

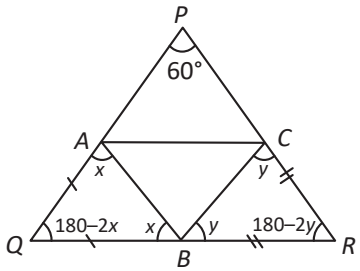
$\angle RPS = x + 2x$ ($\because$ the exterior angle of a triangle is equal to the sum of its two opposite non – adjacent interior angles.)

$x + 2x = 96 \Rightarrow x = 32°$

$\angle PTR = 2x = 2 \times 32° = 64°$

6. (a)

Explanation



Let $\angle ABQ = \angle BAQ = x$ (because $QA = QB$)

$\angle RBC = \angle RCB = y$ (because $CR = BR$)

From triangle AQB

$\angle AQB = 180° - 2x$

From triangle RCB

$\angle BRC = 180° - 2y$

From triangle PQR

$60° + 180° - 2x + 180° - 2y = 180°$

$240° = 2x + 2y$

$120° = x + y$

We have

$X + \angle ABC + y = 180°$

$\angle ABC + 120° = 180°$

$\angle ABC = 180° - 120° = 60°$

7. (b)

Explanation:

Since $\Delta ABC \sim \Delta PQR$

The ratio of the perimeter will be equal to the ratio of the corresponding sides.

Ratio of perimeters of triangle ABC and PQR = 42: 70 = 3: 5

The ratio of corresponding sides of ABC and PQR = 3:5

$$\frac{\text{Side of } ABC}{\text{Side of } PQR} = \frac{3}{5}$$

$$\frac{7.5}{\text{Side of } PQR} = \frac{3}{5}$$

$$\frac{7.5 * 5}{3} = \text{Side of } PQR$$

Side of PQR = 12.5cm.

8. (b)

Explanation:

$X°$ = 50° (angle formed by 2 chords meeting at circumference will be same at any point on circumference)

9. (c)

Explanation:

From triangle PQR

$\angle PQR = \angle PRQ = 55°$ (because $PQ = PR$)

From triangle PQR

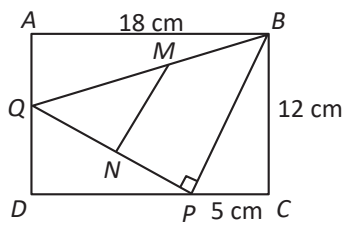
$55 + 55 + \angle QPR = 180$

$\angle QPR = 70°$

$\angle QPR = \angle QTR = 70°$ (angle formed by 2 chords meeting at circumference will be same at any point on circumference)

10. (b)

Explanation:



$\angle BCP = 90°$

$AB = CD = 18$cm

$PC = 18 - 13 = 5$cm

$BP^2 = BC^2 + PC^2 = 12^2 + 5^2 = 144 + 25 = 169$

$BP = 13$ cm

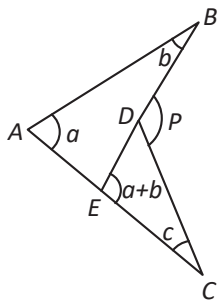
From triangle PQB

N and M are midpoints of QP and QB

Therefore, $MN = \frac{1}{2} \times BP = \frac{1}{2} \times 13 = 6.5$ cm

11. (d)

Explanation:



Extending the line BD

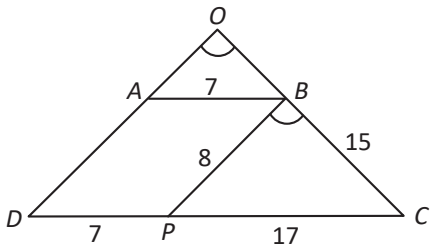
$\angle DEC = \angle a + \angle b$ ($\because$ An exterior angle of a triangle is equal to the sum of its two opposite non – adjacent interior angles.)

$\angle BDC = \angle DEC + \angle DCE$

$P = a + b + c$

12. (d)

Explanation:



Let P be a point on DC such that $AD \| BP$.

$DC = 24$cm

$PC = 24 - 7 = 17$cm

From triangle BPC

$17^2 = 8^2 + 15^2$

$289 = 64 + 225$

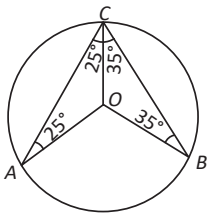
$289 = 289$

i.e., BPC is a right – angled triangle, and $\angle PBC = 90°$.

$\angle PBC = \angle DOC = 90°$. ($OD$ and BP are parallel lines cut by a transverse line OC)

13. (d)

Explanation:



In triangle OAC

$OA = OC$ (radius)

Therefore $\angle OAC = \angle OCA = 25°$

In triangle OBC

$OC = OB$ (radius)

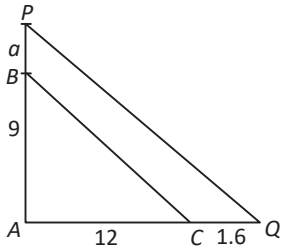
$\angle OBC = \angle OCB = 35°$

$\angle ACB = 25 + 35 = 60°$

$\angle AOB = 2\angle ACB = 2 \times 60 = 120°$

14. (b)

Explanation:



Let AB be height of pole = 9m

AC = length of shadow of pole = 12m

Let $BP = a$ = height of the boy

CQ = length of shadow of boy = 1.6m

From triangles ABC and APQ we have

$$\frac{AB}{AC} = \frac{AP}{AQ}$$

$$\frac{9}{12} = \frac{9+a}{12+1.6} = \frac{9+a}{13.6}$$

$122.4 = 12a + 108$

$12a = 122.4 - 108 = 14.4$

$a = 14.4/12 = 1.2$m

Area and Perimeter

18

INTRODUCTION

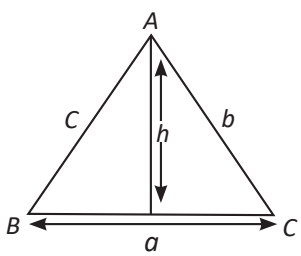
The concept of **area** refers to the measure of space taken up by a flat surface or shape. It is usually measured in square units, such as square meters (m^2), square centimeters (cm^2), or square inches (in^2). Understanding area is essential for tasks ranging from calculating the paint needed to cover a wall, to the planning and development of land, and even to the design of sports fields and gardens.

The **perimeter** measures the length around a shape or the boundary of a space. It is expressed in linear units such as meters (m), centimetres (cm), or inches (in). Knowing how to calculate the perimeter is vital for activities such as fencing , framing a picture, or even creating the layout for a track or athletic field.

AREA AND PERIMETER FOR DIFFERENT SHAPES

TRIANGLE

Consider the triangle below:



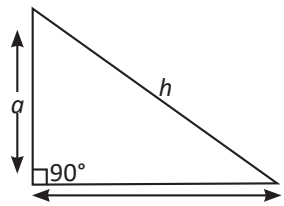
Formulae given below are valid for all types of triangles.

- Area = $\frac{1}{2}$ (base × height) = $\frac{1}{2}$ ah
- A = $\sqrt{s(s-a)(s-b)(s-c)}$

  where s = $\frac{1}{2}$ $(a + b + c)$ or semi-perimeter of the triangle
- Perimeter (P) = $a + b + c = 2s$

This formula is known as Heron's Formula

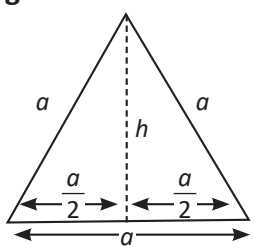
Right Angled Triangle



In a right angled triangle, by Pythagoras theorem (Hypotenuse)^2 = sum of squares of sides i.e. $h^2 = a^2 + b^2$

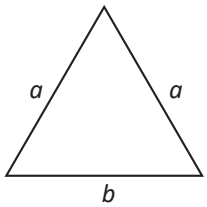
- Area = $\frac{1}{2}$ (base × height)

Equilateral Triangle



- Area (A) of an equilateral triangle = $\frac{\sqrt{3}}{4}$ (side)^2 = $\frac{\sqrt{3}}{4}$ a^2
- Perimeter (P) of an equilateral triangle
  = 3 × (side) = $3a$
- Altitude (h) of an equilateral triangle
  = $\frac{\sqrt{3}}{2}$ × (side) = $\frac{\sqrt{3}}{2}$ a
- Area (A) of an equilateral triangle = $\frac{(\text{altitude})^2}{\sqrt{3}}$

Isosceles Triangle



- Area (A) of an isosceles triangle = $\frac{b}{4}\sqrt{4a^2 - b^2}$
- Perimeter (P) of an isosceles triangle = $(2a + b)$
- Height (h) of an isosceles triangle = $\frac{1}{2}\sqrt{4a^2 - b^2}$

EXAMPLES

E1: What is the perimeter (in cm) of an equilateral triangle whose height is 3.46 cm? (Take $\sqrt{3}$ = 1.73).

(a) 12 (b) 9

(c) 6 (d) 10.4

Ans: (a)

Explanation:

Height of an equilateral triangle = $\frac{\sqrt{3}}{2}$ ×(side)

$\Rightarrow$ 3.46 = (1.73/2) × (side)

$\Rightarrow$ Side = (2 x 3.46)/ 1.73 = 4

Therefore, Perimeter = 3(Side) = 3(4) = 12 cm.

E2: The difference between the two perpendicular sides of a right-angled triangle is 17 cm and its area is 84 cm². What is the perimeter (in cm) of the triangle?

(a) 65 (b) 49

(c) 72 (d) 56

Ans: (d)

Explanation:

Let the two perpendicular sides of right angled triangle be called base (b) and height (*h*).

According to question, $b - h = 17$ ...(i)

and, Area = $\frac{1}{2}$ $bh = 84$

$\Rightarrow bh = 168$ ...(ii)

On solving (i) and (ii), we get b = 24 and *h* =7.

(Hypotenuse)² = sum of squares of sides

$\Rightarrow$ (Hypotenuse)² = $24^2 + 7^2 = 625$

$\Rightarrow$ Hypotenuse = 25.

Perimeter of Triangle = Sum of sides = 7 + 24 + 25 = 56

E3: The sum of three sides of an isosceles triangle is 20 cm, and the ratio of an equal side to the base is 3 : 4, The altitude of the triangle is:

(a) $3\sqrt{3}$ cm (b) $4\sqrt{5}$ cm

(c) $3\sqrt{5}$ cm (d) $2\sqrt{5}$ cm

Ans: (d)

Explanation:

Ratio of equal side to base = 3 : 4

Let the sides be 3*x*, 3*x*, 4*x*

Sum of three sides = 10*x* = 20 cm

$\Rightarrow x$ = 2 cm

Sides of triangle are: 6 cm, 6 cm, 8 cm

Height (h) of an isosceles Δ = $\frac{1}{2}\sqrt{4a^2 - b^2}$ where a is equal side and b is base

$\Rightarrow h = \frac{1}{2}\sqrt{4\times6^2 - 8^2} = \frac{\sqrt{80}}{2} = \frac{4\sqrt{5}}{2} = 2\sqrt{5}$ cm

E4: Imagine you are a landscape designer tasked with creating a beautiful triangular garden in a city park. The city officials have specified that the garden should have a triangular shape to maximize aesthetics. The dimensions of the triangular garden are as follows:

- The length of the first side is 52 meters.
- The length of the second side is 56 meters.
- The length of the third side is 60 meters.

What is the area of the park?

(a) 1300 m2 (b) 1344 m2

(c) 1384 m2 (d) 1396 m2

Ans: (b)

Explanation:

Let *a* = 52 m, *b* = 56 *m* and *c* = 60 m.

Perimeter of the triangle = (*a* + *b* + *c*) units

= (52 + 56 + 60) m = 168 m

$\therefore s = \frac{1}{2}(a+b+c) = \frac{1}{2}\times168 = 84$ m

(*s* – *a*) = (84 – 52) m = 32 m,

(s – *b*) = (84 – 56) m = 28 m

and (*s* – *c*) = (84 – 60) m = 24 m

By Heron's formula, the area of the given triangle, *A* = $A = \sqrt{s(s-a)(s-b)(s-c)}$

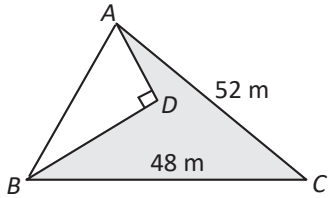
= $\sqrt{84\times32\times28\times24}$ m²

= $\sqrt{14\times6\times16\times2\times14\times2\times6\times4}$ m²

= (14 × 6 × 4 × 2 × 2) m² = 1344 m².

E5: You are an architect working on designing a modern building with a unique triangular courtyard in the center. The triangular courtyard has an additional smaller triangular feature within it, creating a shaded region. In the smaller triangle *AD* = 12 m and *BD* = 16m.

What is the area of the shaded region?



(a) 480 m2 (b) 96 m2

(c) 384 m2 (d) 576 m2

Ans: (c)

Explanation:

By Pythagoras theorem, in ΔADB

$AB = \sqrt{AD^2 + BD^2}$

$= \sqrt{12^2 + 16^2} = \sqrt{144 + 256} = \sqrt{400} = 20$

$AB = 20$ m.

$\therefore$ area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{1}{2}(20 + 48 + 52) = 60$

Area $= \sqrt{60(60-20)(60-48)(60-52)}$

$= \sqrt{60 \times 40 \times 12 \times 8}$

$= \sqrt{(12 \times 5) \times (5 \times 8) \times 12 \times 8}$

$= 5 \times 8 \times 12 = 480$ m2.

also area of $\Delta ADB = \frac{1}{2}(AD)(BD)$

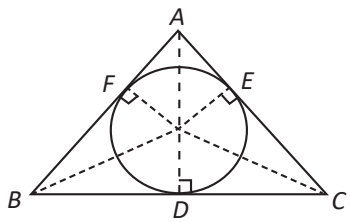
$= \frac{1}{2} \times 12 \times 16 = 96\ m^2$

$\therefore$ Shaded area $= ar(\Delta ABC) - ar(\Delta ADB)$

$= 480 - 96 = 384\ m^2$

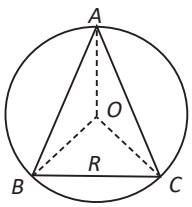
AREA OF INCIRCLE AND CIRCUMCIRCLE OF A TRIANGLE:

The incircle of a triangle is the largest circle that can fit inside the triangle, touching all three sides. This circle is often referred to as the "inscribed circle" or the "incircle." The center of the incircle is called the incentre (I), and the radius of the incircle is called the inradius (r).



- **Area of incircle of a triangle = r.s**, where r is the radius of the incircle and s is the half of the perimeter of the triangle.

The circumcircle of a triangle, also known as the circumscribed circle, is the unique circle that passes through all three vertices of the triangle. The center of this circle is called the circumcenter (O), and the radius is known as the circumradius (R).



- **Area of circumcircle of a triangle = $\frac{abc}{4R}$**, where R is the radius of the circumcircle and a, b, c are the length of sides of the triangle.

EXAMPLES

E6: In a triangle ABC, the ratio of sides is $a : b : c = 4 : 5 : 6$. The ratio of the radius of the circumcircle to that of the incircle is

(a) $\frac{16}{9}$ (b) $\frac{16}{7}$

(c) $\frac{11}{7}$ (d) $\frac{7}{16}$

Ans: (b)

Explanation:

We have $R = \frac{abc}{4\Delta}$ and $r = \frac{\Delta}{s}$

$\Rightarrow \frac{R}{r} = \frac{abc}{4\Delta} \times \frac{s}{\Delta} = \frac{abc}{4(s-a)(s-b)(s-c)}$

Since $a : b : c = 4 : 5 : 6$

$\frac{a}{4} = \frac{b}{5} = \frac{c}{6} = k$ (say)

$\Rightarrow a = 4k$, $b = 5k$ and $c = 6k$

$\Rightarrow s = \frac{1}{2}(4k + 5k + 6k) = \frac{15k}{2}$

Thus $\frac{R}{r} = \frac{(4k)(5k)(6k)}{4\left(\frac{15k}{2} - 4k\right)\left(\frac{15k}{2} - 5k\right)\left(\frac{15k}{2} - 6k\right)} = \frac{16}{7}$

E7: If the sides of the triangle are 5K, 6K, 5K and radius of incircle is 6 then value of K is equal to

(a) 4 (b) 5

(c) 6 (d) 7

Ans: (a)

Explanation:

Let the sides of triangle be: a= 5K, b= 6K and c= 5K

$s = \frac{1}{2}(a + b + c) = \frac{5K + 6K + 5K}{2} = 8K$

$r = \frac{\Delta}{s} = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$

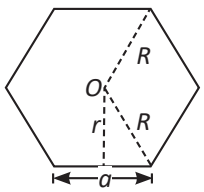
$r = \sqrt{\frac{(8K - 5K)(8K - 6K)(8K - 5K)}{8K}}$

$r = \frac{3K}{2} \Rightarrow K = \frac{2r}{3} = \frac{2 \times 6}{3} = 4$

REGULAR POLYGON

A regular polygon is one in which all sides are of equal length, and all angles are equal.

- Perimeter (P) = $n \times a$, where n = number of sides and a = length of each side



Let 'r' be the radius of a circle drawn inside the polygon touching its sides (incircle).

Let 'R' be the radius of a circle drawn outside the polygon passing through its vertices (circumcircle).

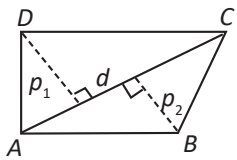
- Area = $\frac{1}{2} \times P \times r = \frac{1}{2} \times n \times a \times r$

  $= \frac{1}{2} \times n \times a \times \sqrt{R^2 - \left(\frac{a}{2}\right)^2}$

- Area of a regular hexagon = $\frac{3\sqrt{3}}{2}$ $(\text{side})^2$
- Area of a regular octagon $= 2(\sqrt{2} + 1)\ (\text{side})^2$

QUADRILATERAL

Consider the quadrilateral below:

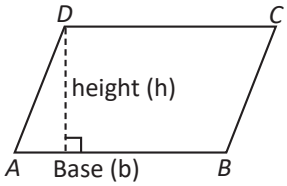


- Area (A) of a quadrilateral = Area of ΔADC + Area of ΔABC

  $= \frac{1}{2} \times$ one diagonal $\times$ (sum of perpendiculars to it from opposite vertices)

  $= \frac{1}{2}\ d\ (p_1 + p_2)$, where d is length of diagonal

PARALLELOGRAM



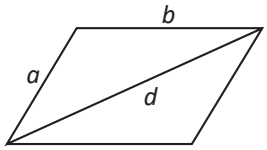
- Area (A) of a parallelogram

  = (base) $\times$ (altitude corresponding to the base)

  = b $\times$ h

- Area (A) of a parallelogram

  $= 2\sqrt{s(s-a)(s-b)(s-d)}$ where a and b are adjacent sides, d is the length of the diagonal connecting the ends of the two sides (a and b) and $s = \frac{a+b+d}{2}$



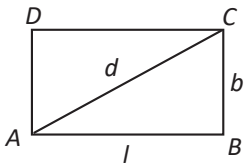
- In a parallelogram, the sum of the squares of the diagonals

  = 2 (the sum of the squares of the two adjacent sides), i.e. $d_1^2 + d_2^2 = 2(a^2 + b^2)$

- Perimeter (P) of a parallelogram

  = 2 $(a + b)$, where a and b are adjacent sides of the parallelogram

RECTANGLE



- Area (A) of rectangle = length $\times$ breadth = $l \times b$

  = $(l \times \sqrt{d^2 - l^2})$ if one side (l) and diagonal (d) are given

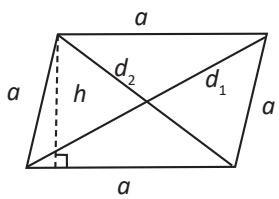
- Perimeter (P) of rectangle

  = 2 (length + breadth) = 2 (l + b)

  = $2(l + \sqrt{d^2 - l^2})$, if one side (l) and diagonal (d) are given

- Diagonal of a rectangle = $\sqrt{l^2 + b^2}$

RHOMBUS



- Area (A) of a rhombus = $a \times h$ (i.e. base $\times$ height)
- Area (a) of a rhombus = $\frac{1}{2}$ $\times$ product of its diagonals

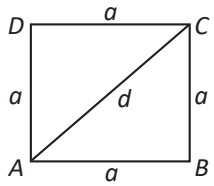
  = $\frac{1}{2}(d_1 \times d_2)$

- Perimeter (P) of a rhombus = 4 $\times$ (side) = 4a
- Perimeter (P) of a rhombus

  = $2\sqrt{d_1^2 + d_2^2}$, where d_1 and d_2 are two diagonals

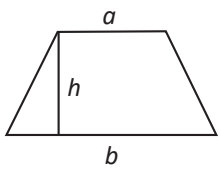
- Side (a) of a rhombus = $\frac{1}{2} \times \sqrt{d_1^2 + d_2^2}$

SQUARE



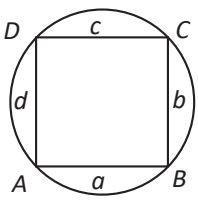
- Area (A) of a square = $(\text{side})^2 = a^2$
- Area (A) of a square = $\frac{(\text{Diagonal})^2}{2} = \frac{d^2}{2}$
- Area (A) of a square = $\frac{(\text{Perimeter})^2}{16} = \frac{P^2}{16}$
- Perimeter (P) of a square = 4 × (side) = 4a
- Perimeter (P) of a square
  $= \sqrt{16 \times area} = 2\sqrt{2} \times (\text{diagonal})$
- Length (d) of the diagonal of a square
  $= \sqrt{2} \times \text{Side} = \sqrt{2}a$
- Length (d) of the diagonal of a square
  $= \sqrt{2 \times \text{area}} = \frac{\text{Perimeter}}{2\sqrt{2}}$

TRAPEZIUM (TRAPEZOID)



- Area (a) of a trapezium
  $= \frac{1}{2} \times$ (sum of parallel sides) × (perpendicular distance between the parallel sides)
  $= \frac{1}{2} \times (a+b) \times h$

CYCLIC QUADRILATERAL



- Area of Cyclic quadrilateral
  $= \sqrt{s(s-a)(s-b)(s-c)(s-d)}$, where s = $\frac{a+b+c+d}{2}$
- **Ptolemy's Theorem:** In cyclic quadrilateral the product of the diagonals equals the sum of the products of opposite sides i.e.
  $AC \times BD = (AB \times CD) + (AD \times BC)$

EXAMPLES

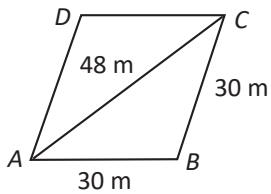
E8: A rhombus field has a side length of 30 meters, and its longer diagonal is 48 meters. If the field is divided into smaller sections for 18 goats to graze, how much area does each goat get?

(a) 54 sq. m (b) 864 sq. m

(c) 1296 sq. m (d) 48 sq. m

Ans: (d)

Explanation:



$s = (30 + 30 + 48)/2 = (108)/2 = 54$ m

area of rhombus = $2(\Delta ABC)$

$= 2\sqrt{s(s-a)(s-b)(s-c)}$

$= 2\sqrt{54(54-30)(54-30)(54-58)}$

$= 2\sqrt{9 \times 6 \times 24 \times 24 \times 6}$

$= 2 \times 3 \times 6 \times 24$

= 864 sq. m

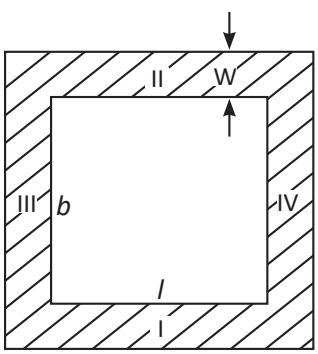
$\therefore$ area for 1 goat to graze

$= \frac{864}{18} = 48$ sq. m

PATHS AROUND RECTANGLE AND A SQUARE

Pathway outside a Rectangle Space

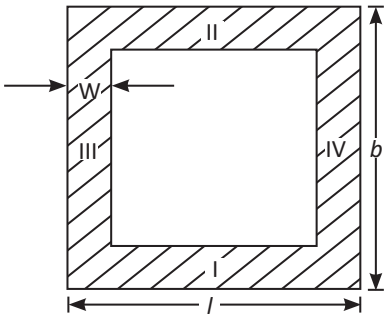
Suppose a rectangular garden 'l' m long and 'b' m broad is surrounded by a path 'w' m wide.



- Length of outer rectangle = l + 2w
- Breadth of outer rectangle = b + 2w
- Area of Path = Area of Outer rectangle – Area of Inner rectangle = (l + 2*w*)(b + 2*w*) – (lb) = 2 w (*l* + b + 2w) sq. m
- Perimeter of Path = Perimeter of inner rectangle + Perimeter of outer rectangle
  = 2(l + b) + 2[(l +2w) + (b + 2w)]
  = 4(l + b + 2w) m

Pathway Inside a Rectangle Space

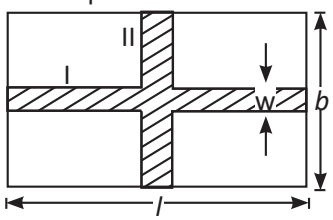
Suppose a rectangular garden 'l' m long and 'b' m broad is surrounded by a path 'w' m wide constructed inside it along its boundary.



- Length of Inner rectangle = l – 2w
- Length of Outer rectangle = b – 2w
- Area of Path = Area of Outer rectangle – Area of Inner rectangle
  = (*lb*) – (*l* – 2*w*)(*b* – 2*w*) = 2*w* (*l* + b – 2*w*) sq. m.
- Perimeter of Path = Perimeter of Outer rectangle + Perimeter of inner rectangle
  = 2(*l* + *b*) + 2[(*l* – 2*w*) + (*b* – 2*w*)]
  = 4(*l* + *b* – 2*w*) *m*.

Pathways Running Across The Middle of a Rectangular Space:

Consider a rectangular park that is '*l*' m long and 'b' m broad. Two paths 'w' m wide each are perpendicular to each other inside the park.



- Area of the path
  = (Area of 2 rectangular paths) – (Common Square of both paths)
  = (l.w + b.w) – (w.w)
  = w (*l* + b – w) sq. m.
- Clearly, from the figure, the area of the paths does not change on shifting their locations as long as they are perpendicular to each other.

EXAMPLES

E9: In a bustling urban neighborhood, a square park with each side measuring 18 meters is getting a makeover. The city planners have decided to enhance the park's aesthetics by laying a pavement 3 meters wide around its inside perimeter. Calculate the total area of the pavement that will be laid around the inside of the park.

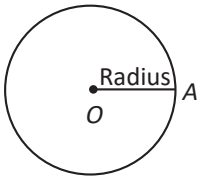
(a) 42 sq. m (b) 126 sq. m
(c) 252 sq. m (d) 180 sq. m

Ans: (d)

Explanation:

Area of the pavement inside the park
= 2w (*l* + b – 2w)
= 2 × 3 (18 + 18 – 2 × 3)
(Here, *l* = *b* = 18) = 180 sq m.

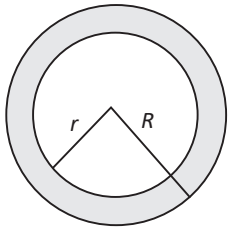
CIRCLES



If r is the radius of a circle, then

- Perimeter or Circumference = 2π*r* or π*d*, where *d* = 2*r* is the diameter of the circle.
- Area = πr^2 or $\frac{\pi d^2}{4}$
- Area of semi-circle = $\frac{\pi r^2}{2}$
- Area of a quadrant of a circle = $\frac{\pi r^2}{4}$

Area enclosed by Two Concentric Circles



If R and r are radii of two con-centric circles, then:

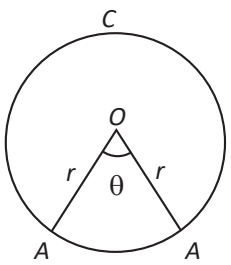
- Area enclosed by the two circles (Area of ring)
  = Area of outer circle – Area of inner circle
  = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2) = \pi(R + r)(R - r)$

Note: Rotations made by a wheel

- Distance moved by a rotating wheel in one revolution is equal to the circumference of the wheel.
- The number of revolutions completed by a rotating wheel in one minute

$$= \frac{\text{Distance moved in one minute}}{\text{Circumference}}$$

Sector of a Circle



- If the arc subtends an angle of θ at the centre, then its arc length (l)

$$= \frac{\theta}{360} \times 2\pi r = \frac{\theta}{180} \times \pi r$$

- If the arc subtends an angle θ, then area of the corresponding sector *A*

$$= \frac{\theta}{360} \times \pi r^2$$

Note:

Facts about clock

- Angle of rotation by minute hand in 60 minutes = 360°
- Angle of rotation by minute hand in one minute = 360/60 = 6°
- Angle of rotation by hour hand in 12 hours = 360°
- Angle of rotation by hour hand in one hour = 360/12 = 30°

EXAMPLES

E10: The wheels of a car have a diameter of 80 cm. If the car is moving at a speed of 66 km/h, how many full rotations do each wheel complete in a span of 10 minutes?

(a) 4350 (b) 4375

(c) 4400 (d) 4425

Ans: (b)

Explanation:

Speed of the car = 66 km/hr

- Distance travelled by the car in 1 hour = 66 km
- Distance travelled by the car in 10 min.

$= \left(\frac{66}{60} \times 10\right)$ km = 11 km = 11× 1000× 100 cm

We have, Radius of car wheels = 40 cm

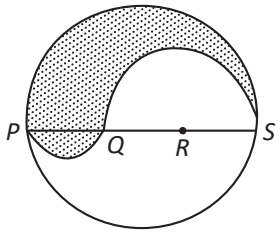
- Circumference of the wheels = $2 \times \frac{22}{7} \times 40$ cm
- Distance travelled by the car when its wheels take one complete revolution = Circumference of wheel
- Number of revolutions made by the wheels in 10 minutes

$$= \frac{\text{Distance covered by the car in 10 minutes}}{\text{Distance covered by the car when its wheels make one complete revolution}}$$

$$= \frac{11 \times 1000 \times 100}{2 \times \frac{22}{7} \times 40} = 4375$$

Hence, each wheel makes 4375 revolutions in 10 min

E11: Consider the diameter *PQRS* of a circle with a radius of 6 cm. Each of the segments *PQ*, *QR*, and *RS* are of equal length. Two semicircles are constructed, one on *PQ* and the other on *QS*, both using their respective diameters. If the perimeter of the shaded region is kπ cm, then value of *k* is:



(a) 8 (b) 10

(c) 12 (d) 14

Ans: (c)

Explanation:

PS = Diameter of a circle of radius 6 cm = 12 cm

$\therefore$ *PQ* = *QR* = *RS* = 12/3 = 4 cm

QS = *QR* + *RS* = (4 + 4) cm = 8 cm

Hence, required perimeter

= Arc of semi-circle of radius 6 cm

+ Arc of semi-circle of radius 4 cm

+ Arc of semi-circle of radius 2 cm

= $(\pi \times 6 + \pi \times 4 + \pi \times 2)$ cm = 12π cm

E12. Consider the following statements in respect of a rectangular sheet of length 20 cm and breadth 8 cm:

1. **It is possible to cut the sheet exactly into 4 square sheets.**
2. **It is possible to cut the sheet into 10 triangular sheets of equal area.**

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

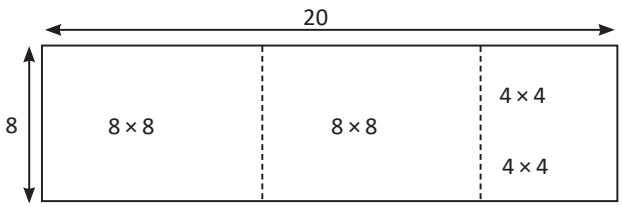
(c) Both 1 and 2 (d) Neither 1 nor 2

Ans: (c)

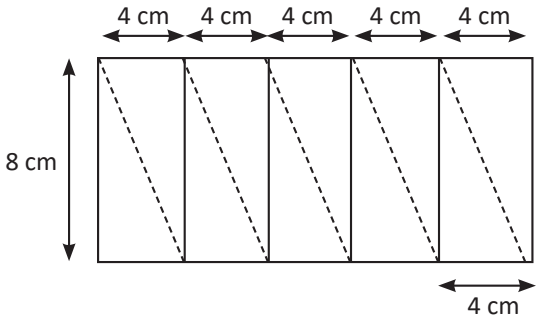
Explanation:

It is given that the rectangle is of dimensions 20 cm × 8 cm.

Statement-I: We need to cut the given rectangle into 4 squares(of any area, not identical)



Statement II: The given rectangle can be cut into 10 triangles of equal area as shown below.



So, both statements 1 and 2 are correct.

E13. There are eight equidistant points on a circle. How many right-angled triangles can be drawn using these points as vertices and taking the diameter as one side of the triangle? **(UPSC CSAT 2022)**

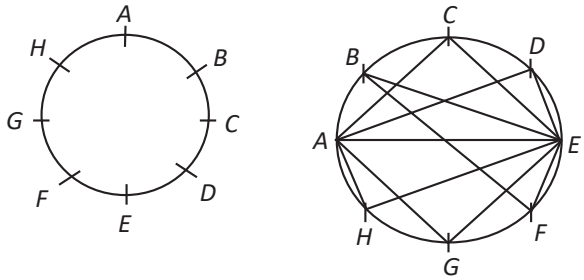
(a) 24 (b) 16

(c) 12 (d) 8

Ans: (a)

Explanation:

In the following figure are drawn eight equidistant points on a circle - A, B, C, D, E, F, G, and H.



When we consider AE as the diameter and one side of the right-angled triangle, we can draw 6 right-angled triangles.

Similarly, if we consider BF as the diameter and one side of the right-angled triangle, we can draw 6 right angled triangles. The same can be done when we consider CG and DH as the diameter and one side of the right-angled triangle. Hence, the total number of right-angled triangles that can be drawn = 6 + 6 + 6 + 6 = 24.

E14. AB is a vertical trunk of a huge tree with A being the point where the base of the trunk touches the ground. Due to a cyclone the trunk has been broken at C which is at a height of 12 metres, broken part is partially attached to the vertical portion of the trunk at C. If the end of the broken part B touches the ground at D which is at a distance of 5 metres from A, then the original height of the trunk is

(UPSC CSAT 2016)

(a) 20 m (b) 25 m

(c) 30 m (d) 35 m

Ans: (b)

Explanation:

Pythagoras theorem $CD^2 = AC^2 + AD^2$

$= 122 + 52 \Rightarrow 144 + 25 = 169$

$CD^2 = 169 \Rightarrow CD = 169 = 13$

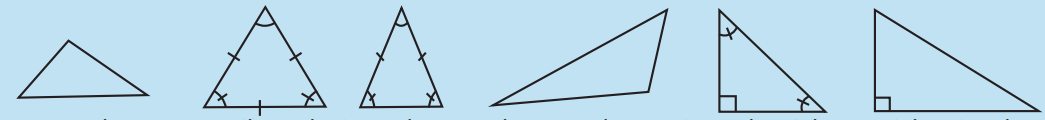
Decimal height = $AC + CB$

= 12 + 13 = 25 meters.

Hence, the correct answer is (b).

Formulae Summary

- Triangle and its properties:



acute scalene triangle | equilateral triangle | isosceles triangle | obtuse scalene triangle | isosceles right triangle | right triangle

- Sum of three angles of triangle = 180°
- Three types of triangle based on angle–
  - **Acute** – All angles less than 90°
  - **Right** – One angle equal to 90° and $P^2 + B^2 = H^2$
  - **Obtuse** – One angle greater than 90°
- Three types of triangle based on sides
  - **Scalene triangle** – no sides are equal
  - **Isosceles triangle** - two sides are equal
  - **Equilateral triangle** – all sides are equal
- Sum of two sides is always greater than third side in a triangle.
- Difference of two sides is always less than third side in a triangle.
- **Sum of interior angles of a triangle. = 180°**
- Exterior angle is always equal to sum of two opposite angles in a triangle.
- Perimeter of triangle = Sum of sides
- Semi - perimeter, s = 1/2 of perimeter
- Area of triangle using Heron's Formula = $\sqrt{(s)(s-a)(s-b)(s-c)}$,s = 1/2 of perimeter
- Area of Triangle = 1/2× base × height
- Important Pythagorean Triplets:
  - Numbers that follow $P^2 + B^2 = H^2$
  - [3,4,5], [5, 12, 13], [6, 8, 10], [7, 24, 25], [9, 40, 41], [20, 21, 29]
- **Quadrilateral**
  - Total number of sides = 4
  - Sum of interior angles = 360°
  - 5 major types of Quadrilateral

  Trapezium
  - Two sides are parallel and two sides are non-parallel
  - Area = $\frac{1}{2}$ × sum of parallel sides × distance between parallel sides = $\frac{1}{2} \times (a + b) \times h$
  - Perimeter = $a + b + c + d$

  Parallelogram
  - Opposite sides are equal and parallel
  - Opposite angles are equal
  - Perimeter = $2(a + b)$
  - Area = Base × Perpendicular Height
  - $(D1)^2 + (D2)^2 = 2\ (a^2 + b^2)$
  - Where $D1$ = Diagonal 1 and D2 = Diagonal 2

Rectangle

- Opposite sides are equal, parallel and all angles are of 90°
- Perimeter = $2(l + b)$
- Area = lb
- Diagonal $= \sqrt{l^2 + b^2}$
- Diagonals are equal
- Where l= Length and b = Breadth of rectangle

Square

- All sides are equal
- All angles are of 90°
- Perimeter = 4a
- Diagonal = $\sqrt{2}\,a$
- Area = a^2

Rhombus

- All sides are equal
- Opposite angles are equal
- Perimeter = $4a$
- Area = $\frac{1}{2}$ × product of diagonals

Circle

- Radius = r = Diameter/2
- Circumference = $2 \times \pi \times r$
- Area = $\pi \times r^2$
- Area of Arc = $(\theta/360) \times \pi \times r^2$

Solid (3D) Figures

Cube

- All sides are equal with length a
- Base of Cube is a square
- Volume of cube = a^3
- Length of Diagonal = $\sqrt{3}\,a$
- Lateral surface area = Perimeter of base × Height = $4a^2$
- Total surface area = Lateral Surface Area + 2 × Base Area = $6a^2$
- Open area = Lateral Surface Area + Base Area = $5a^2$

Cuboid

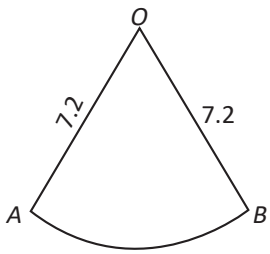
- Base of Cuboid is a rectangle
- Volume = Length × Breadth × Height
- Let Length = l, Breadth = b, Height = h, then
- Length of Diagonal $= \sqrt{l^2 + b^2 + h^2}$
- Lateral Surface Area = $2(l + b)h$
- Open area = $2(l + b)h + lb$
- Total Surface Area = $2(l + b)h + lb = 2(lb + bh + lh)$.

Cylinder

- Base of cylinder is a circle with radius r and having height h
- Volume = $\pi r^2 h$
- Curved Surface Area = $2\pi rh$
- Total Surface Area = Curved Surface Area + 2 ×Base area = $2\pi rh + 2\pi r^2 = 2\pi r\,(h + r)$

PRACTICE QUESTIONS

1. **The perimeter of sector *AOB* = 32.4 cm and Radius = 7.2 cm. Find the area of the sector *AOB*.**

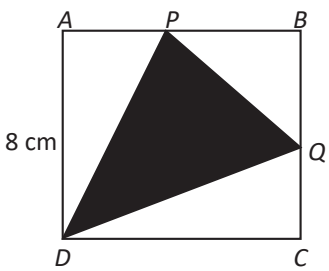


(a) 44.8 cm^2 (b) 52.6 cm^2
(c) 64.8 cm^2 (d) 48.6 cm^2

2. ***ABCD* is a square of side 4cm. *PQRS* are the midpoints of AB, BC, CD, DA respectively. *PR* and *QS* intersect at point O, find the ratios of the area of triangle *AOB* to the area of triangle *ABC*.**

(a) 3:4 (b) 1:2
(c) 2:3 (d) 3:8

3. ***ABCD* is a square of side 8cm and *P* & *Q* are the midpoints of *AB* and *BC* respectively. Find the area of the triangle *PQD*.**

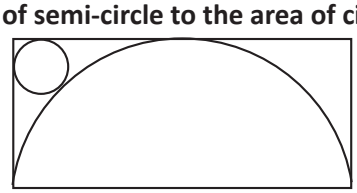


(a) 44 cm^2 (b) 36 cm^2
(c) 24 cm^2 (d) 34.6 cm^2

4. ***AB* is a chord of a circle of radius 28cm. The chord subtends a right-angle triangle at the centre of the circle. What is the area of the minor segment formed?**

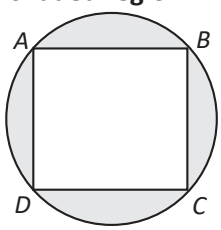
(a) 224 cm^2 (b) 180 cm^2
(c) 200 cm^2 (d) 256 cm^2

5. **A semi-circle and a circle are inscribed in a rectangle as shown in the figure below. What is the ratio of the area of semi-circle to the area of circle.**



(a) $1:2(\sqrt{2}-1)^4$ (b) $1:(\sqrt{2}+1)^2$
(c) $(\sqrt{2}+1)^4:1$ (d) $2(\sqrt{2}-1)^2:1$

6. **A square of side 14 cm in inscribed in a circle touching its circumference as shown in the figure below. Find the area of the shaded region.**



(a) 122 sq cm (b) 130 sq cm
(c) 112 sq cm (d) 146 sq cm

7. **The area of two similar triangles *ABC* and *DEF* are 144 cm^2 and 81 cm^2 respectively. If the longest side of the larger triangle *ABC* is 24 cm, then what is the length of the longest side of the smaller triangle *DEF*?**

(a) 18 cm (b) 15 cm
(c) 12 cm (d) 11 cm

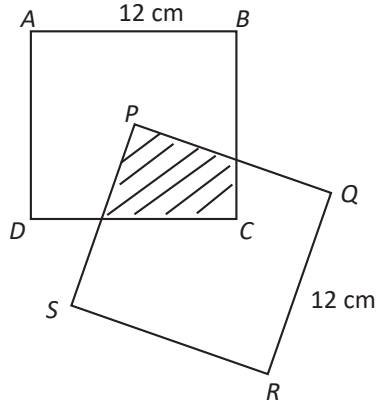
8. **The cost of fencing a circular plot at the rate of Rs. 20 per metre is Rs. 5280. What will be the cost of flooring the plot at the rate of Rs. 25 per square metre?**

(a) Rs. 285060 (b) Rs. 138600
(c) Rs. 358650 (d) Rs. 218730

9. **A rectangular garden 70m long and 65m broad has a path 4m wide inside it. Find the cost of tiling the path at Rs. 21.50 per square meter.**

(a) Rs. 19321 (b) Rs. 22301
(c) Rs. 21100 (d) Rs. 21844

10. **In the figure given below *ABCD* and *PQRS* are squares each having a side of 12cm. Find the area of the shaded region. Where P is at the centre of Square *ABCD*.**



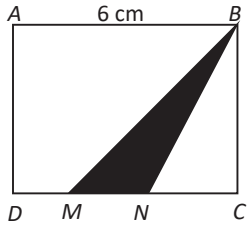
(a) 24 cm^2 (b) 30 cm^2
(c) 36 cm^2 (d) 48 cm^2

11. The area of Rhombus having one side 17cm and one diagonal of 30cm is?

(a) 240 cm^2 (b) 300 cm^2

(c) 280 cm^2 (d) 260 cm^2

12. *ABCD* is a square of side 6cm and *DM* = *MN* = *NC*. Find the area of the unshaded part.



(a) 24 cm^2 (b) 30 cm^2

(c) 28 cm^2 (d) 26 cm^2

13. The length of a rectangular plot of ground is three times its breadth and its area is 30000 square kilometers. How long will it take a person to walk around it at the rate of 4 km/hr?

(a) 12 min (b) 20 min

(c) 18 min (d) 15 min

14. How many times will a wheel of diameter 98 cm rotate to cover a distance of 308 m?

(a) 100 (b) 107

(c) 114 (d) 96

15. The perimeter of a square is equal to twice the perimeter of a rectangle of length 9 cm and breadth 6 cm. What is the circumference of a semicircle whose diameter is equal to the side of the square?

(a) 38.57 cm (b) 25.29 cm

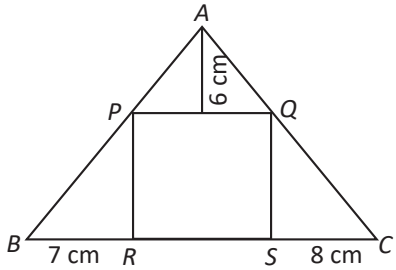
(c) 33.42 cm (d) None of the above

16. The side of a square exceeds the side of the another square by 4 cm and the sum of the areas of the two squares is 400 cm^2. What is the side of the smaller square?

(a) 8 cm (b) 16 cm

(c) 12 cm (d) 10 cm

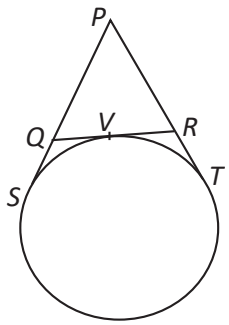
17. Find the area of square *PQSR*.



(a) 64 cm^2 (b) 49 cm^2

(c) 81 cm^2 (d) 90 cm^2

18. If PS = 15cm, then find the perimeter of the triangle *PQR*



(a) 15 cm (b) 12.5 cm

(c) 30 cm (d) 45 cm

ANSWERS

1. (*c*)	2. (*b*)	3. (*c*)	4. (*a*)	5. (*a*)	6. (*c*)	7. (*a*)	8. (*b*)	9. (*d*)	10. (*c*)
11. (*a*)	12. (*b*)	13. (*a*)	14. (*a*)	15. (*a*)	16. (*c*)	17. (*d*)	18. (*c*)		

EXPLANATIONS

1. (c)

Explanation:

Perimeter of the sector = 32.4cm and radius = 7.2 cm

Length of the arc *AB* = 32.4 – 7.2 – 7.2 = 18cm

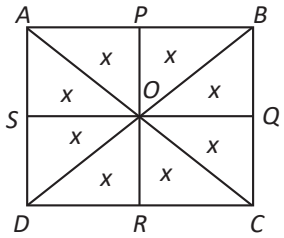
Area of the sector = $\frac{lr}{2}$

Where l = length of the arc and r is radius

Area of the sector = $\frac{18 * 7.2}{2} = 64.8\,cm^2$

2. (b)

Explanation:



Here areas of all the small triangle will be same. Let the area of each small triangle be x cm^2.

Ratio of area of AOB to area of ABC = $2x : 4x = 1 : 2$

3. (c)

Explanation:

AB = 8cm

$AP = PB = BQ = QC$ = 4cm

$$\text{Area of } APD = \frac{1}{2} * 4 * 8 = 16\,\text{cm}^2$$

$$\text{Area of } BPQ = \frac{1}{2} * 4 * 4 = 8\,\text{cm}^2$$

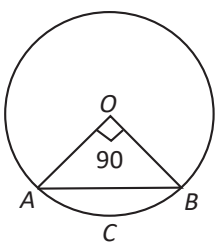
$$\text{Area of } DQC = \frac{1}{2} * 4 * 8 = 16\,\text{cm}^2$$

Area of $ABCD$ = 8 × 8 = 64 cm^2

Shaded area = 64 – 16 – 8 – 16 = 24 cm^2

4. (a)

Explanation:



Area of the sector $AOBC$ = $\frac{90}{360} * \pi * r^2$ (θ = 90º)

= $\frac{90}{360} * \pi * 28^2$ = 616 cm^2

Area of the triangle AOB = (28×28)/2

= 392 cm^2

Area of the segment ACB = Area of the sector AOBC – Area of the triangle AOB

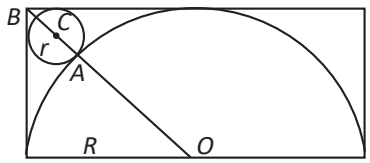
= 616 – 392 = 224 cm^2

5. (a)

Explanation:

Let the radius of the semi-circle and the smaller circle be R and r units respectively.

Area of the semi-circle = $\frac{\pi R^2}{2}$; Area of the circle = πr^2



$OB = \sqrt{2}\,R$

$AB = AC + BC \Rightarrow AB = r + \sqrt{2}\,r$

$OB = AO + AB \Rightarrow \sqrt{2}\,R = R + r + \sqrt{2}\,r$

$\Rightarrow r(\sqrt{2} + 1) = R(\sqrt{2} - 1)$

$$\Rightarrow r = \frac{\sqrt{2}-1}{\sqrt{2}+1}\,R = (\sqrt{2}-1)^2 R$$

$$\text{Required ratio} = \frac{(\pi R^2/2)}{\pi r^2} = \frac{R^2}{2(\sqrt{2}-1)^4 R^2} = 1 : 2(\sqrt{2}-1)^4$$

6. (c)

Explanation:

Side of the square = 14 cm

Diameter of the circle = Diagonal of the square $= 14\sqrt{2}$

Radius of the circle $= 14\sqrt{2}/2 = 7\sqrt{2}$

Area of the shaded region = Area of the circle – Area of the square

$= \pi r^2 - \text{side}^2 = \frac{22}{7} * 7\sqrt{2} \times 7\sqrt{2} - 14^2 = 308 - 196$

= 112 sq cm

7. (a)

Explanation:

Let the length of the longest side of the smaller triangle be x cm

For similar triangles, (Ratio of corresponding sides)^2 = Ratio of areas

$$\Rightarrow \left(\frac{24}{x}\right)^2 = \frac{144}{81} \Rightarrow x = 24 * \frac{9}{12} = 18\,\text{cm}$$

8. (b)

Explanation:

Let r be the radius of the plot.

Circumference of the plot

= 5280/20 = 264 m = $2 \times r \times (\frac{22}{7})$

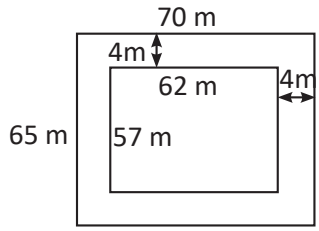
$$\Rightarrow r = \frac{264 * 7}{2 * 22} = 42 \text{ m}$$

Area of the circle = (22/7) × 42 × 42 = 5544 sq m

Cost of flooring = 5544 × 25 = 138600

9. (d)

Explanation:



Dimensions of the garden = 70m × 65m

The width of the path is given as 4m.

Thus, dimensions of the inner rectangle formed = 62m × 57m

Area of the path = 70 × 65 – 62 × 57

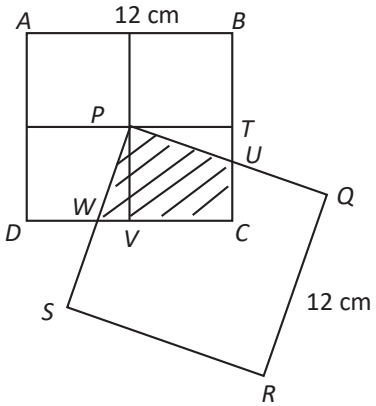
= 4550 – 3534 = 1016 sq m

Cost of tiling the path = (21.50) × 1016

= Rs 21844

10. (c)

Explanation:



$\angle PTU = 90^\circ$

let $\angle PUT = a$, $\angle TPU = (90 - a)$

$\angle PVW = 90^\circ$

$PT = PV$

We have, $\Delta PTU \cong \Delta PVW$

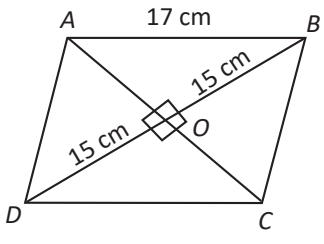
Area $PUCW$ = Area $PTCV$

Area $PTCV$ = (Area $ABCD$)/4

= (12 × 12)/4 = 36cm^2

11. (a)

Explanation:



The sides of Rhombus will be equal.

Let the 2 diagonals meet at point 'O'.

The diagonals of the Rhombus cut each other at right angles and divide each other in two halves.

$\angle AOB = 90^\circ$

$17^2 = 15^2 + OA^2$

$289 = 225 + OA^2$

$OA^2 = 289 - 225 = 64$

$OA = 8$

$OA = OC = 8$cm

$AC = 8 + 8 = 16$ cm

Area of rhombus $= \frac{1}{2} d_1 d_2$

Where d_1 and d_2 are diagonals

Area of rhombus $= \frac{1}{2} \times 30 \times 16 = 240 \text{ cm}^2$

12. (b)

Explanation:

$AB = BC = CD = DA = 6$cm

$DM = MN = NC = 6/3 = 2$cm

Area of triangle $MBN = \frac{1}{2} * MN * BC$

$= \frac{1}{2} * 2 * 6 = 6 \text{ cm}^2$

Area of square $ABCD = 6 \times 6 = 36\text{cm}^2$

Areas of unshaded part = 36 – 6 = 30cm^2

13. (a)

Explanation:

Let the breadth of a rectangle is x m. Then, the length will be $3x$.

According to the question, $3x \times x = 3 \times 10000$ m$^2 \Rightarrow x = 100$ m

Thus, length = 300m, Breadth = 100m

Perimeter = 2(300 + 100) = 800 m = 0.8 km

Time taken to cover 800m = Distance/speed = 0.8/4 = 0.2 hr = 0.2×60 = 12 min

14. (a)

Explanation:

Circumference of the wheel = 2× 22/7 ×49 = 308 cm

Distance covered in 1 revolution = 308 cm = 3.08 m

Number of revolutions required to cover 308m = 308/3.08 = 100

15. (a)

Explanation:

Perimeter of a rectangle = 2 (9 + 6) = 30 cm

Perimeter of a square = 2×30 = 60 cm

Side of the square = 60/4 = 15 cm

Radius of the semicircle, r = side of square/2 = 15/2 cm

Circumference of the semicircle

$= r\times\frac{22}{7} + 2\times r = \frac{15}{2} \times \frac{22}{7} + 2\times\frac{15}{2} = 38.57$ cm

16. (c)

Explanation:

Let side of square = x cm

Side of another square = $(x + 4)$ cm

$x^2 + (x + 4)^2 = 400 \Rightarrow 2(x^2 + 4x - 192) = 0$

$\Rightarrow (x + 16)(x - 12) = 0 \Rightarrow x = -16$ or 12

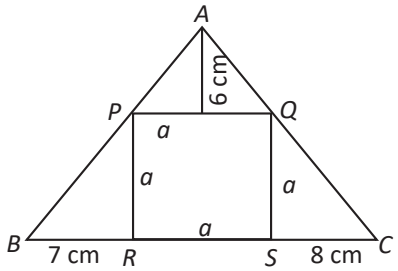
Length can not be negative. ⇒ Side of smaller square = 12 cm

Side of another square = 16 cm

17. (d)

Explanation:

Since $PQSR$ is square (given). Let the side of the square be 'a' cm



Area of $PQSR = a^2$

$\Delta ABC \sim \Delta APQ$

Therefore

$\frac{6}{a} = \frac{6+a}{7+a+8}$

$42 + 6a + 48 = 6a + a^2$

$a^2 = 90\text{cm}^2$

18. (c)

Explanation:

$PS(PQ + QS) = PT(PR + RT) = 15$cm (tangents)

$QS = QV$ and $VR = RT$ (tangents)

Perimeter of triangle $PQR = PQ + QV + VR + PR$

Replacing QV with QS and VR with RT

Perimeter of triangle $PQR = PQ + QS + RT + PR$

$= 15 + 15 = 30$cm

Surface Area and Volume

19

INTRODUCTION

The chapter on Surface Area and Volume is an essential component of geometry, focusing on calculating the surface area and volume of three-dimensional shapes such as cubes, cylinders, spheres, and cones. These concepts are crucial for practical applications in various fields, including engineering, architecture, and science. Understanding how to determine the space an object occupies (volume) and the area that covers its surface (surface area) is fundamental for problem-solving in real-life scenarios.

SOLIDS

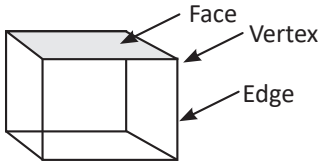
In geometry and mathematics, a solid is a three-dimensional object that occupies space. Unlike two-dimensional shapes that only have length and width, solids have a third dimension, which is typically referred to as height or depth. Here are some key characteristics of solids:

Faces, Edges, and Vertices

Faces: These are the flat surfaces of the solid. A face is a two-dimensional shape that bounds the solid. For example, a cube has six faces.

Edges: Edges are the line segments where two faces meet. They form the boundaries of the faces. A cube has 12 edges.

Vertices: Vertices are the points where three or more edges meet. A cube has eight vertices.



Volume

The volume of a solid is the amount of space it occupies. It is measured in cubic units.

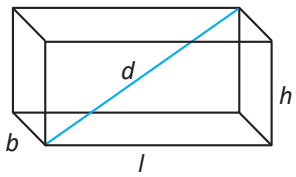
Surface Area

The surface area of a solid is the total area of its faces. It is measured in square units.

Examples of solids include cubes, cuboids, spheres, cylinders, cones, pyramids, prisms, and more. Each type of solid has unique characteristics and formulas for calculating volume and surface area.

CUBOID

A cuboid, also known as a rectangular prism, is a 3D solid that has six rectangular faces, twelve edges, and eight vertices. All the angles in a cuboid are right angles, and opposite faces are congruent (having the same size and shape). A match box, a brick, a book, etc., are all examples of a cuboid.



If l, b and h denote the length, breadth and height of the cuboid and d denotes the body diagonal, then:

- Body Diagonal of cuboid,

  $d = \sqrt{l^2 + b^2 + h^2}$

- Volume = $l \times b \times h = \sqrt{A_1 \times A_2 \times A_3}$

  where, A_1 = area of base or top; A_2 = area of one side face, and A_3 = area of other side face

- Total Surface Area = 2 (lb + bh + lh)

  = $(l + b + h)^2 - d^2$

- Area of four walls of a room = 2 (l + b) h

EXAMPLES

E1. The length (l), breadth (b) and height (h) of a cuboid are in the ratio 6 : 4 : 5. If the total surface area of the cuboid is 2368 cm2 ; then $l + b + h$ is

(a) 60 cm (b) 70 cm

(c) 80 cm (d) 90 cm

Ans: (a)

Explanation:

Let length (l) = 6k cm, breadth (b) = 4k cm and height (h) = 5k cm,

Total surface area $= 2(l \times b + b \times h + h \times l)$

$= 2(6k \times 4k + 4k \times 5k + 5k \times 6k)\text{cm}^2$

$= 2(24k^2 + 20k^2 + 30k^2)\ \text{cm}^2$

$=148k^2\ \text{cm}^2$

Given : Total surface = 2368 cm^2

$\therefore\ 148\ k^2 = 2368$

$\Rightarrow k^2 = \dfrac{2368}{148} = 16$

and $k = \sqrt{16} = 4$

$\therefore$ length (l) = $6k$ cm = 6×4 cm = 24 cm,

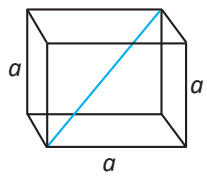
Breadth (b) = $4k$ cm = 4×4 cm = 16 cm and

Height (h) = $5k$ cm = 5×4 cm = 20 cm

$\therefore\ l + b + h = 24 + 16 + 20$ cm = 60 cm

CUBE

A cube is a solid that is characterized by having six square faces, twelve edges, and eight vertices. All angles in a cube are right angles, and all the faces are congruent (identical and of the same size). The cube is a special case of a cuboid (rectangular prism) where all sides are equal in length.



If 'a' be the edge of a cube, then

- Body diagonal of the cube, $d = \sqrt{3}a$
- Volume of the cube = $(\text{edge})^3 = a^3$

$$= \left(\frac{\text{diagonal}}{\sqrt{3}}\right)^3 = \left(\frac{d}{\sqrt{3}}\right)^3 = \left(\sqrt{\frac{\text{Surface Area}}{6}}\right)^3$$

- Total surface area of the cube = $6\ (\text{edge})^2 = 6a^2$
- For two cubes
  - (a) Ratio of volumes = $(\text{ratio of sides})^3$
  - (b) Ratio of surface areas = $(\text{ratio of sides})^2$
  - (c) $(\text{Ratio of surface areas})^3 = (\text{ratio of volumes})^2$

EXAMPLES

E2. A cube of side length of 12 cm is cut into 8 identical small cubes. Then which of the following is correct?

(a) Volume of each small cube = 216 cm^3

(b) Side of each small cube = 6 cm

(c) Surface area of each small cube = 216 cm^2

(d) All of the above are correct

Ans: (d)

Explanation:

Since, side (edge) of the given solid cube = 12 cm.

$\therefore$ Volume of given solid cube = $(\text{edge})^3$

$= (12\ \text{cm})^3 = 1728\ \text{cm}^3$ Ans.

As the given cube is cut into 8 identical cubes.

$\Rightarrow$ Vol. of 8 small cubes obtained = Vol. of given cube = 1728 cm^3

$\Rightarrow$ Vol. of each small cube $= \dfrac{1728}{8} = 216\ \text{cm}^3$

If edge (side) of each small cube = x cm

$(\text{edge})^3$ = Volume

$\Rightarrow x3 = 216 = 6 \times 6 \times 6 = 6^3 \Rightarrow x = 6$ cm

$\therefore$ Side of each small cube = 6 cm

Surface area of each small cube

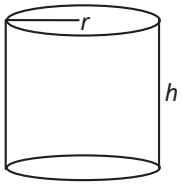
$= 6 \times (\text{edge})^2$

$= 6 \times (6\ \text{cm})^2 = 216\ \text{cm}^2$

RIGHT CIRCULAR CYLINDER

A right circular cylinder is characterized by its circular base and its height, which is perpendicular to the base. The term "right" indicates that the axis (height) of the cylinder is perpendicular to the center of the circular base.

Rotating a rectangle around one of its sides can create a cylindrical shape.



If 'r' is the radius of base and 'h' is the height of the cylinder

- Volume of cylinder
  = Area of the base × height
  $= \pi r^2 \times h = \pi r^2 h$
- Area of the curved surface
  = Circumference of the base × height
  $= 2\pi r \times h = 2\pi rh$

- Area of the total surface
  = Area of the curved surface + Area of the two circular ends
  $= 2\pi rh + 2\pi r^2 = 2\pi r\,(h + r)$
- For two cylinders
  When radii are equal: Ratio of volumes
  = Radii of curved surface areas = Ratio of height
  When heights are equal
  (a) Ratio of volumes =
  (Ratio of curved surface areas)^2 = (Ratio of radii)^2
  (b) Ratio of curved surface areas = Ratio of radii

EXAMPLES

E3. The ratio of radii of two right circular cylinders is 3 : 4. Also, the ratio of their height is 6 : 5. Then the ratio of their curved surface areas is :

(a) 3 : 4 (b) 6 : 5
(c) 1 : 1 (d) 9 : 10

Ans: (d)

Explanation:

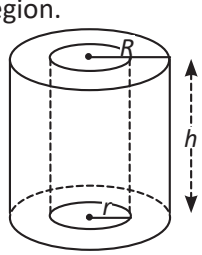
If the radii of two cylinders be r_1 and r_2,
let $r_1 = 3x$ and $r_2 = 4x$.
Similarly, if the heights of two cylinders be h_1 and h_2, let $h_1 = 6y$ and $h_2 = 5y$.
Ratio between their C.S.A.

$$= \frac{2\pi r_1 h_1}{2\pi r_2 h_2} = \frac{2\pi \times 3x \times 6y}{2\pi \times 4x \times 5y} = \frac{9}{10}$$

= 9 : 10

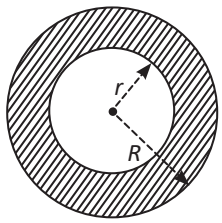
HOLLOW RIGHT CIRCULAR CYLINDER

A hollow right circular cylinder resembles a regular right circular cylinder but with a tubular or hollow interior. It consists of an outer cylinder and an inner cylinder having the same axis, the inner cylinder is removed to create a tubular or hollow region.



Let external radius = R, Internal radius = r and height = h

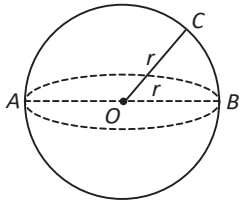
- Outer curved surface area = $2\pi Rh$
- Inner curved surface area = $2\pi rh$
- Total curved surface area = Outer curved surface area + Inner curved surface area
  $= 2\pi Rh + 2\pi rh = 2\pi h(R + r)$
- Area of cross-section = $\pi R^2 - \pi r^2$
  $= \pi(R^2 - r^2)$



- Total surface area = C.S.A. of hollow cylinder + Area of 2 circular end rings.
  $= 2\pi h(R + r) + 2\pi(R^2 - r^2)$
- Volume = Volume of outer cylinder – Volume of inner cylinder
  $= \pi R^2 h - \pi r^2 h = \pi(R^2 - r^2)h$

SPHERE

A sphere is perfectly round and symmetrical in all directions. It is often defined as the set of all points in space that are equidistant from a given point called the center. The distance from the center to any point on the surface of the sphere is the radius.



If r = radius of the sphere, then

- Volume of sphere = $\frac{4}{3}\pi r^3$
- Surface area = $4\pi r^2$
- For two spheres
  (a) (Ratio of radii)^2 = Ratio of surface areas
  (b) (Ratio of radii)^3 = Ratio of volumes

EXAMPLES

E4. A right circular cylinder has been designed to perfectly fit a sphere inside it. If the radius of sphere is 'r', then the ratio of surface area of sphere to curved surface area of cylinder is:

(a) 1:2 (b) 2:1
(c) 1:1 (d) 1:3

Ans: (c)

Explanation:

Surface area of the sphere = $4\pi r^2$

According to the condition,

the height of the cylinder = diameter of the sphere

$\Rightarrow h = 2r$

$\therefore$ C.S.A. of the cylinder

$= 2\pi rh = 2\pi r \times 2r = 4\pi r^2$

Required ratio = $\frac{4\pi r^2}{4\pi r^2}$ = 1 Hence, 1 : 1

E5. A spherical ball of iron of diameter 3cm is melted and recast into three smaller spherical balls of different diameters. The diameters of two of these balls are 1 cm and 1.5 cm. Then the diameter of the third ball is:

(a) $\sqrt[3]{\frac{181}{64}}$ cm (b) $\frac{\sqrt[3]{181}}{2}$ cm

(c) $\frac{181}{64}$ cm (d) $\frac{724}{192}$ cm

Ans: (b)

Explanation:

The diameter of a spherical ball = 3cm

$\Rightarrow$ radius of it = 1.5 cm = $\frac{3}{2}$ cm

So, volume of it $=[4/3\times\pi\times(3/2)^3$ cm^3

$= \frac{108\pi}{24}$ cm^3

$= \frac{9\pi}{2}$ cm^3

This spherical ball is melted and recast into three small spherical balls. The diameters of two of these are 1 cm and 1.5 cm respectively.

So, volume of the two spherical balls.

$$= \left[\frac{4}{3}\pi\times\left\{\left(\frac{1}{2}\right)^3+\left(\frac{3}{4}\right)^3\right\}\right] cm^3$$

$$= \left[\frac{4}{3}\pi\left(\frac{1}{8}+\frac{27}{64}\right)\right] cm^3$$

$$= \left(\frac{4}{3}\pi\times\frac{35}{64}\right) cm^3 = \frac{140\pi}{192} cm^3$$

Let r be the radius of the third small spherical ball.

Thus, volume of the third ball = volume of the big spherical ball – sum of volume of two small spherical balls.

$$\Rightarrow \frac{4}{3}\pi r^3 = \frac{9\pi}{2} - \frac{140\pi}{192}$$

$$\Rightarrow \frac{4}{3}r^3 = \frac{9}{2} - \frac{140}{192} = \frac{864-140}{192} = \frac{724}{192}$$

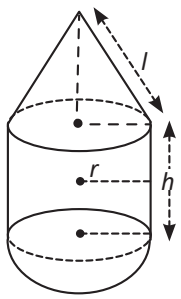
$$\Rightarrow r^3 = \frac{724\times3}{4\times192} = \frac{181}{64}$$

$$\Rightarrow r = \sqrt[3]{\frac{181}{64}} \text{ cm}$$

Hence, diameter of the 3rd spherical ball = 2r

$$= 2\times\frac{\sqrt[3]{181}}{\sqrt[3]{64}} = 2\times\frac{\sqrt[3]{181}}{4} = \frac{\sqrt[3]{181}}{2} \text{ cm}$$

E6. Consider the given figure, a combination of hemispherical base, cylindrical middle portion and conical upper end. The total surface area of combination of these solids is (for given dimensions):



(a) $\pi r(2r + l)$ (b) $2\pi r(2r + h)$

(c) $\pi r(2r + 2h + l)$ (d) $\pi r(2r + \sqrt{h^2 + r^2})$

Ans: (c)

Explanation:

Required surface area

= C.S.A. of the hemisphere

+ C.S.A. of the cylinder + C.S.A. of the cone

$= 2\pi r^2 + 2\pi rh + \pi r\ell = \pi r(2r + 2h + \ell)$

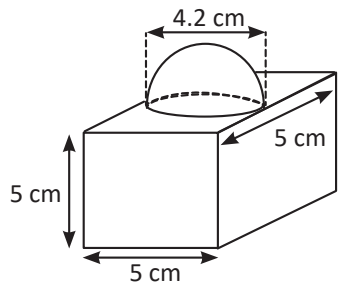
HEMISPHERE

A hemisphere is a 3D object that is half of a sphere. It is formed by cutting a sphere along a plane passing through its center.

- Volume of hemisphere $= \frac{2}{3}\pi r^3$
- Area of curved surface = $2\pi r^2$
- Total surface area of hemisphere = $3\pi r^2$

EXAMPLES

E7. A miniature model of an architectural dome is constructed with a cubical base and a hemispherical top. The edge of the base is 5 cm and the hemispherical top has a diameter of 4.2 cm. Then the total surface area of the model is:



(a) 160 cm^2 (b) 167.46 cm^2

(c) 180 cm^2 (d) 163.86 cm^2

Ans: (d)

Explanation:

The total surface area of the cube

= 6 × $(edge)^2$

= 6 × 5 × 5 cm^2 = 150 cm^2.

Note that the part of the cube where the hemisphere is attached is not included in the surface area.

So, the surface area of the block

= TSA of cube – base area of hemisphere + CSA of hemisphere

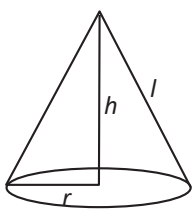
= 150 – $\pi r^2 + 2\pi r^2 = (150 + \pi r^2)$ cm^2

$= 150\ cm^2 + \frac{22}{7} \times \frac{4.2}{2} \times \frac{4.2}{2}\ cm^2$

= (150 + 13.86) cm^2 = 163.86 cm^2

RIGHT CIRCULAR CONE

A right circular cone is characterized by a circular base and a pointed top, or apex, or vertex, directly above the center of the base. The axis of the cone is the line segment connecting the apex to the center of the circular base.



If r = radius of base, h= height, l = slant height

- l = slant height = $\sqrt{h^2 + r^2}$
- Volume of cone

$= \frac{1}{3} \times$ area of the base × height

$= \frac{1}{3} \times \pi r^2 h$

- Area of curved surface = $\pi r l$

$= \pi r \sqrt{h^2 + r^2}$

- Total surface area of cone = Area of the base + area of the curved surface

$= \pi r^2 + \pi r l = \pi r(r + l)$

- For two cones
  - (a) When radii are equal then Ratio of volumes = Ratio of heights
  - (b) When heights are equal then Ratio of volumes = (ratio of radii)^2

EXAMPLES

E8. A circus tent is in the form of a cylinder surmounted by a cone. The total height of the tent above the ground is 13m. The height of the cylindrical portion is 8m and radius is 28m. The radius of the conical part is the same as that of the cylindrical part. Find the cost of painting the inner part at the rate of Rs. 10 per m^2.

(a) 39000 (b) 39050

(c) 39072 (d) 39100

Ans: (c)

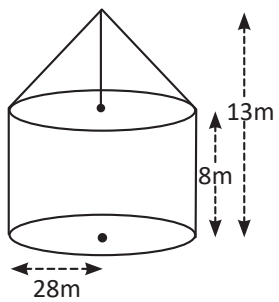
Explanation:

According to the given statement, the rough sketch of the circus tent will be as shown:

For the cylindrical portion : r = 28 and h = 8 m

∴ Curved surface area = $2\pi rh$

$= 2 \times \frac{22}{7} \times 28 \times 8 m^2 = 1408 m^2$



For conical portion : r = 28 m and h = 13 – 8 = 5 m

∴ $\ell^2 = h^2 + r^2 \Rightarrow \ell^2 = 5^2 + 28^2 = 809$

$\Rightarrow \ell = \sqrt{809}\ m$ = 28.4 m

∴ Curved surface area = $\pi r \ell$

$= \frac{22}{7} \times 28 \times 28.4\ m^2 = 2499.2\ m^2$

$\therefore$ Total inner curved surface area of the tent.

= C.S.A. of cylindrical portion + C.S.A. of the conical portion

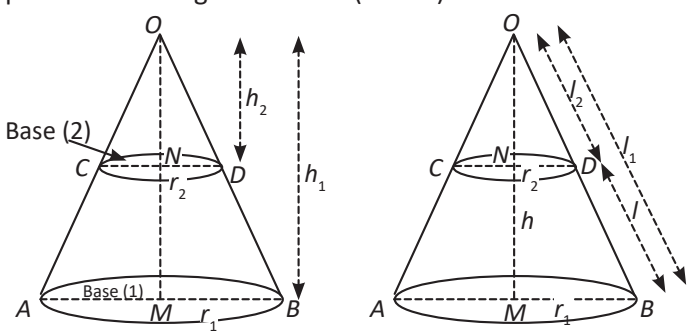
= 1408 m^2 + 2499.2 m^2 = 3907.2 m^2

Cost of painting the inner surface

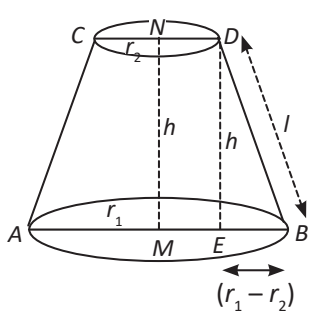
= 3907.2 × Rs. 10 = Rs. 39072

FRUSTUM OF A CONE

A frustum is a three-dimensional geometric shape that results from cutting a cone into two parts with a plane parallel to its base. The portion that remains after the cut is called the frustum. The frustum of a cone has two parallel and congruent bases (circles).



- Height of the frustum (h)
  = (height of the cone OAB) – (height of the cone OCD)
  = $h_1 - h_2$
- Slant height of the frustum = (slant height of the cone OAB) - (slant height of then cone OCD)
  $l = l_1 - l_2$
- r_2 and r_2 be the radii of the two bases ($r_1 > r_2$)

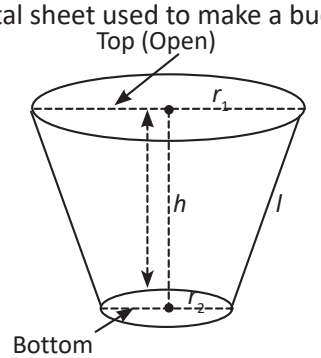


- Slant height of the frustum $l = l_1 - l_2 = \sqrt{h^2 + (r_1 - r_2)^2}$
- Volume $V = \frac{1}{3}\pi h\{r_1^2 + r_1 r_2 + r_2^2\}$

$= \frac{h}{3}(\pi r_1^2 + \pi r_2^2 + \pi r_1 r_2) = \frac{h}{3}\{\pi r_1^2 + \pi r_2^2 + \sqrt{(\pi r_1^2)(\pi r_2^2)}\}$

$= \frac{h}{3}\{A_1 + A_2 + \sqrt{A_1 A_2}\}$, where A_1 and A_2 denote the areas of circular bases of the frustum

- Curved surface area = $\pi (r_1 + r_2) l$
- Total surface area = (Curved surface area) + (Area of two circular ends)
  $= \pi l(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$
- Height of the cone of which the frustum is a part $= \frac{hr_1}{r_1 - r_2}$
- Slant height of the cone of which the frustum is a part $= \frac{lr_1}{r_1 - r_2}$
- Area of the metal sheet used to make a bucket



The area of the metal sheet used for making the bucket = Outer (or inner) curved surface area + Area of bottom

$= \pi \times l \times (r_1 + r_2) + \pi r_2^2$

EXAMPLES

E9. A bucket is in the shape of a frustum of a right circular cone. The diameter of the circular ends is 8 m at the top and 4 m at the bottom. It is 6 m deep. Then its volume is:

(a) 160 m^3 (b) 176 m^3

(c) 180 m^3 (d) 196 m^3

Ans: (b)

Explanation:

Here, Radius of base, R = 4; radius of top, r = 2 and height, h = 6

Volume of the frustum $= \frac{\pi h}{3}(R^2 + r^2 + Rr)$

$= \frac{22 \times 6}{7 \times 3}(4^2 + 2^2 + 4 \times 2)$

$= \frac{44}{7}(16 + 4 + 8) = 176\,m^3.$

Formulae Summary

1. Cuboid:
   - Surface Area (SA): The total area of all six faces. $SA = 2(lw + lh + wh)$
   - Volume: The space occupied by the cuboid. Volume= $l \times w \times h$
2. Cube:
   - Surface Area: $SA = 6a^2$
   - Volume: Volume = a^3
3. Cylinder:
   - Lateral Surface Area (LSA): The area of the curved surface. $LSA = 2\pi rh$
   - Total Surface Area: $TSA = LSA + 2\pi r^2 = 2\pi rh + 2\pi r^2$
   - Volume: Volume = $\pi r^2 h$
4. Sphere:
   - Surface Area: SA = $4\pi r^2$
   - Volume: Volume= $\frac{4}{3} \times \pi r^3$
5. Hemisphere:
   - Lateral Surface Area: LSA = $2\pi r^2$
   - Total Surface Area: TSA = $3\pi r^2$
   - Volume: Volume = $\frac{2}{3} \times \pi r^3$
6. Cone:
   - Lateral Surface Area: LSA = πrl (where l is the slant height)
   - Total Surface Area: TSA = LSA + $\pi r^2 = \pi rl + \pi r^2$
   - Volume: Volume = $\frac{1}{3} \times \pi r^2 h$ (h is the height of the cone)

PRACTICE QUESTIONS

1. **A cylindrical overhead tank of radius $\sqrt{6}$ m and height 7 m is to be filled from an underground tank of size 6.5m x 5m x 5.5m. How much portion of the underground tank is still filled with water after filling the overhead tank completely?**

   (a) $\frac{31}{43}$ (b) $\frac{23}{28}$

   (c) $\frac{11}{17}$ (d) $\frac{17}{65}$

2. **A piece of tin is in the form of a rectangle having length 18 cm and width 12 cm. This is used to construct a closed cube. The side of the cube is:**

   (a) 8 cm (b) 4 cm

   (c) 6 cm (d) 5 cm

3. **A rectangular water tank measures 15m × 10m at the top and is 40 m deep. It is completely filled with water. If the water is drawn out lowering the level by 5m, then how much water has been drawn out?**

   (a) 750, 000 liters (b) 650, 000 liters.

   (c) 550, 000 liters (d) 850, 000 liters

4. **A rectangular tank of dimension 12m × 10m × 8m is to be built using bricks to make the outer dimension 12.4m × 10.4m × 8.4m approximately. How many bricks of the size 20cm × 10cm × 5cm are required to build the tank?**

   (a) 123000 (b) 123264

   (c) 150000 (d) 120000

5. **The length of the cuboid is increased by 10%, its breadth is kept unchanged. By what percentage should the height be decreased so that the volume of the cuboid remains unchanged.**

   (a) 300/11% (b) 200/11%

   (c) 100/11% (d) 35%

6. **Find the number of bricks, each measuring 25 cm × 12 cm × 5 cm, required to construct a wall 15 m long, 6m high and 0.25 m thick, while the sand and cement mixture occupies 5% of the total volume of the wall.**

   (a) 13650 (b) 12800

   (c) 15425 (d) 14250

7. **The ratio of surface areas of a cube and a sphere is 3 : 2. What would be the ratio of their volumes?**

   (a) $(3/4)\sqrt{\pi}$ (b) $\sqrt{\pi}$

   (c) $2\sqrt{\pi}$ (d) π

8. **If the three sides of a cube are increased by 30%, 20%, and 10%, respectively, to transform it into a cuboid, what is the percentage change in the surface area?**

   (a) 33.67% (b) 43.67%

   (c) 23.67% (d) 40%

9. **In a swimming pool measuring 85 m × 45 m, 125 men take a dip. If the average displacement of water by a man is 10 cubic metres, what will be the rise in water level?**

   (a) 32.68 cm (b) 25.42 cm

   (c) 50.65 cm (d) None of the above

10. **A copper sphere of diameter 30 cm is drawn into a wire of diameter 50 mm. What is the length of the wire as formed?**

    (a) 720 cm (b) 700 cm

    (c) 680 cm (d) 750 cm

11. **A drawing room is 8 m long, 7 m broad and 5 m high. Find the length of paper 2.5 m wide to cover its walls allowing 10 sq. m for doors:**

    (a) 51 m (b) 59 m

    (c) 56 m (d) 52 m

12. **A tank 5 m long and 3 m wide and 4 m deep is dug in a field 10 m long and 8 m wide. If the earth dugout is evenly spread over the field, the rise in level of the field will be:**

    (a) 17/14 m (b) 11/15 m

    (c) 13/12 m (d) 12/13 m

13. **Three equal cubes are placed adjacently in a row. Find the ratio of total surface area of the new cuboid to that of the sum of the surface areas of the three cubes?**

    (a) 8:11 (b) 12:13

    (c) 7:9 (d) 6:7

14. **Three metal cubes *A*, *B* and *C* with edges 6 cm, 8 cm and 10 cm respectively are melted together and formed into a single cube. Find the side of the resulting cube?**

    (a) 18 cm (b) 16 cm

    (c) 14 cm (d) 12 cm

15. **If the radius and the height of a cone are doubled, what is the percentage change in the volume of the cone.**

    (a) 300% (b) 400%

    (c) 700% (d) 800%

ANSWERS

1. (*d*)	2. (*c*)	3. (*a*)	4. (*b*)	5. (*c*)	6. (*d*)	7. (*a*)	8. (*b*)	9. (*a*)	10. (*a*)
11. (*c*)	12. (*d*)	13. (*c*)	14. (*d*)	15. (*c*)					

EXPLANATIONS

1. (d)

Explanation:

Water capacity of overhead tank = Volume of cylinder

$= (22/7)(\sqrt{6})^2 \times 7 = 132\ m^3$

$= 132{,}000$ litres

Water capacity of underground tank = Volume of tank

$= 6.5 \times 5 \times 5.5 = 178.75\ m^3$

$= 178750$ litres

Water left in underground tank after filling overhead tank = 178750-132000 = 46750 litres

Portion of tank still filled $\frac{46,750}{1,78,750} = \frac{17}{65}$

2. (c)

Explanation:

Let the side of a cube be s cm.

Total surface area of the cube will be equal to the area of the rectangular tin.

Thus, $6s^2 = 18 \times 12 \Rightarrow s^2 = 36$ cm

$\Rightarrow s = 6$ cm

3. (a)

Explanation:

Length (l) = 15 meters

Width (b) = 10 meters

Depth (d) = 40 meters

Change in depth = $\Delta d = 5$m

Change in volume = $l \times b \times \Delta d$

$= 15 \times 10 \times 5$

$= 750\ m^3 = 750 \times 1000$

$= 750{,}000$ liters.

($1\ m^3 = 1000$ liters)

4. (b)

Explanation:

Volume of all the bricks required (VB) = (outer volume - inner volume)

Outer volume = 12.4m × 10.4m × 8.4m ×

$= 1083.264 m^3$

Inner volume = 12m × 10m × 8m = 960 m^3

$V_B = 1083.264 - 960 = 123.264\ m^3$

Volume of one brick (v) = 20cm × 10cm × 5cm

$= 1000\ cm^3 = 10^{-3}\ m^3$

$1cm^3 = 10^{-6}\ m^3$

Number of bricks required

$= (123.264\ m^3) / (10^{-3}) = 123264$

5. (c)

Explanation:

Let l, b, h be the length, breadth and height of the cuboid respectively.

volume = l × b × h

New length = $l + l\ (10/100) = 1.1\ l$

Let the height is decreased by x %

New height = $h - (h \times (x)/100) = h(100 - x)/100$

New volume = $1.1\ l \times b \times h(100 - x)/100$

New volume = old volume.

$l \times b \times h = 1.1\ l \times b \times h\ (100 - x)/100$

x = 100/11

6. (d)

Explanation:

Length, breadth and height of the wall are 15m, 0.25m and 6m respectively.

Volume of the wall = $15 \times 6 \times 0.25 \times (100)\ cm^3$

Sand and cement mixture occupies 5% of the total volume of the wall.

So, Volume of wall occupied by brick = 95% of the volume = $0.95 \times 15 \times 6 \times 0.25 \times (100)\ cm^3$

Volume of each brick = $25 \times 12 \times 5\ cm^3$

So, number of bricks required = $(0.95 \times 15 \times 6 \times 0.25 \times (100^3) / (25 \times 12 \times 5) = 14250$

7. (a)

Explanation:

Let the side of the cube be 'a' and radius of the sphere be 'r'.

Surface area of sphere = $4\pi r^2$

Surface area of the cube = $6a^2$

Ratio of the surface areas = $6a^2/ 4\pi r^2 = 3/2$

$a / r = \sqrt{\pi}$

The volume of the cube (V1) = a^3

The volume of the sphere (V2) = $(4/3)\pi r^3$

$$\frac{V1}{V2} = \frac{a^3}{(4/3)\pi r^3}$$

Now putting the value of a/r

$$= \frac{V1}{V2} = \frac{a^3}{(4/3)\pi r^3} \quad \frac{3(\sqrt{\pi})^3}{4\pi} = \frac{3\sqrt{\pi}}{4}$$

8. (b)

Explanation:

Let 'a' be the edge of the original cube

Original surface area (S_1) = $6a^2$

Length of the cuboid formed (l) = $a + a \times \frac{30}{100} = 1.3a$

Breath of the cuboid formed (b) = $a + a \times \frac{20}{100} = 1.2a$

Height of the cuboid formed (h) = $a + a \times \frac{10}{100} = 1.1a$

Surface area of the cuboid (S_2) = $2(lb + bh + hl)$

= $2(1.3a \times 1.2a + 1.2a \times 1.1a + 1.1a \times 1.3a)$

= $8.62a^2$

$\Delta S = S_2 - S_1 = 8.62a^2 - 6a^2 = 2.62a^2$

Percentage increase in surface area

$$= \frac{\Delta S}{S_1} \times 100 = \frac{2.62a^2}{6a^2} \times 100 = 43.67\ \%$$

9. (a)

Explanation:

Average displacement of water by one man = 10 m^3

Displacement of water by 125 men = 125×10 = 1250 m^3

Let rise in water level be 'h' m

Then, 85×45×h = 1250 ⇒ h = 32.68 cm

10. (a)

Explanation:

Let the length of the wire be x cm

Volume of sphere = Volume of wire

$$\Rightarrow \frac{4}{3}\pi r^3 = \pi r^2 l$$

$$\Rightarrow \frac{4}{3} * \frac{22}{7} * 15^3 = \frac{25}{10} * \frac{25}{10} * \frac{22}{7} * x$$

$$\Rightarrow x = \frac{4*15*15*15*10*10}{3*25*25} = 720 \text{ cm}$$

11. (c)

Explanation:

Net area to be covered = Area of 4 walls – area of the door = 2(length + breadth)×height – area of the door = 2×(8 + 7)×5 – 10 = 140 sq m

Length of the paper required = 140/2.5 = 56m

12. (d)

Explanation:

Volume of the earth dug out = 5×3×4 = 60 m^3

Area of the remaining portion = 10×8 – 5×3 = 65 m^2

Let the rise in the level of the field be 'h'

Then, 65×h = 60 ⇒ h = 60/65 = 12/13 m

13. (c)

Explanation:

Let edge of cube be a units

Length of the new cuboid = 3a

Breadth of the cuboid = Height of the cuboid = a

Total surface area of the new cuboid

= $2(l \times b + b \times h + l \times h) = 2(3a^2 + a^2 + 3a^2) = 14a^2$

Sum of surface areas of 3 cubes = $3 \times 6a^2 = 18a^2$

Thus, Required ratio = $14a^2/18a^2 = 7/9$

14. (d)

Explanation:

Volumes of the cubes A, B and C = 6^3, 8^3 and 10^3 respectively

Volume of the resultant cube = Sum of volumes of cubes A, B and C = 216 + 512 + 1000 = 1728

Side of the resultant cube = $\sqrt[3]{1728}$ = 12 cm

15. (c)

Explanation:

Let the original radius be r and height be h.

Original volume $(V_1) = \frac{1}{3}\pi r^2 h$

New radius = $2r$

New height = $2h$

New Volume $(V_1) = \frac{1}{3}\pi(2r)^2(2h)$

$$\frac{V_2}{V_1} = \frac{\frac{1}{3}\pi(2r)^2(2h)}{\frac{1}{3}\pi r^2 h} = 8$$

$V_2 = 8V_1$

$\Delta V = V_2 - V_1 = 8V_1 - V_1 = 7V_1$

% increase in volume = $\frac{V}{V_1} \times 100 = \frac{7V_1}{V_1} \times 100 = 700\%$

Ratio of the surface areas = $6a^2/ 4\pi r^2 = 3/2$

$a / r = \sqrt{\pi}$

The volume of the cube (V1) = a^3

The volume of the sphere (V2) = $(4/3)\pi r^3$

$$\frac{V1}{V2} = \frac{a^3}{(4/3)\pi r^3}$$

Now putting the value of a/r

$$= \frac{V1}{V2} = \frac{a^3}{(4/3)\pi r^3} \quad \frac{3(\sqrt{\pi})^3}{4\pi} = \frac{3\sqrt{\pi}}{4}$$

8. (b)

Explanation:

Let 'a' be the edge of the original cube

Original surface area (S_1) = $6a^2$

Length of the cuboid formed (l) = $a + a \times \frac{30}{100} = 1.3a$

Breath of the cuboid formed (b) = $a + a \times \frac{20}{100} = 1.2a$

Height of the cuboid formed (h) = $a + a \times \frac{10}{100} = 1.1a$

Surface area of the cuboid (S_2) = $2(lb + bh + hl)$

= $2(1.3a \times 1.2a + 1.2a \times 1.1a + 1.1a \times 1.3a)$

= $8.62a^2$

$\Delta S = S_2 - S_1 = 8.62a^2 - 6a^2 = 2.62a^2$

Percentage increase in surface area

$$= \frac{\Delta S}{S_1} \times 100 = \frac{2.62a^2}{6a^2} \times 100 = 43.67\ \%$$

9. (a)

Explanation:

Average displacement of water by one man = 10 m^3

Displacement of water by 125 men = 125×10 = 1250 m^3

Let rise in water level be 'h' m

Then, 85×45×h = 1250 ⇒ h = 32.68 cm

10. (a)

Explanation:

Let the length of the wire be x cm

Volume of sphere = Volume of wire

$$\Rightarrow \frac{4}{3}\pi r^3 = \pi r^2 l$$

$$\Rightarrow \frac{4}{3} * \frac{22}{7} * 15^3 = \frac{25}{10} * \frac{25}{10} * \frac{22}{7} * x$$

$$\Rightarrow x = \frac{4*15*15*15*10*10}{3*25*25} = 720 \text{ cm}$$

11. (c)

Explanation:

Net area to be covered = Area of 4 walls – area of the door = 2(length + breadth)×height – area of the door = 2×(8 + 7)×5 – 10 = 140 sq m

Length of the paper required = 140/2.5 = 56m

12. (d)

Explanation:

Volume of the earth dug out = 5×3×4 = 60 m^3

Area of the remaining portion = 10×8 – 5×3 = 65 m^2

Let the rise in the level of the field be 'h'

Then, 65×h = 60 ⇒ h = 60/65 = 12/13 m

13. (c)

Explanation:

Let edge of cube be a units

Length of the new cuboid = 3a

Breadth of the cuboid = Height of the cuboid = a

Total surface area of the new cuboid

= $2(l \times b + b \times h + l \times h) = 2(3a^2 + a^2 + 3a^2) = 14a^2$

Sum of surface areas of 3 cubes = $3 \times 6a^2 = 18a^2$

Thus, Required ratio = $14a^2/18a^2 = 7/9$

14. (d)

Explanation:

Volumes of the cubes A, B and C = 6^3, 8^3 and 10^3 respectively

Volume of the resultant cube = Sum of volumes of cubes A, B and C = 216 + 512 + 1000 = 1728

Side of the resultant cube = $\sqrt[3]{1728}$ = 12 cm

15. (c)

Explanation:

Let the original radius be r and height be h.

Original volume $(V_1) = \frac{1}{3}\pi r^2 h$

New radius = $2r$

New height = $2h$

New Volume $(V_1) = \frac{1}{3}\pi(2r)^2(2h)$

$$\frac{V_2}{V_1} = \frac{\frac{1}{3}\pi(2r)^2(2h)}{\frac{1}{3}\pi r^2 h} = 8$$

$V_2 = 8V_1$

$\Delta V = V_2 - V_1 = 8V_1 - V_1 = 7V_1$

% increase in volume = $\frac{V}{V_1} \times 100 = \frac{7V_1}{V_1} \times 100 = 700\%$

Data Interpretation

20

INTRODUCTION

Data refers to a collection of raw and unprocessed facts, observations, or values, often in the form of numbers, text, or multimedia, obtained from various sources.

Data interpretation is the process of making sense of, analyzing, and deriving meaningful insights from raw data. It involves transforming data into a comprehensible and actionable form, allowing individuals to extract valuable insights, draw conclusions, make informed decisions, and identify patterns or trends.

REPRESENTATION OF DATA

Representing data effectively is crucial for communication and analysis. The choice of representation depends on the nature of the data and the message you want to convey. Here are various ways to represent data:

1. Tables:
   - Ideal for presenting structured data with rows and columns.
   - Commonly used in reports, scientific papers, and business documents.
2. Charts and Graphs:
   - Visual representations of data to highlight patterns and trends.
   - Types include:
     (i) Bar charts: Compare quantities across categories.
     (ii) Line graphs: Show trends over time.
     (iii) Pie charts: Display parts of a whole.
     (iv) Scatter plots: Visualize relationships between two variables.

In this chapter we will learn to interpret data mainly from:

Tables, Pie Charts, Bar Graphs, Line Graphs, Caselets and Mixed graphs.

GRAPHS

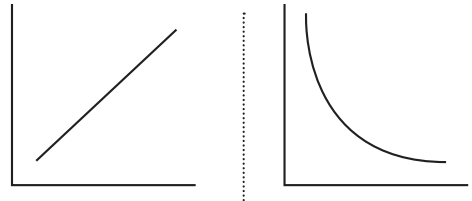
The study of graphs, in the context of data visualization and mathematics, involves understanding the properties, types, and applications of graphs. Graph theory is a branch of discrete mathematics that deals with the study of graphs.

When you're dealing with a graph of two quantities, you're essentially creating a visual representation of the relationship between those two variables. Variation in two quantities refers to how the values of one variable change in relation to changes in the values of another variable. This variation can be visualized and analyzed using different methods. Here are some common scenarios and ways to represent variation in two quantities:

1. **Direct Proportionality:** When one variable increases, the other variable also increases proportionally, or when one variable decreases, the other variable decreases proportionally.

   The relationship can be represented by a straight line passing through the origin.
2. **Inverse Proportionality:** When one variable increases, the other variable decreases proportionally, and vice versa.

   The relationship can be represented by a hyperbolic curve on a graph.

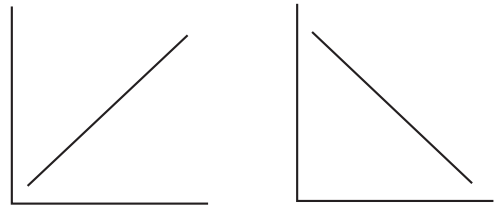


Direct proportion Inverse proportion

3. **Positive Correlation:** When an increase in one variable is associated with an increase in the other variable.

   Represented by a positively sloped line on the graph.
4. **Negative Correlation:** When an increase in one variable is associated with a decrease in the other variable.

   Represented by a negatively sloped line on a scatter plot.



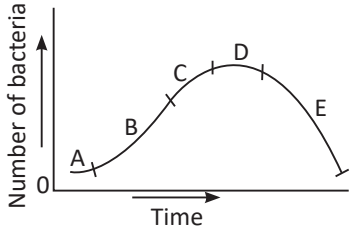
Positive Correlation Negative Correlation

QUESTIONS

Directions (Q1 to Q3) : Read the passage given below, study the graph and answer the questions that follow.

(UPSC CSAT 2011)

During a party, a person was exposed to contaminated water. A few days later, he developed a fever and loose motions. He suffered for some days before going to a doctor for treatment. On starting the treatment, he soon became better and recovered completely a few days later. The following graph shows different phases of the person's disease condition as regions A, B, C, D and E of the curve



Q1: Which region/regions of the curve correspond/ corresponds to the incubation phase of the infection?

(a) Only A

(b) Only B

(c) Only B and C

(d) No part of the curve indicates the incubation phase

Ans: (a)

Explanation:

In the incubation phase, a disease takes hold and produces symptoms (bacteria will start growing). So, region A corresponds to the incubation phase.

Q2: Which region of the curve indicates that the person began showing the symptoms of infection?

(a) A (b) B

(c) C (d) D

Ans: (b)

Explanation:

In region B, the count of the bacteria has increased so symptoms of infection would be visible.

Q3: Which region of the curve indicates that the treatment yielded effective relief?

(a) C (b) D

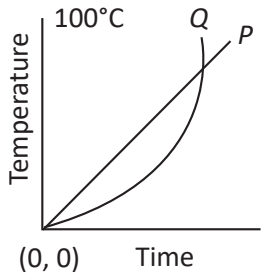
(c) E (d) None of these

Ans: (c)

Explanation:

Effective relief from treatment means bacteria count must go down; region E is indicative of this.

Q4: The variation in temperatures from 0°C to 100°C of two liquids P and Q with respect to time is shown in the graph given below.



Which one of the following statements is correct?

(a) During heating, liquid P remained hotter than liquid Q throughout.

(b) At no point of time during heating did the two liquids have the same temperature.

(c) P attained the temperature of 100°C faster than Q.

(d) Q attained the temperature of 100°C faster than P.

Ans: (d)

Explanation:

Option (a) is wrong as Q crossed P and became hotter than P thereafter.

Option (b) is wrong as at the point of intersection, P and Q have same temperature

Option (c) is also wrong as after crossing P, Q attained the temperature of 100°C faster than P (graph of Q is to the left of P at 100°C)

So, option (d) is correct.

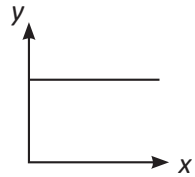
BASIC TYPES OF GRAPHS

To understand and analyze a graph, we need to understand the basic nature of graphs.

1. **Graph parallel to x-axis:**

A straight line that runs parallel to the x-axis is a horizontal line.

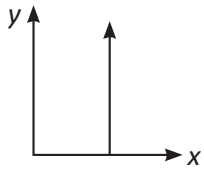
- The equation of a horizontal line is of the form y = k, where "k" is a constant.
- **Interpretation:** Every point on the line has the same y-coordinate, meaning the value of y does not change as x varies. It represents a constant value on the vertical axis.



2. **Graph parallel to y-axis:**

A straight line that runs parallel to the y-axis is vertical line

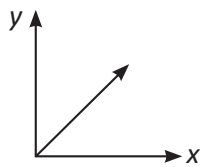
- The equation of a vertical line is of the form x = h, where "h" is a constant.
- **Interpretation:** Every point on the line has the same x-coordinate, meaning the value of x does not change as y varies. It represents a constant value on the horizontal axis.



3. **Graph passing through origin:**

A graph passing through the origin (0, 0) typically indicates a relationship where the dependent variable (usually represented on the y-axis) is directly proportional to the independent variable (usually represented on the x-axis).

In mathematical terms, if a graph passes through the origin, it is of the form $y = mx$, where m is the slope of the line. This equation signifies that the value of y is directly proportional to the value of x, and the line passes through the origin because when $x = 0$, y is also 0.



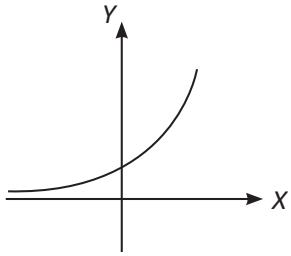
4. **Exponentially increasing graph:**

An exponentially increasing graph represents a relationship where the dependent variable (usually represented on the y-axis) grows at an accelerating rate with respect to the independent variable (usually represented on the x-axis). In other words, as the x-values increase, the y-values grow at a rate that is proportional to the current value of the function.

The general form of an exponential function is often written as: $y = a \cdot b^x$ a is a constant multiplier (the initial value when $x = 0$), and b is the base of the exponential term, and it's greater than 1 for exponential growth.

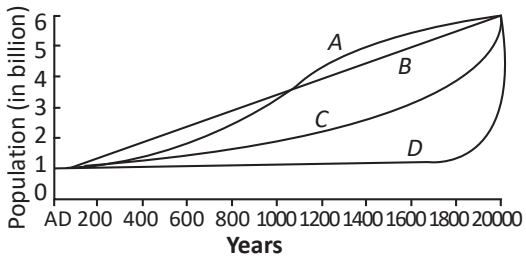
Steep Growth: As x increases, the function's growth becomes steeper and steeper, indicating rapid and accelerating growth.

No Upper Bound: Exponential growth continues indefinitely. There is no upper limit to the values of y as x increases.



QUESTIONS

Q5: The following diagram has four curves namely *A*, *B*, *C* and *D*. Study the figure and answer the questions that follow. **(UPSC CSAT 2011)**



Which curve indicates exponential growth?

(a) A (b) B

(c) C (d) D

Ans: (c)

Explanation:

In curve C as the value of x (years) increases, the value of y (population) increases continuously. Also, the increment is slow initially and rapid after some years. So, it represents exponential growth.

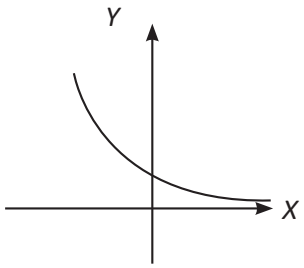
5. **Exponentially decreasing graph:**

An exponentially decreasing graph represents a relationship where the dependent variable (usually represented on the y-axis) decreases at a rate that is proportional to its current value with respect to the independent variable (usually represented on the x-axis). In other words, as the x-values increase, the y-values decrease at an accelerating rate.

The general form of an exponential function is often written as: $y = a \cdot b^x$ a is a constant multiplier (the initial value when x=0),and b is the base of the exponential term, and it's a fraction between 0 and 1 for exponential decay.

Rapid Decay: As x increases, the function's decay becomes steeper and steeper, indicating rapid and accelerating decay.

Approaches but Never Reaches Zero: Exponential decay continues indefinitely but never quite reaches zero. It gets arbitrarily close to zero as x approaches infinity.

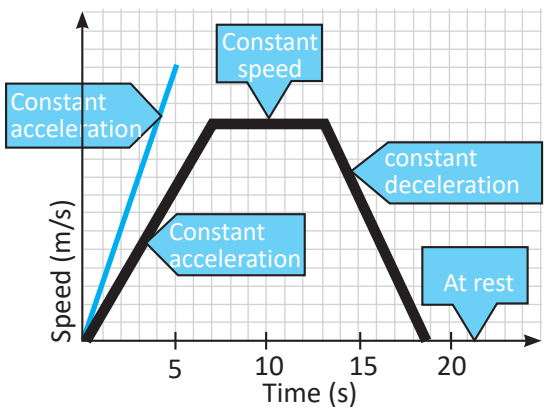


SOME SPECIAL GRAPHS

1. **Speed-Time Graph:**

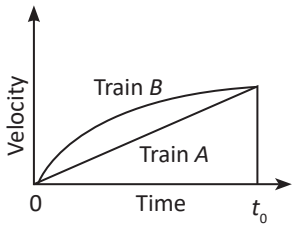
A speed-time graph, also known as a velocity-time graph, is a graphical representation of the motion of an object as a function of time. In this graph, the speed (or velocity) of the object is plotted on the y-axis, and time is plotted on the x-axis. Here are some key features and interpretations of a speed-time graph:

- **Constant Speed:** A straight horizontal line on the graph indicates constant speed. The slope of the line represents the magnitude of the constant speed. If the line is at a higher position on the y-axis, it indicates a higher speed.
- **Acceleration and Deceleration:** An inclined line (not horizontal) indicates acceleration or deceleration. A positive slope represents acceleration, and a negative slope represents deceleration. The steeper the slope, the greater the acceleration or deceleration.
- **Instantaneous Speed:** The slope of the tangent to the curve at any point represents the instantaneous speed of the object at that particular moment.
- **Area Under the Curve:** Area under the speed- time graph represents the distance travelled by a body.



QUESTIONS

Q6: Consider the following velocity-time graph. It shows two trains starting simultaneously on parallel tracks. **(UPSC CSAT 2011)**



With reference to the above graph, which one of the following statements is not correct?

(a) Train B has an initial acceleration greater than that of train A.

(b) Train B is faster than train A at all times.

(c) Both trains have the same velocity at time to.

(d) Both trains travel the same distance in time to units.

Ans: (d)

Explanation:

Option (a) is correct: The slope of train B's graph is greater than that of train A initially.

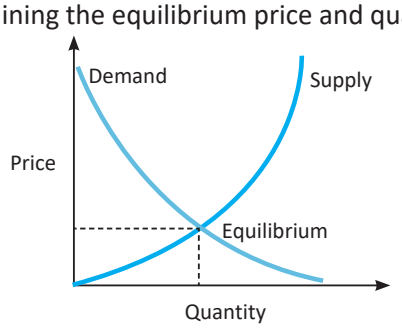
Option (b) is correct: Graph of train B is always above train A, that means its velocity is always greater than that of train A.

Option (c) is correct : At time to both graphs have the same y-coordinate that means the same velocity.

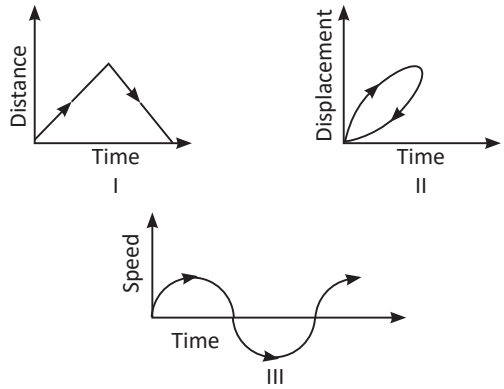
Option (d) is wrong: Area under speed time graph gives distance. Clearly the area under graph B is greater than that of train A.

2. **Supply-Demand Curve:**

The supply and demand curve is a fundamental concept in economics used to illustrate the relationship between the quantity of a good or service that producers are willing to sell (supply) and the quantity that consumers are willing to buy (demand) at different prices. The point where the supply and demand curves intersect is the market equilibrium, determining the equilibrium price and quantity.



Q7: Which of the following graphs can possibly represent the motion of a car on a road? (UPSC CSAT 2012)



(a) I only (b) I and II both
(c) II and III both (d) None of these

Ans: (d)

Explanation:

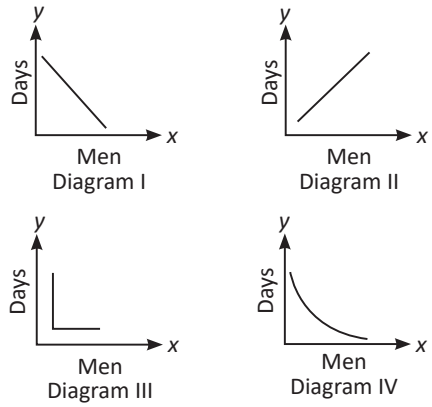
All the three graphs are impossible.

(i) This graph shows that distance first increases with time and then decreases. However distance can never decrease with time.

(ii) At a particular time the body can't be at two different places. In this graph on drawing a vertical line at time 't', it will cut at two points (representing two different displacements at same time).

(iii) Speed can never be negative.

Q8: Consider the following diagrams. *x* men, working at constant speed, do a certain job in *y* days. Which one of these diagrams shows the relation between x and *y*? (UPSC CSAT 2013)



(a) Diagram I (b) Diagram II
(c) Diagram III (d) Diagram IV

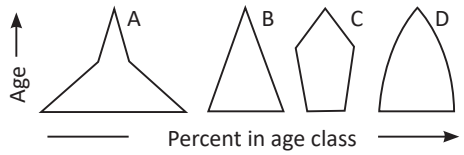
Ans: (d)

Explanation:

Time taken to finish a work is inversely proportional to the number of men working.

So, the graph must be hyperbolic.

Q9: Consider the four age pyramids given below namely A, B, C and D representing four different countries. (UPSC CSAT 2011)



Which one of them indicates the declining population?

(a) A (b) B
(c) C (d) D

Ans: (c)

Explanation:

Here, the base of the pyramid represents the number of children (less aged population) and top represents senior citizens. Pyramid C has the smallest base, so fewer people are born i.e. the population is aging.

DATA TABLE

Data tables are structured arrangements of information organized in rows and columns. They are commonly used to present and organize data in a systematic and easily readable format.

QUESTIONS

Q10. Consider the following information regarding the performance of a class of 1000 students in four different tests. (UPSC CSAT 2012)

Marks/Tests	I	II	III	IV
Average marks	60	60	70	80
Range of marks	30 to 90	45 to 75	20 to 100	0 to 100

If a student scores 74 marks in each of the four tests, then in which one of the following tests his performance is the best comparatively?

(a) Test I (b) Test II
(c) Test III (d) Test IV

Ans: (b)

Explanation:

From the table we can see that average marks obtained by students are minimum in tests I and II (i.e. 60). Average marks of a given student is 74. So, his performance is best in either test I or test II. To decide among test I and II we need to consider the range of marks. Clearly in test II the maximum possible marks is lower (75) than test I (90) and our student has obtained 74 in the IInd test which is closer to the highest marks. Hence, his performance is best in test II comparatively.

Q11. Four cars are hired at the rate of ₹ 6 per km plus the cost of diesel at ₹ 40 per L. In this context, consider the details given in the following table.

(UPSC CSAT 2013)

Cars	Mileage (km/L)	Hours	Total Payment (₹)
A	8	20	2120
B	10	25	1950
C	9	24	2064
D	11	22	1812

Which car maintained the maximum average speed?

(a) Car A (b) Car B

(c) Car C (d) Car D

Ans: (a)

Explanation:

Average speed of a car = distance/total time.

Let 's' be the average speed, 't' the time taken, 'm' the mileage, 'd' the total distance and 'p' the total payment.

$$\text{Total payment, } p = (\text{rate per km} \times d) + \left(\text{cost of diesel per } L \times \frac{d}{m}\right)$$

$$p = (6 \times d) + \left(40 \times \frac{d}{m}\right) = d\left(6 + \frac{40}{m}\right)$$

$$\text{So, } d = \frac{pm}{6m + 40}$$

Thus, average speed = d/Hours

So, Average Speed of

(a) Car A $= \frac{(2120 \times 8)}{(6 \times 8 + 40)(20)} = \frac{106}{11}$

(b) Car B $= \frac{(1950 \times 10)}{(6 \times 10 + 40)(25)} = \frac{39}{5}$

(c) Car C $= \frac{(2064 \times 9)}{(6 \times 9 + 40)(24)} = \frac{387}{47}$

(d) Car D $= \frac{(1812 \times 11)}{(6 \times 11 + 40)(22)} = \frac{453}{53}$

Hence Avg. Speed of car A is the greatest among all the cars.

Q12. The following table shows the marks obtained by two students in different subjects: **(UPSC CSAT 2014)**

	Student A	Maximum marks	Students B	Maximum marks
English	60	100	80	150
Psychology	70	100	70	100
History	50	100	60	100
Sanskrit	30	50	15	25

The difference in the mean aggregate percentage marks of the students is:

(a) 2.5% (b) 13.75%

(c) 1.25% (d) Zero

Ans: (d)

Explanation:

Total marks obtained by student

A = 60 + 70 + 50 + 30 = 210

Total maximum marks for student

A = 350

% of marks of student

A = (210/350) × 100 = 60%

Total marks obtained by student B

= 80 + 70 + 60 + 15 = 225

Total maximum marks for student

B = 375

% of marks of student

B = (225/375) × 100 = 60%

Difference between the aggregate %

= 60 – 60 = 0

Q13. The following table shows the number of Science, Arts and Commerce students in a group of boys and girls:

	Science	Arts	Commerce	Total
Boys	20	–	15	80
Girls	6	40	–	–
Total	–	–	20	–

The number of students in the group is:

(a) 127 (b) 130

(c) 131 (d) 135

Ans: (c)

Explanation:

Boys in Arts = 80 –20 – 15 = 45

Total science students = 20 + 6 = 26

Total Arts students = 45 + 40 = 85

Girls in commerce = 20 – 15 = 5

Total girls = 6 + 40 + 5 = 51

Completed table is as follows:

	Science	Arts	Commerce	Total
Boys	20	45	15	80
Girls	6	40	5	51
Total	26	85	20	131

So, total number of students = 80 + 51 = 131.

PIE CHART

A pie chart is a circular statistical graphic that is divided into slices to illustrate numerical proportions. Each slice represents a proportionate part of the whole, and the entire circle represents 100% of the data. Pie charts are commonly used to show the distribution of a categorical variable or the relative contribution of different components to a whole.

Key features of a pie chart

Slices: Each slice of the pie represents a category or group. The size of each slice is proportional to the percentage or fraction it represents.

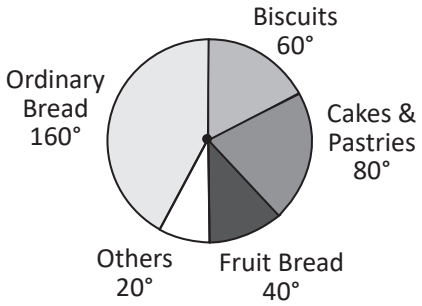
Central Angle: The central angle of each slice is proportional to the percentage it represents. The sum of all central angles in the pie chart is 360 degrees.

Let's consider an example of a pie chart representing the sales of a bakery in a day. In this scenario, we'll consider five different categories of bakery items: Normal bread, Fruit bread, Cakes & Pastries, Biscuits and Others. Consider the data of these items with sales (in ₹). To draw the pie chart of the following data we need to calculate the central angles for each of these items.

Ordinary bread	320
Fruit bread	80
Cakes & Pastries	160
Biscuits	120
Others	40

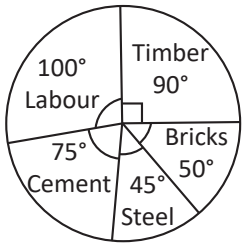
Item	Sales (in ₹)	In Fraction	Central Angle
Ordinary Bread	320	$\frac{320}{720}=\frac{4}{9}$	$\frac{4}{9}\times360° = 160°$
Biscuits	120	$\frac{120}{720}=\frac{1}{6}$	$\frac{1}{6}\times 360° = 60°$
Cakes & pastries	160	$\frac{160}{720}=\frac{2}{9}$	$\frac{2}{9}\times360° = 80°$
Fruit Bread	80	$\frac{80}{720}=\frac{1}{9}$	$\frac{1}{9}\times360° = 40°$
Others	40	$\frac{40}{720}=\frac{1}{18}$	$\frac{1}{18}\times360° =20°$

Below is the pie chart representation:



QUESTIONS

Q14. The pie chart below gives the expenditure on different items in a house construction.



The percentage expenditure on labour is:

(a) (250/9)% (b) (125/6)%

(c) (125/9)% (d) 20%

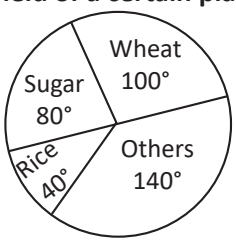
Ans: (a)

Explanation:

The percentage expenditure on labour

= (100/360) × 100 % = (250/9)%

Q15. The following pie chart shows the annual agricultural yield of a certain place.



The production of wheat exceed that of rice by

(a) 50% (b) 75%

(c) 100% (d) 150%

Ans: (d)

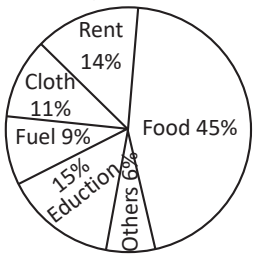
Explanation:

Central angle of wheat = 100° and central angle of rice = 40°

So, % by which wheat production exceeds that of rice will be = $\frac{(100-40)}{40}\times100\%$

$=\frac{(3)}{2}\times100\%=150\%$

Directions (Q16 – Q19): The following pie chart shows the expenditure incurred by a family in a month over different heads (or items).



Study the above chart carefully and answer the following questions

Q16. What is the proportion of expenditure incurred on education and food?

(a) 3 : 1 (b) 1 : 3

(c) 3 : 5 (d) 5 : 3

Ans: (b)

Explanation:

Ratio of expenditure incurred on education and food = 15 : 45 = 1 : 3

Q17. What is the percentage of expenditure incurred on rent with respect to expenditure on fuel?

(a) 135.56% (b) 172.56%

(c) 155.56% (d) 167.56%

Ans: (c)

Explanation:

% of expenditure incurred on rent with respect to expenditure on fuel = $\frac{14}{9}\times 100\%$ = 155.56%

Q18. Expenditure on which three heads constitute and an angle of 108° at origin?

(a) Fuel, Cloth, Others

(b) Fuel, Education, Others

(c) Cloth, Rent, Others

(d) Education, Rent, Others

Ans: (b)

Explanation:

Total sum of central angles should be 108°.

360° = 100%

So, 108° = $\frac{108}{360}\times 100\% = 30\%$

Education (15%), Fuel (9%) and others (6%) makes a total of 30%.

Q19. If the total income of the family is ₹ 25000, then how much has been spent on rent and food?

(a) ₹ 17250 (b) ₹ 16750

(c) ₹ 14750 (d) ₹ 8500

Ans: (c)

Explanation:

Total expenditure on rent and food

= $\frac{(45+14)}{100}\times 25000 = \frac{59}{100}\times 25000$ = ₹14750

BAR GRAPH

A bar graph is a graphical representation of data in which rectangular bars or columns are used to represent different categories or groups. Each bar's length or height is proportional to the quantity it represents. Bar graphs are commonly used to display and compare the values of different variables, making it easy to visualize patterns, trends, and relationships within the data.

Here Are The Key Components of A Bar Graph

X-Axis and Y-Axis: The graph has two axes – a horizontal axis (X-axis) and a vertical axis (Y-axis). The X-axis typically represents the categories or groups being compared, while the Y-axis represents the values or frequencies associated with each category.

Bars: The bars in a bar graph are vertical or horizontal rectangles that correspond to each category on the X-axis. The length or height of each bar represents the value of the variable being measured. The bars are usually separated from each other, allowing for easy visual comparison.

Title: A title is provided at the top of the graph to describe the content or purpose of the graph. It helps viewers understand what the graph is illustrating.

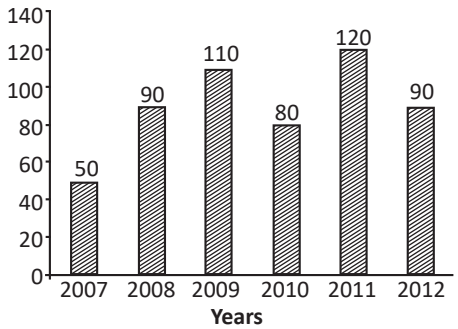
Labels: Each axis is labeled to provide information about what is being measured and the units of measurement. Labels on the X-axis indicate the categories or groups, while labels on the Y-axis indicate the values or frequencies.

Scale: The scale on the Y-axis is important for accurately interpreting the values represented by the bars. It ensures that the differences in bar lengths are proportional to the differences in the data values.

Double bar graph: A type of bar graph that compares two sets of data side by side is known as a double bar graph, also known as a double-bar chart. It is useful for visually representing and comparing the values of two different categories or groups across various subcategories or time periods.

QUESTIONS

Q20. In the bar diagram given below is data pertaining to the production of cycles in each year by a company. After studying the diagram, find out the percentage drop in cycle production between the years 2011 and 2012.



(a) 20% (b) 30%

(c) 15% (d) 25%

Ans: (d)

Explanation:

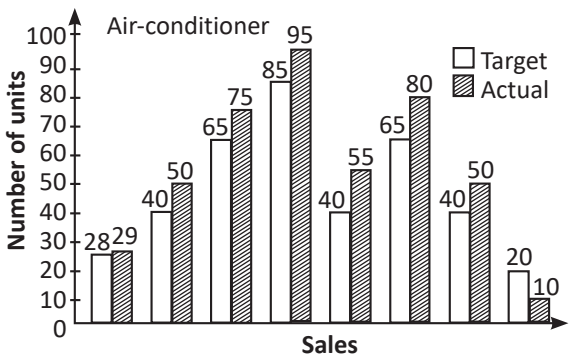
Cycle production in year 2011 = 120 units

Cycle production in year 2012 = 90 units

% drop between year 2011 and 2012

$$=\frac{(120-90)}{120}\times 100\% = \frac{30}{120}\times 100\% = 25\%$$

Q21. A company manufacturing air conditioners set a monthly target. The target and realized values are shown in the bar chart.



Consider the following statements based on the chart.

I. **The targeted sales on a monthly basis have been achieved.**

II. **The overall target value has been exceeded by 7.5 %.**

III. **The sales department deserves a pat on the back.**

Which of the above statements is/are correct?

(a) Only I (b) Only II

(c) I and II (d) II and III

Ans: (b)

Explanation:

From the bar graph, it is clear that the monthly target set by the company is achieved by the sales team every month except the last one. So, statement I is not true. Now we are left with option (b) and option (d). Hence, statement II has to be correct. The sales team could not achieve the target in the last month, so statement III is also not correct.

Q22. The misery index is the sum of a country's unemployment and inflation rate. The higher the index, the more miserable is the country to live in. The figure given below is the misery index for various countries in Europe.

Which of the following conclusions can be drawn from the misery index given above?

I. **Britain is the most miserable country to live in.**

II. **The inflation rate in Spain is less than that in Belgium and Britain.**

III. **Italy and France seem to have almost identical unemployment.**

IV. **The higher the misery index, the higher the inflation rate.**

Select the correct answer using the codes given below.

Codes

(a) Only I (b) II and III

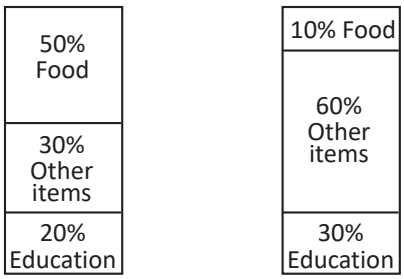
(c) I, II, III and IV (d) None of these

Ans: (b)

Explanation:

The shaded portion in the graph shows inflation rate. The size of the shaded portion seems less in Spain than Belgium and Britain. So, statement II is correct. The unshaded portion which represents the unemployment rate is of almost the same size for Italy and France, which makes statement III correct. Clearly, statement IV is incorrect as the inflation rate seems highest in Britain while its misery index is lower.

Q23. The proportion of expenditure on various items by two families A and B are represented in the following bar chart. **(UPSC CSAT 2015)**



Family A
Total expenditure:
₹ 20000 per month

Family B
Total expenditure:
₹ 100000 per month

From these charts, we can conclude that

(a) family A spent more money on food than family B

(b) family B spent more money on food than family A

(c) family A and family B spent the same amount on food

(d) the expenditure on food by family A and family B cannot be compared

Ans: (c)

Explanation:

Expenditure of family A on food

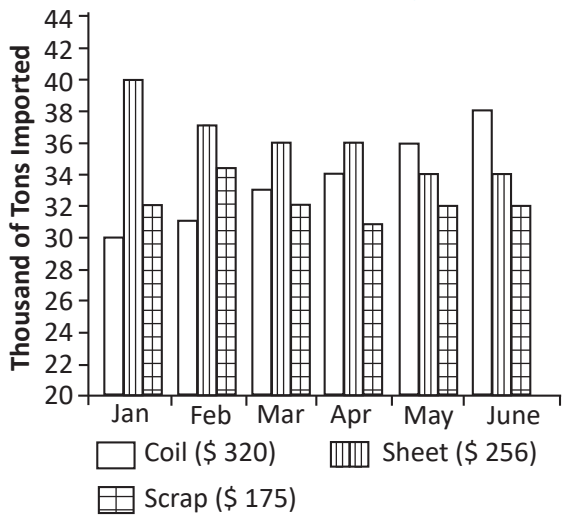
= 50% 0f ₹ 20000 = ₹ 10000 per month

Expenditure of family B on food

= 10% of ₹ 100000 = ₹ 10000 per month

Thus expenditure of both the families is the same.

Directions (Q24 – Q26): The following three questions are based on the graph given below which shows imports of three different types of steel over a period of six months of a year. Study the graph and answer three questions that follow. **(UPSC CSAT 2018)**



The figures in the brackets indicate the average cost per ton over a six month period.

Q24. By how much (measured in thousands of tons) did the imports of sheet steel exceed the imports of coil steel in the first three months of the year?

(a) 11 (b) 15

(c) 19 (d) 23

Ans: (c)

Explanation:

Imports of sheet steel (in thousands of tons) in the first three months = 40 + 37 + 36 = 113.

Imports of coil steel (in thousands of tons) in the first three months = 30 + 31 + 33 = 94.

Imports of sheet steel exceed the imports of coil steel in the first three months of the year by (113 – 94) = 19 thousand tons.

Q25. What was the approximate total value (in $) of sheet steel imported over the six months period?

(a) 45555 (b) 50555

(c) 55550 (d) 65750

Ans: (c)

Explanation:

Imports of sheet steel (in thousands of tons) in six months = 40 + 37 + 36 + 36 + 34 + 34 = 217.

Average cost per ton of sheet steel over a six month period = $ 256.

Total value (in $)

= 217 × 256 = $55552 (approximately $55550).

Q26. What was the approximate ratio of sheet steel and scrap steel imports in the first three months of the year?

(a) 1:1 (b) 1.2 : 1

(c) 1.4 : 1 (d) 1.6 : 1

Ans: (b)

Explanation:

Imports of sheet steel (in thousands of tons) in the first three months = 40 + 37 + 36 = 113.

Imports of scrap steel (in thousands of tons) in the first three months = 32 + 34 + 32 = 98.

Ratio of sheet steel and scrap steel imports in first three months of the year = $\frac{113}{98} = 1.15 : 1$

Approximate ratio ≅ 1.2 : 1

LINE GRAPH

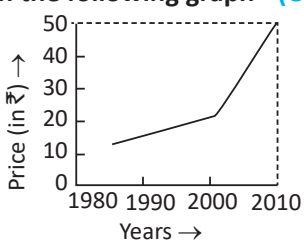
A line graph is a type of chart that displays data points using lines to connect them. It is particularly useful for showing trends and changes in data over a continuous interval or time span. Line graphs are commonly used in various fields such as science, economics, and finance to visualize the relationship between two variables.

Line graphs have several properties that make them effective for representing and visualizing data. Here are the key properties of a line graph:

1. **Continuous Data Representation:** Line graphs are well-suited for representing continuous data, where the independent variable (usually time) progresses in a continuous manner.
2. **Trend Visualization:** One of the primary purposes of a line graph is to show trends and patterns in data over time or another continuous scale. The slope of the line between data points provides a visual indication of the rate of change in the dependent variable.
3. **Data Points:** Each data point on the line graph represents a specific value for the dependent variable at a particular point in time or along the continuous scale. These points are marked and connected by lines to form the overall graph.
4. **Clarity and Simplicity:** Line graphs are generally clear and easy to understand. The simplicity of the lines makes it easy for viewers to identify trends and make comparisons between different data sets.
5. **Comparison of Multiple Data Series:** Line graphs can accommodate multiple lines on the same graph, making it easy to compare trends and patterns between different data series. Each line is usually associated with a different color or pattern, and a legend may be included to identify each series.

QUESTIONS

Q27. Year wise variation of the price of certain commodity is shown in the following graph (UPSC CSAT 2017)



The price of the commodity in the year 1990

(a) must have been ₹ 10

(b) must have been ₹ 12

(c) must have been anywhere between ₹ 10 and ₹ 20

(d) is higher than that of the year 1991

Ans: (c)

Explanation:

The price of the commodity is ₹10 before 1990 and ₹20 till many years after 1990, and the graph is increasing initially with constant slope. So, the price lies between ₹10 and ₹20.

Q28. Average hourly earnings per year (E) of the workers in a firm are represented by figures A and B as follows. (UPSC CSAT 2017)

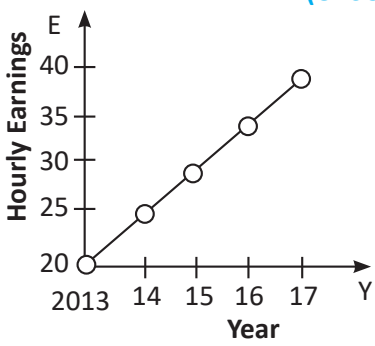


Fig. A

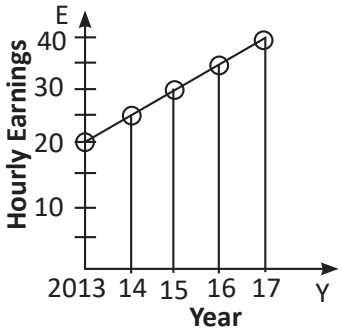


Fig. B

From the figures, it is observed that the

(a) values of E are different

(b) ranges (i.e. the difference between the maximum and minimum) of E are different

(c) slopes of the graphs are same

(d) rates of increase of E are different

Ans: (c)

Explanation:

Both the graphs represent the same variables on the x and y axis. Slope of line graph = $\frac{\Delta y}{\Delta x}$

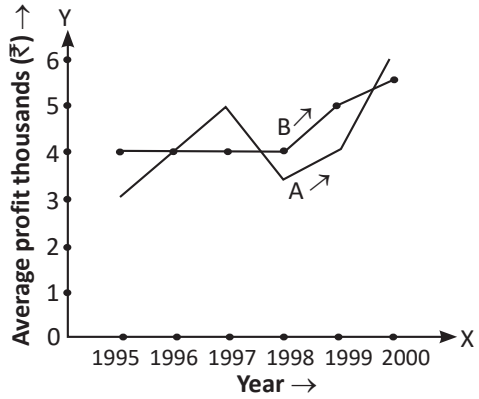
Slope in figure A = $\frac{30-25}{2015-2014}=5$

Slope in figure B = $\frac{35-25}{2016-14}=5$

So, both figures have the same slope.

Range of E for both the graph is same = 40 – 20 = 20

Directions (Q29 – Q32): The following graph shows the average profit of two fruit-sellers, A and B in thousands (₹) per year from the year 1995 to 2000. Consider the graph and answer the questions that follow. **(UPSC CSAT 2014)**



Q29. In which year is the average profit of A and B the same?

(a) 1995 (b) 1996
(c) 1997 (d) 1998

Ans: (b)

Explanation:

Average profit of A and B will be the same at the point of intersections. The intersection corresponds to the year 1996.

Q30. What is the difference between the average profit of B and A in the year 1998?

(a) –₹100 (b) –₹1000
(c) +₹600 (d) –₹300

Ans: (c)

Explanation:

In year 1998, graph of B is above A, i.e. Average profit of B is greater than average profit of A. Hence, their difference is positive.

Q31. How much more average profit did A make in the year 2000 than in the year 1999?

(a) ₹200 (b) ₹1000
(c) ₹1500 (d) ₹2000

Ans: (d)

Explanation:

In year 2000, average profit of A = ₹6000

In year 1999, average profit of A = ₹4000

Average profit of A was greater in year 2000 than 1999 by (₹6000 - ₹4000) = ₹2000

Q32. What is the trend of the average profit of B from the year 1997 to the year 2000?

(a) Non-increasing (b) Non-decreasing
(c) Steady (d) Fluctuating

Ans: (b)

Explanation:

Graph line of B is parallel to the x-axis between 1997 to 1998 (constant), has increasing slope between 1998 to 1999 (showing increasing nature) and further has increasing slope between 1999 to 2000 (increasing). Thus it is non-decreasing.

CASELETS

"Caselets" typically refer to short, concise case studies that are used for teaching and analysis purposes. These are usually brief scenarios or situations that present a business problem, challenge, or decision-making opportunity.

Here's a general process for interpreting data from caselets:

1. **Read and Understand the Caselet:** Begin by thoroughly reading the caselet to understand the scenario, the key players, and the context of the business problem or situation.

   Identify the main issues, challenges, or decisions that the caselet presents.
2. **Identify Relevant Data:** Extract and identify relevant data points, facts, and figures provided in the caselet. Pay attention to numerical data, percentages, trends, and any other quantitative information.
3. **Analyze Relationships:** Look for relationships and correlations between different pieces of data. Consider how changes in one variable may be associated with changes in another. Identify any cause-and-effect relationships or patterns in the data.

QUESTIONS

Directions (Q33 – Q35): Read the following passage and answer the three questions that follow. **(UPSC CSAT 2011)**

In a survey regarding a proposed measure to be introduced, 2878 persons took part of which 1652 were males. 1226 persons voted against the proposal of which 796 were males. 1425 persons voted for the proposal, 196 females were undecided.

Q33: How many females voted for the proposal?

(a) 430 (b) 600
(c) 624 (d) 640

Ans: (b)

Q34. How many males were undecided?

(a) 31 (b) 227

(c) 426 (d) 551

Ans: (a)

Q35. How many females were not in favour of the proposal?

(a) 430 (b) 496

(c) 586 (d) 1226

Ans: (a)

Explanation(Q33 – Q35): In the survey, 2878 persons took part of which 1652 were males.

∴ Number of females who took part in survey = 2878 - 1652 = 1226

Now, 1226 persons voted against the proposal of which 796 were males.

∴ Number of females which are against the proposal = 1226 - 796 = 430

⇒ Number of females who voted in favour of the proposal = (Total number of females) – (Number of females voted against the proposal) – (Number of females who were undecided) = 1226 – 430 – 196 = 600.

⇒ Number of males who were undecided = (Total number of males) – (Number of males voted in favour of the proposal) – (Number of males who voted against the proposal) = 1652 – (1425 - 600) – 796 = 31

[Number of males voted in favour of the proposal= Number of persons voted for the proposal - Number of females who voted in favour of the proposal = 1425 - 600 = 825]

This information can be summarized as

	Males	Females	Total
Favour	825	600	1425
Against	796	430	1226
Undecided	31	196	227
Total	1652	1226	2878

MIXED GRAPHS

Mixed graphs in data interpretation refer to the use of different types of graphs or charts within the same visual representation to convey multiple dimensions of data. By combining various graph types, such as bar graphs, line graphs, and pie charts, in a single presentation, mixed graphs provide a comprehensive view of complex data sets.

Here's how you can interpret mixed graphs effectively:

1. **Understand Graph Types:** Before interpreting a mixed graph, ensure that you understand the types of graphs included and their individual characteristics. For example, bar graphs are useful for comparing quantities, line graphs for trends over time, and pie charts for proportions.
2. **Identify Data Series:** Different parts of the mixed graph may represent distinct data series or categories. Identify and understand what each part of the graph represents, whether it's a specific time period, a group, or a variable.
3. **Analyze Relationships:** Analyze how different elements in the mixed graph interact or relate to each other. For example, observe if changes in one variable correlate with changes in another, or if certain categories dominate others in a pie chart.
4. **Interpret Trends and Comparisons:** If the mixed graph includes line graphs, pay attention to trends over time. If it includes bar graphs, analyze comparisons between different categories. Interpret the data to derive insights into patterns, variations, or anomalies.

QUESTIONS

Directions (Q36 – Q40): Study the two figures given below and answer the five items that follow. **(UPSC CSAT 2013)**

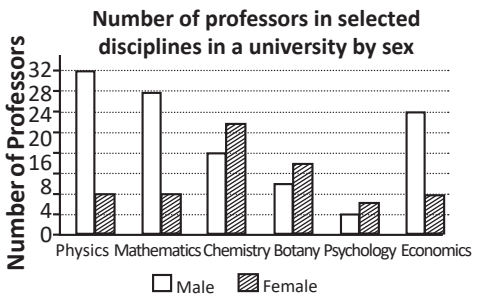


Figure 1

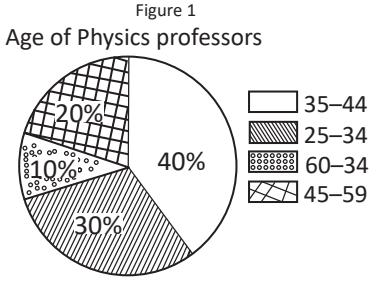


Figure 2

Q36. How many Physics professors belong to the age group 35-44?

(a) 18 (b) 16

(c) 14 (d) 12

Ans: (b)

Explanation:

From figure 1, Total number of Physics professors = 32 (males) + 8 (females) = 40. From figure 2, 40% Physics professors are in the age group 35-44.

So, 40% of 40 = $\frac{40}{100}\times 40 = 16$.

Q37. Which one of the following disciplines has the highest ratio of males to females?

(a) Physics (b) Mathematics

(c) Chemistry (d) Economics

Ans: (a)

Explanation:

Chemistry, Botany and Psychology departments have more females than males, so we need not to calculate the ratio for it. Remaining options, Physics, Mathematics and Economics departments have the same number of females, so among these three departments, the department with the most number of males will have the highest ratio of males to females. Among these three, the Physics department has the maximum number of males.

Q38. What percentage of all Psychology professors are females?

(a) 40% (b) 50%

(c) 60% (d) 70%

Ans: (c)

Explanation:

Number of female Psychology professors is 6 from figure 1. Total number of Psychology professors is (6 females + 4 males) = 10

Hence, percentage of all psychology professors who are females = $\frac{6}{10}\times 100\% = 60\%$

Q39. If the number of female Physics professors in the age group 25-34 equals 25% of all the Physics professors in that age group, then what is the number of male Physics professors in the age group 25-34?

(a) 9 (b) 6

(c) 3 (d) 2

Ans: (a)

Explanation:

Total number of Physics professors is (32 males + 8 females) = 40. Now out of 40, 30% are in the age group 25 – 34, i.e. 12 (30% of 40 is 12).

Now, the number of female Physics professors in the age group 25-34 is 25% i.e. 3(25% of 12 is 3). Hence, male Physics professors in age group 25-34 is (12 – 3 = 9).

Q40. If the Psychology professors in the university constitute 2% of all the professors in the university, then what is the number of professors in the university?

(a) 400 (b) 500

(c) 600 (d) 700

Ans: (b)

Explanation:

Total number of Psychology professors in the university is (4 males + 6 females) = 10.

Thus, 10 professors make up 2% of the total number of Professors.

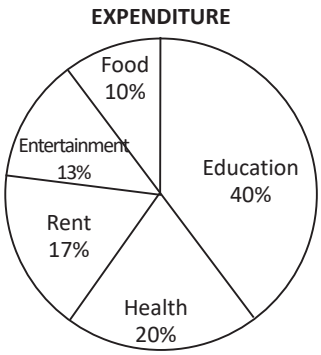
Let the number of all professors be P.

Now, 2% of P = 10.

So, $\frac{2}{100}\times P = 10 \Rightarrow P = \frac{100}{2}\times 10 = 500$

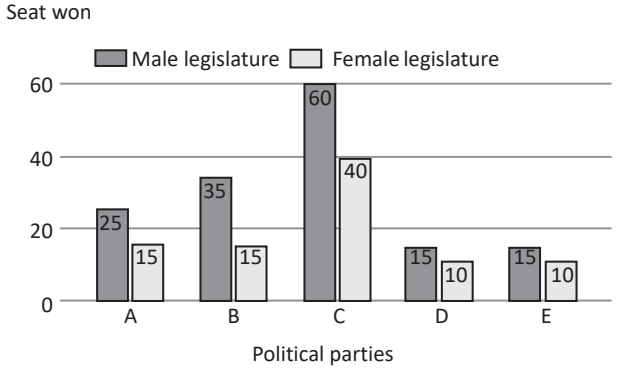
PRACTICE QUESTIONS

Directions (Q1 - Q4): The following pie chart shows the expenditure incurred by Ram in a month over different heads (or items). He earns a monthly income of Rs 100,000.

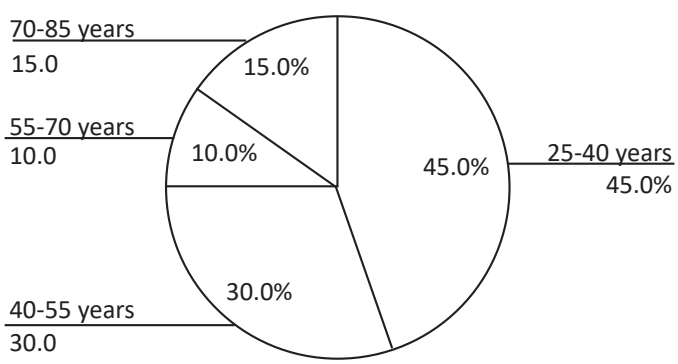


1. **What is the excess amount (in Rs) of money spent on entertainment in comparison to the amount spent on food?**
   (a) 3000 (b) 4000
   (c) 6000 (d) 7000
2. **Ram has decided to save 20% of his monthly income. But he can't reduce his expenditure on food and education. By what amount should Ram reduce his expenditure on other items?**
   (a) 30% (b) 40%
   (c) 50% (d) 45%
3. **What is the ratio of expenditure on health and education to that of expenditure on rent and entertainment?**
   (a) 1 : 2 (b) 3 : 1
   (c) 2 : 1 (d) 4 : 3
4. **What is the percentage of expenditure incurred on rent with respect to expenditure on health?**
   (a) 85% (b) 55%
   (c) 35% (d) 13%

Directions (Q5 - Q7): Study the two figures given below and answer the questions



Age distribution of legislators won from party C.



5. **How many legislators from party C are older than 55 years?**
   (a) 15 (b) 40
   (c) 17 (d) 25
6. **Political Party C has formed a government in alliance with Party A, with Party A offering conditional support. The number of ministerial berths is equal to 15% of the legislature's total strength, to be equally divided between both the parties. Eligibility for ministerial positions is limited to legislators aged 55 or older. In this context, What percentage of C's eligible legislators will become ministers?**
   (a) 26% (b) 48%
   (c) 72% (d) 76%
7. **Considering A & C to be a part of the government. A committee of 15 women members from opposition parties is to be formed in such a way that each party has an equal number of women members. In how many ways can this committee be formed?**
   (a) 3507 (b) 2467
   (c) 2789 (d) 3309

Directions (Q8 - Q10): Study the following table to answer the given questions. Production (in crore units) of four companies over the years

Company	2009	2010	2011	2012	Total
AVC	300	310	360	370	1340
CTU	275	280	281	285	1121
PEN	25	35	35	40	135
SIO	85	89	89	91	354
Total	685	714	765	786	

8. **The production of company CTU in 2012 is approximately what percent of its average production over the given years?**

(a) 104% (b) 101.69%

(c) 97% (d) 104.51%

9. **Which company has less average production in the last two years compared to that of first two years?**

(a) AVC (b) SIO

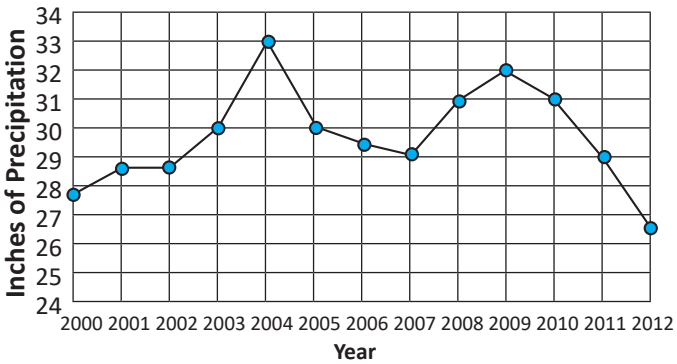
(c) PEN (d) none of these

10. **The total production of the four companies in the first two given years is what percent of that of the last two years?**

(a) 88.8% (b) 90.19%

(c) 84.5% (d) 93.54%

Directions (Q11-Q12): The graph below depicts the precipitation (in inches) year-on-year for almost a decade.



11. **What is the average of the lowest and highest precipitation recorded?**

(a) 30.25 inches (b) 29 inches

(c) 29.75 inches (d) 28 inches

12. **Which one of the following statements is correct with reference to the above graph?**

(a) There was steady rise in the rainfall from the year 2000 to 2005

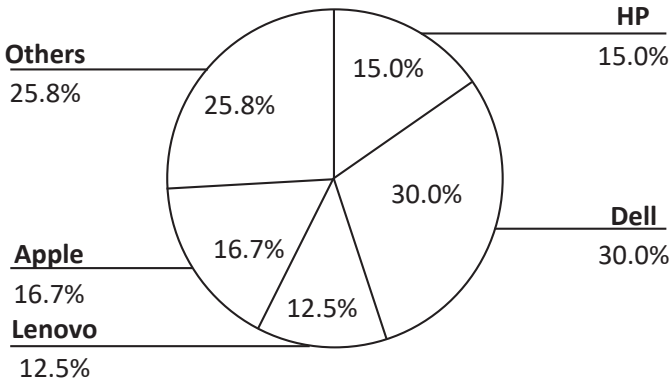
(b) The average precipitation in the interval 2000-2003 is less than the average in 2009-2012

(c) The lowest ever precipitation recorded was in the year 2011

(d) None of the above

Directions (Q23-Q24): Consider the given pie chart which shows the market share of computer companies in India for the year 2020 by value.

Market share by value (Revenue) in 2020.



13. **If the average selling price of Lenovo computers is half the average selling price of Apple . What is the ratio of the number of computer units sold by Apple to that of lenovo in 2020?**

(a) 1 : 2 (b) 2 : 3

(c) 3 : 4 (d) 2 : 5

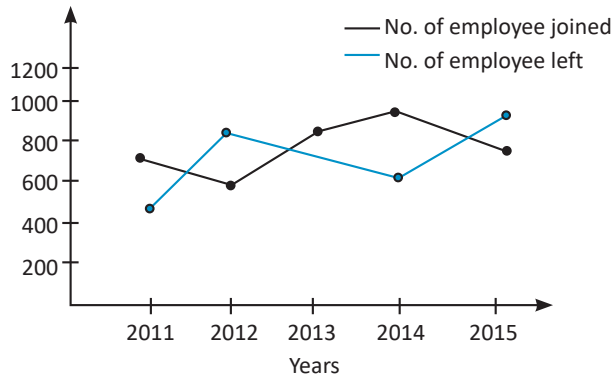
14. **If HP and Lenovo were merged to form a single company, By what percentage would the newly formed company be smaller than Dell in terms of revenue?**

(a) 8.33% (b) 12.5%

(c) 16.67% (d) 7.28%

Directions (Q15-Q16): Study the following line graph which gives the number of employees who joined and left Infosys at the beginning of the year for five years, from 2011 to 2015.

Initial Strength of employees in 2010 = 6000



15. **The number of employees working at infosys office during 2015 was?**

(a) 6200 (b) 6300

(c) 6250 (d) 6500

16. **For which year, the percentage rise/fall in the number of employees who left Infosys compared to the previous year is maximum?**

(a) 2013 (b) 2012

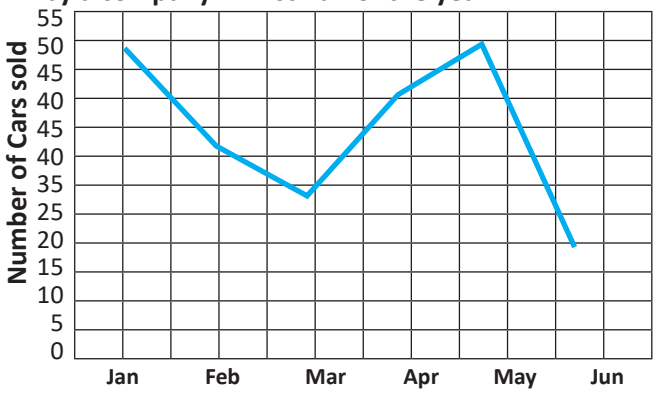
(c) 2014 (d) 2015

17. By what percentage did the number of Infosys employees change (increased/decreased) from 2013 to 2014?

(a) 7% (b) 6%

(c) 5% (d) 2%

18. The following graph depicts the number of cars sold by a company in first half of the year:

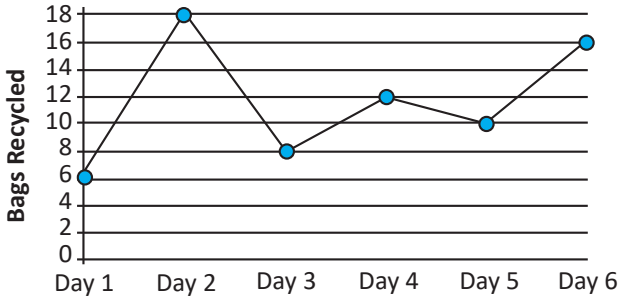


What is the average number of cars sold from February to April?

(a) 32 (b) 27

(c) 30 (d) 35

Directions for Questions (29-30): The graph below shows the number of bags recycled on six days of a week. Study the graph carefully and answer the questions that follow.



19. On which day the minimum number of bags were recycled?

(a) Day 1 (b) Day 4

(c) Day 5 (d) Day 6

20. What is the difference between the number of bags recycled on day 2 and day 6 ?

(a) 10 (b) 4

(c) 2 (d) 12

ANSWERS

1. (*a*)	2. (*b*)	3. (*c*)	4. (*a*)	5. (*d*)	6. (*c*)	7. (*a*)	8. (*b*)	9. (*d*)	10. (*b*)
11. (*c*)	12. (*b*)	13. (*b*)	14. (*a*)	15. (*a*)	16. (*b*)	17. (*c*)	18. (*c*)	19. (*a*)	20. (*c*)

EXPLANATIONS

1. (a)

Explanation:

Monthly salary = 100,000 = 100%

Expenditure on entertainment = 13%

Expenditure on food = 10%

Excess expenditure on entertainment = 13 – 10 = 3%

$$=\frac{3}{100}\times 100000 = \text{Rs. } 3000$$

2. (b)

Explanation:

Expenditure on food and education = 10 + 40 = 50%

Desired savings must come from the remaining 50 % of income i.e. Rs 50000

Desired savings = 20% of Rs 100000

Let x be the percentage reduction on the remaining expenditure

$$20\% \times 100000 = x\% \times 50000 \qquad x = 40\%$$

3. (c)

Explanation:

Expenditure on health(H) and education(E)

= 40% + 20% = 60%

Expenditure on rent(R) and entertainment (Ent)

= 17% + 13% = 30%

$$\frac{(H+E)}{(R+Ent)} = \frac{60\%}{30\%} = \frac{2}{1}$$

4. (a)

Explanation:

Expenditure on rent = 17%

Expenditure on health = 20%

% expenditure on rent with respect to health

$$=\frac{17}{20}\times 100 = 85\%$$

5. (d)

Explanation:

Total seats won by party C (Males + Females)

= 60 + 40 = 100

% of legislators of Party C, above 55 years

= 10% + 15% = 25%

Number of party C legislators who are above 55 years of age = $\frac{25}{100}\times 100 = 25$

6. (c)

Explanation:

Total seats = 240

Number of ministerial berths (15%) = $\frac{15}{100}\times 240 = 36$

Number of ministerial berths to be occupied by Party C = $\frac{36}{2} = 18$

C's eligible legislators to become a minister (above 55 years of age) = 25 (calculated in the last question)

% of eligible legislators of party C, who becomes a minister = $\frac{18}{25}\times 100 = 72\%$

7. (a)

Explanation:

From the bar graph

Number of women in B = 15

Number of women in D = 10

Number of women in E = 10

Number of women to be selected from B, D, E each $= \frac{15}{3} = 5$

Number of ways to select 5 women from each of *B*, *D*, *E* = ${}^{15}C_5 + {}^{10}C_5 + {}^{10}C_5$

${}^{15}C_5 + {}^{10}C_5 + {}^{10}C_5$ = 3003 + 252 + 252 = 3507

8. (b)

Explanation:

Production of company CTU in 2012 = 285 crore units average production of CTU over the given years = 1121/4 = 280.25

So, required percentage = (285/280.25) × 100 = 101.69%

9. (d)

Explanation:

Sum of the production of the companies in the first two years and last two years is as follows:

Company	First two years	Last two years
AVC	610	730
CTU	555	566
PEN	60	75
SIO	174	180

According to the table

All companies has more average production in the last two years compared to the two first year.

10. (b)

Explanation:

Total production of the four companies in first two given years = 685 + 714 = 1399

total production of the four companies in last two given years = 765 + 786 = 1551

Then, required percentage

= (1399/1551) × 100 = 90.19%

11. (c)

Explanation:

Lowest precipitation = 26.5 inches

Highest precipitation =33 inches

Average $= (26.5+33)/2 = 29.75$ inches

12. (b)

Explanation:

Average precipitation in the interval 2000 $2003 = (28+28.5+28.5+30)/4 = 115/4 =$ 28.75

Average precipitation in the interval 2009 $2012 = (32+31+29+26.5)/4 = 118.5/4 =$ 29.625

13. (b)

Explanation:

Share of Apple by value $= 16.7\% = \frac{16.7}{100} \cong \frac{1}{6}$

Share of Lenovo by value $= 12.5\% = \frac{12.5}{100} = \frac{1}{8}$

Let Lenovo sales *n* computers at the rate of Rs *x* and Apple sales *m* computers at the rate of Rs 2*x*.

Ratio of revenues of Apple to Lenovo

$$= \frac{m\times 2x}{n\times x} = \frac{(1/6)}{(1/8)} = \frac{8}{6} = \frac{4}{3}$$

$$\frac{m}{n} = \frac{4}{3\times 2} = \frac{2}{3}$$

14. (a)

Explanation:

Market share of (Lenovo + HP) = 12.5% + 15% = 27.5%

Market share of Dell = 30%

Difference in market share of Dell and (Lenovo + HP) = 30% – 27.5% = 2.5%

$$\frac{2.5\%}{30\%}\times 100 = 8.33\%$$

Thus the new company is 8.33% smaller than Dell.

15. (a)

Explanation:

Given that 6000 employees were working in 2010.

So, the number of employee

In 2011 $\Rightarrow$ 6000 + 700 – 500 = 6200,

In 2012 $\Rightarrow$ 6200 + 600 – 900 = 5900,

In 2013 $\Rightarrow$ 5900 + 900 – 800 = 6000,

In 2014 $\Rightarrow$ 6000 + 1000 – 700 = 6300,

In 2015 $\Rightarrow$ 6300 + 800 – 900 = 6200,

Therefore, the total number of employees working in the year 2015 is 6200.

16. (b)

Explanation:

The percentage rise/fall in the number of employees who left infosys (compared to the previous year) during various years are:

For $2012 = \frac{900-500}{500} * 100 = 80\%$ (rise)

For $2013 = \frac{900-800}{900} * 100 = 11.11\%$ (fall)

For $2014 = \frac{800-700}{800} * 100 = 12.5\%$ (fall)

For $2015 = \frac{900-700}{700} * 100 = 28.57\%$ (rise)

Therefore, the maximum percentage rise/fall is for 2012.

17. (c)

Explanation:

So, we know that the number of employee

In 2011 $\Rightarrow$ 6000 + 700 – 500 = 6200,

In 2012 $\Rightarrow$ 6200 + 600 – 900 = 5900,

In 2013 $\Rightarrow$ 5900 + 900 – 800 = 6000,

In 2014 $\Rightarrow$ 6000 + 1000 – 700 = 6300,

In 2015 $\Rightarrow$ 6300 + 800 – 900 = 6200,

Now, percentage increase/decrease in the strength of the infosys from 2013 to 2014

$$\Rightarrow \frac{6300-6000}{6000} * 100 = 5\%$$

Therefore, the percentage increase/decrease in the strength of Infosys from 2013 to 2014 is 5%.

18. (c)

Explanation:

Number of cars sold in the month of February = 30

Number of cars sold in the month of March = 20

Number of cars sold in the month of April = 40

Average

$$= (30+20+40)/3 = 90/3 = 30$$

19. (a)

Explanation:

Direct observation.

Day1 = 6 bags (minimum)

20. (c)

Explanation:

Number of bags recycled on day 2 = 8

Number of bags recycled on day 6 = 16

Difference = 18 – 16 = 2

Data Sufficiency

21

INTRODUCTION

Data sufficiency is a crucial problem-solving skill that transcends disciplines, enabling individuals to analyze and draw conclusions from limited information. In this chapter, we'll explore the fundamental aspects of data sufficiency, its formats, and its significance in diverse fields and various strategies and tricks to solve data sufficiency questions.

Data sufficiency refers to the ability to determine whether the given information is adequate to solve a problem. Unlike traditional problem-solving, where the aim is to find a definite answer, data sufficiency evaluates the sufficiency of information to arrive at a conclusive solution.

FORMAT OF DATA SUFFICIENCY QUESTIONS

At the core of data sufficiency questions lies a distinct structure. Each question typically comprises a stem followed by two statements, labeled as (1) and (2). These statements provide additional information to address the question posed in the stem. However, they may not necessarily yield a single, definitive answer.

Question Stem: Is the value of x positive?

Statements: (1) $x^2 = 25$ (2) $x + 5 > 0$

Answer Choices: (A) Statement (1) alone is sufficient, but statement (2) alone is not sufficient.

(B) Statement (2) alone is sufficient, but statement (1) alone is not sufficient.

(C) Both statements together are sufficient, but neither statement alone is sufficient.

(D) Each statement alone is insufficient.

(E) Statements (1) and (2) together are insufficient.

THE TWO STEP APPROACH

A two-step approach to solve data sufficiency questions involves systematically analyzing each statement and then combining the information from both statements. Here's a step-by-step guide:

Step 1: Evaluate Each Statement Independently

(a) Determine if the statement is sufficient on its own:
- If the statement provides enough information to answer the question definitively, select the corresponding answer choice (e.g., Statement (1) alone is sufficient).
- If the statement is not sufficient on its own, proceed to the next step.

(b) Identify any key restrictions or conditions in the statement:
- Pay attention to inequalities, absolute values, or any specific constraints mentioned in the statement.

Step 2: Combine the Information from Both Statements

(a) Assess whether combining the statements is sufficient:
- If combining the information from both statements allows you to answer the question definitively, select the corresponding answer choice (e.g., Both statements together are sufficient).
- If the combined information is still insufficient, choose the appropriate answer choice.

Let's consider the following data sufficiency question and apply the two-step approach:

Question: Is the value of x positive?

(1) $x^2 = 25$

(2) $x + 5 > 0$

Two-Step Approach

Step 1: Evaluate Each Statement Independently

(1) $x^2 = 25$

Evaluate sufficiency: This statement is not sufficient on its own since x could be either 5 or -5.

Identify key restrictions: x can be positive or negative.

(2) $x + 5 > 0$

Evaluate sufficiency: This statement alone is not sufficient because it only tells us that $x > -5$, but it doesn't guarantee x is positive.

Identify key restrictions: x must be greater than -5.

Step 2: Combine the Information from Both Statements

Combining the information, we know from Statement (1) that x could be either 5 or -5, and from Statement (2) that x must be greater than -5.

Combining these restrictions, we find that x must be 5, and therefore, positive.

Conclusion: Both statements together are sufficient.

Following Strategies can prove helpful while applying the two step approach:

Backsolving Techniques: Backsolving is a powerful approach to tackle Data Sufficiency questions by leveraging the answer choices. By strategically plugging answer choices into the question stem, you can often deduce the sufficiency of the information without the need for complex calculations.

Substitution Strategies: Substitution is a versatile strategy that involves replacing variables or expressions with specific numbers to simplify a problem.

Estimation Methods: Estimation is not just a quick calculation technique but a strategic approach to gauge the sufficiency of information.

Process of Elimination: The process of elimination is a systematic strategy to narrow down answer choices by eliminating those that are implausible or inconsistent. By strategically discarding options that don't align with the given information, you'll increase your chances of arriving at the correct answer with greater efficiency.

EXAMPLES

Directions (E1 to E8) : Each of the following questions is followed by two statements. Mark

(a) If statement-I alone is sufficient to answer the question.

(b) If statement-II alone is sufficient to answer the question.

(c) If both statements-I and II together are necessary to answer the question.

(d) If both statements-I and II together are not sufficient to answer the question.

E1: Is 'b' positive?

(I) a + b is positive.

(II) a – b is positive

Ans: (d)

Explanation:

This question can easily be solved using a combination of substitution and elimination strategy.

If we look at Statement I then we will get

If $a = 2$ and $b = 3$, $a + b > 0$. Here $b > 0$.

If $a = 5$ and $b = -1$, $a + b > 0$. Here $b < 0$

Hence statement I alone is not sufficient and can be eliminated.

Now if we look at Statement II only then,

If $a = 4$ and $b = 3$, $a - b > 0$. Here $b > 0$.

If $a = 4$ and $b = -2$, $a - b > 0$. Here $b < 0$.

Hence II alone is not sufficient.

Now by using statements I and II together,

If $a = 4$ and b = 3, $a - b > 0$ and $a + b > 0$,

here $b > 0$

If $a = 4$ and $b = -2$, a – b > 0 and $a + b > 0$,

here $b < 0$.

Hence I and II together are also insufficient.

Therefore the only option now remains is option (d).

E2: ΔABC and ΔPQR are congruent.

(I) Areas of ΔABC and ΔPQR are same.

(II) ΔABC and ΔPQR are right angled triangles.

Ans: (d)

Explanation:

Consider Statement-I alone.

Areas of two non-congruent triangles can also be the same. Hence, I alone is not sufficient.

Consider Statement-II alone, Two right angled triangles are not always congruent, hence, II alone is not sufficient.

Now using both I and II, we have two right angled triangle with same area, we may have different combination as only product of base and height is same (eg. Sides may be (2 and 6) or (3 and 4)).

Hence even by using both the statement we cannot say that triangles are congruent.

E3: The average weight of students in class is 50 kg. What is the number of students in the class?

(I) The heaviest and the Lightest members of the class weigh 60 kg and 40 kg respectively.

(II) Exclusion of the heaviest and the lightest members from the class does not change the average weight of the students.

Ans: (d)

Explanation:

Statement-I alone – it only tells us about the highest and lowest weight of members of the class. We cannot infer the number of students in the class from this. Hence this statement alone is not sufficient.

Statement-II alone – It also is not sufficient to answer anything.

Both statements combined also cannot determine the number of students as the average weight of students (after excluding highest and lowest) cannot determine the total number.

So, option (d) is the correct answer.

E4: Consider three real numbers, *X*, *Y* and *Z*. Is *Z* the smallest of these numbers?

(I) X is greater than at least one of Y and *Z*.

(II) Y is greater than at least one of *X* and *Z*.

Ans: (c)

Explanation:

Statement-I alone: *X* may be greater than *Z*. But doesn't say anything about Z being smallest.

Statement-II alone: *Y* may be greater than *Z*. But says nothing about Z being smallest.

Both statements combined:

If $X > Y$, then $Y > Z$ hence $X > Y > Z$.

If $X > Z$, then Y > X hence $Y > X > Z$. Also If $X > Z$, and $Y > Z$, then also Z is the smallest. In either of these cases, Z is the least and hence the question can be answered using both statements together.

E5: How many people are watching TV programme *P*?

(I) Number of people watching TV programme *Q* is 1000 and the number of people watching both the programmes, *P* and Q, is 100.

(II) Number of people watching either P or Q or both is 1500.

Ans: (c)

Explanation:

Statement-I alone: using the formula

$n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$

We get $n(P \cup Q) = n(P) + 1000 - 100$.

We get 2 unknowns and 1 equation. Hence Statement I alone is not insufficient enough.

Statement II alone: n(P U Q) = 1500 given, but from II alone, n(P) cannot be determined.

Both statements combined:

1500 = n(P) + 1000 – 100

hence, n(P) = 1500 – 900 = 600

Both the statements combined can solve the question.

E6: Is Country X's GDP higher than country Y's GDP?

(I) GDP's of countries X and Y have grown over the past 5 years at compounded annual rate of 5% and 6% respectively.

(II) 5 years ago, the GDP of country *X* was higher than that of country *Y*.

Ans: (d)

Explanation:

Statement-I alone: We have no idea of their GDPs 5 years ago. Hence we can't compare its present value.

Statement II alone: GDP of country X was higher than country Y 5 years ago, but we don't know how much they have grown in 5 years. So, this statement alone is not sufficient.

Both statements combined:

Even after combining both statements nothing conclusive can be said which country has higher GDP as values of GDPs from 5 years ago are not known.

E7: On a given day a boat ferried 1500 passengers across the river in 12 hrs. How many round trips did it make?

(I) The boat can carry 200 passengers at any time.

(II) It takes 40 minutes each way and 20 minutes for waiting time at each terminal.

Ans: (b)

Explanation:

Statement-I alone: We cannot make out the number of rounds from maximum capacity as no information is given that boat ferried in maximum capacity.

Statement-II alone: Using time of travel and waiting period at each terminal, the number of round trips can be calculated.

E8: *F* and *M* are father and mother of *S*, respectively. *S* has four uncles and three aunts. F has two siblings. The siblings of *F* and *M* are unmarried. How many brothers does *M* have?

(I) F has two brothers.

(II) M has five siblings.

Ans: (a)

Explanation:

Statement-I alone: F and M are father and mother of S respectively. S has 4 uncles and 3 aunts who are unmarried. Statement-I tells us F has 2 unmarried brothers. Then definitely rest two uncles must be brothers of M. 3 aunts must be M's siblings (F has only two siblings).Rest aunt of s should be M `s sibling. So, Statement-I is sufficient to solve this question.

Directions (E9 to E11) : Each of the following questions is followed by two statements. Mark

(a) if the question can be answered by one of the statements alone but not by the other.

(b) if the question can be answered by using either statement alone.

(c) if the question can be answered by using both the statements together but cannot be answered using either statement alone.

(d) if the question cannot be answered even by using both the statements I and II.

E9. A game consists of tossing a coin successively. There is an entry fee of ₹10 and an additional fee of ₹ 1 for each toss of the coin. The game is considered to have ended normally when the coin turns heads on two consecutive throws. In this case the player is paid ₹100. Alternatively, the player can choose to terminate the game prematurely after any of the tosses. Ram has incurred a loss of ₹ 50 by playing this game. How many times did he toss the coin?

(I) The game ended normally.

(II) The total number of tails obtained in the game was 138.

Ans: (b)

Explanation:

Let the number of times the coin is tossed be '*n*'.

So, total amount spent by Ram = (10 + n × 1) = 10 + n

First he paid ₹ 10 entry fees and then ₹ 1 for each toss, but he suffered ₹ 50 loss.

Statement I alone: The game ended normally, so Ram is being paid ₹ 100 at last.

Then the amount spent by him,

(10 + n) – 100 = 50 ⇒ n =140.

Statement II alone: For this statement we have 2 cases.

Case (1): Game ended normally – this case is similar to statement I (Total loss might have been ₹ 148 and in the last two tosses head must have occurred, loss becomes ₹ 150 then profit of ₹ 100, so effective loss of ₹ 50).

Case (2): Game ended prematurely – this case is not possible with 138 tails as a loss of only ₹ 50 is given. Hence both the statements are sufficient enough.

E10. If A and B run a race, then A wins by 60 seconds. If B and C run the same race, then B wins by 30 seconds. Assuming that C maintains a uniform speed, what is the time taken by C to finish the race?

(I) A and C run the same race and A wins by 375 metres.

(II) The length of the race is 1 km.

Ans: (c)

Explanation:

Statement-I alone: In the race of A and C, A wins by 375 m also A wins by 60 seconds from B, and B wins by 30 seconds from C, so A wins by (60 + 30) = 90s from C.

Speed of c = (375/90) m/s. Time can't be calculated because we don't know the length of the track.

Statement-II alone: Length of race track = 1 Km. from it nothing is concluded.

Both statements combined:

Speed from statement-I is (375/90) m/s and length of the race track is 1 km. So time can be calculated.

E11. Four candidates for an award obtain distinct scores in a test. Each of the four casts a vote to choose the winner of the award. The candidate who gets the largest number of votes wins the award. In case of a tie in the voting process, the candidate with the highest score wins the award. Who wins the award?

(I) The candidates with top three scores each vote for the top scorer amongst the other three.

(II) The candidate with the lowest score votes for the player with the second highest score.

Ans: (a)

Explanation:

Assume A, B, C, D get score (distinct) 4, 3, 2, 1 respectively.

Statement-I alone: With the given condition A will give vote to B, B will give vote to A, C will give vote to A. Now either A will win (when D votes for A) or there will a tie when D gives vote to B. Even then A will win (maximum score). So we will get the answer.

Statement-II alone: With the given condition, D will vote for B. No other information is given, so nothing can be concluded from this statement.

Both statements combined:

On combining both the statements, A will get two votes (from B and C) and B will also get two votes (from A and D). In this tie, A will be declared the winner.

E12. Nandini paid for an article using currency notes of denominations ₹ 1, ₹ 2, ₹ 5 and ₹ 10 using at least one note of each denomination. The total number of five and ten rupee notes used was one more than the total number of one and two rupee notes used. What was the price of the article?

(I) Nandini used a total of 13 currency notes.

(II) The price of the article was a multiple of ₹ 10.

Ans: (d)

Explanation:

Let the number of 1, 2, 5, 10 ₹ notes be *p*, *q*, *r* and s respectively. According to the given condition we have $r + s = p + q + 1$.

Statement I alone: There is no other condition, and 13 currency notes will give many different possibilities, hence it is not sufficient.

Statement II alone: Multiple of 10 will also give many values.

Even if you combine the statement, we can have various values.

PRACTICE QUESTIONS

1. **Decide whether the data provided in the statements are sufficient to answer the question.**

   Three persons, A, B and C are hired to complete a work. Find the time taken by A alone to complete the work.

   Statement I: B and C together can complete 50% of the work in 5 days. If all of them together complete the work, then they together received Rs. 3960 as a wage out of which A received Rs. 1800.

   Statement II: A, B and C together can complete the whole work in 60/11 days.

   (a) The data in statement I alone are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.

   (b) The data in statement II alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question

   (c) The data in statement I alone and in statement II alone are sufficient to answer the question.

   (d) The data given in both statements I and II together are not sufficient to answer the question.

2. **Decide whether the data provided in the statements are sufficient to answer the question.**

   A shopkeeper sold an article to a customer. Find the profit percentage earned by the shopkeeper.

   Statement I: The shopkeeper marked the article 50% above its cost price.

   Statement II: The respective ratio of the cost price to the selling price of the article is 5:6.

   Statement III: The shopkeeper gave a discount of 20% and sold the article for Rs. 3600.

   (a) The data in statement I and II together are necessary to answer the question, while data in statement III alone is not sufficient to answer the question.

   (b) The data in statement II and III together are necessary to answer the question, while data in statement I alone is not sufficient to answer the question.

   (c) The data in statement I and III together are necessary to answer the question, while data in statement II alone is not sufficient to answer the question.

   (d) The data in either statement II alone or data in statement I and III together are necessary to answer the question.

3. **Decide whether the data provided in the statements are sufficient to answer the question.**

   Distance between City P and City Q is 720 km. Raj started travelling from City P to City Q and Anjali started travelling from City Q to City P simultaneously with different speeds. Find the speed of Raj.

   Statement I: Raj and Anjali meet 4 hours after they started travelling, and the distance travelled by Raj is 240 km more than distance travelled by Anjali before they meet.

   Statement II: The ratio of speed of Raj and Anjali is 2:1.

   Statement III: After meeting for the first time, Raj and Anjali take 2 hours and 8 hours respectively to cover the remaining distance.

   (a) The data in statement I and II together are necessary to answer the question, while data in statement III alone is not sufficient to answer the question.

   (b) The data in statement II and III together are necessary to answer the question, while data in statement I alone is not sufficient to answer the question

   (c) The data in statement I and III together are necessary to answer the question, while data in statement II alone is not sufficient to answer the question.

   (d) The data in either statement I alone or data in statement II and III together are necessary to answer the question.

4. **Present age of C is 20% less than the present age of A. Find the present age of B.**

   Statement I: Difference between the ages of A and B is 6 years. Present age of C is 24 years.

   Statement II: Present average age of A, B and C is 30 years. Present average age of A and B is 33 years.

   (a) The data in statement I alone are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.

   (b) The data in statement II alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question

   (c) The data either in statement I alone or in statement II alone are sufficient to answer the question.

   (d) The data given in both statements I and II together are not sufficient to answer the question.

5. **A bag contains red, blue and green balls. Numbers of red and green balls in the bag are 20% and 40% respectively less than the number of blue balls in the bag. Find the total number of balls in the bag.**

Statement I: Probability of drawing two blue balls from the bag is 1/6.

Statement II: Probability of drawing a green ball from the bag is 1/4.

(a) The data in statement I alone are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.

(b) The data in statement II alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.

(c) The data either in statement I alone or in statement II alone are sufficient to answer the question.

(d) The data given in both statements I and II together are not sufficient to answer the question.

6. **A bag contains certain number of blue, grey and black balls. Find the total number of balls in the bag if the probability of drawing a grey ball from the bag is 1/8.**

Statement I: The bag contains '2x + 1' blue, 'x - 1' grey and '3x' black balls.

Statement II: The bag contains blue, grey and black balls in the ratio 3:1:4 respectively.

(a) The data in statement I alone are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.

(b) The data in statement II alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.

(c) The data either in statement I alone or in statement II alone are sufficient to answer the question.

(d) The data given in both statements I and II together are not sufficient to answer the question.

7. **Average cost price of a book and a copy is Rs. 750. Discount given on book is 10%. The copy is marked 20% above its cost price. Find the selling price of copy if the cost price of book is Rs. 100 more than that of copy.**

Statement I: Discount percentage given on copy is same as the percentage by which the book is marked above its cost price.

Statement II: Selling price of book is Rs. 270 more than that of copy.

(a) The data in statement II alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.

(b) The data either in statement I alone or in statement II alone are sufficient to answer the question.

(c) The data given in both statements I and II together are not sufficient to answer the question.

(d) The data in both statements I and II together are necessary to answer the question.

8. **Mixture A containing milk and water in the ratio of 9:7, respectively is mixed with mixture B containing milk and water in the ratio of 5:11, respectively. If the ratio of water to milk in the final mixture is 25:23, respectively then find the total quantity of mixture A.**

Statement I: Quantity of mixture B is 400 ml.

Statement II: Quantity of final mixture is 1.2 litres.

(a) The data in statement I alone are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.

(b) The data in statement II alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.

(c) The data either in statement I alone or in statement II alone are sufficient to answer the question

(d) The data given in both statements I and II together are not sufficient to answer the question.

9. **Ratio of simple interest to compound interest on a certain amount of sum at R% rate of interest after 2 years will be 30:33. Find the simple interest earned if the same amount is invested at (R+8)% p.a. for 4 years.**

Statement I: Compound interest on the given sum after 2 years at R% rate of interest will be Rs. 528.

Statement II: Simple interest on the given sum after 2 years at R% rate of interest will be Rs. 480

(a) The data in statement I alone are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.

(b) The data in statement II alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.

(c) The data either in statement I alone or in statement II alone are sufficient to answer the question.

(d) The data given in both statements I and II together are not sufficient to answer the question.

10. **Three friends A, B and C took a contract of Rs. 26250 for completing a certain amount of work. Find the share of amount received by B on completion of the work.**

Statement I: A and C working together can complete the whole work in 36 days.

Statement II: A and B working together can complete the whole work in 30 days. If A and B started the work and worked on it for 20 days and then rest of the work is completed by C alone in 40 days.

(a) The data in statement II alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.

(b) The data either in statement I alone or in statement II alone are sufficient to answer the question.

(c) The data given in both statements I and II together are not sufficient to answer the question.

(d) The data in both statements I and II together are necessary to answer the question.

11. **Six persons M, N, O, P, Q and R have different heights. Who is the tallest among all?**

Statement I: M is taller than N and P. Q is taller than O, who is taller than M. Neither N nor P is the shortest.

Statement II: Q is taller than N. Number of persons taller than O is equal to number of persons shorter than P. M is the 3rd tallest. R is the shortest.

(a) The data in statement I alone are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question

(b) The data in statement II alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question

(c) The data either in statement I alone or in statement II alone are sufficient to answer the question

(d) The data given in both statements I and II together are not sufficient to answer the question

12. **The ratio of the present ages of 'A' and 'B' is 4:5, respectively. Find the present age of 'C'.**

Statement I: The sum of the present ages of 'B' and 'C' is 50 years.

Statement II: The present age of 'C' is 87.5% more than that of 'A'.

(a) The data in statement II alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.

(b) The data either in statement I alone or in statement II alone are sufficient to answer the question.

(c) The data given in both statements I and II together are not sufficient to answer the question.

(d) The data in both statements I and II together are necessary to answer the question.

13. **Find the cost price of the article.**

Statement I: If shopkeeper marked the article 80% above the cost price, and sold it after giving a discount of 40% such that he made a profit of Rs. 48.

Statement II: The shopkeeper sold the article at a profit of 40%. If the cost price and selling price are increased by 30% and 20% respectively then he had earned Rs. 12 less.

(a) The data in statement I alone are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.

(b) The data in statement II alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.

(c) The data either in statement I alone or in statement II alone are sufficient to answer the question.

(d) The data given in both statements I and II together are not sufficient to answer the question.

14. **There are four persons Amar, Abi, Aadhi and Geetha. Find the age difference between Abi and Amar.**

Statement I: Age of Aadhi, 5 years hence from now will be equal to the age of Abi, 8 years ago from now. Amar is two years younger than Geetha.

Statement II: Ratio between present ages of Amar and Abi is 5:11, respectively.

(a) The data in statement I alone are sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.

(b) The data in statement II alone are sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.

(c) The data either in statement I alone or in statement II alone is sufficient to answer the question.

(d) The data given in both statements I and II together is not sufficient to answer the question.

15. A, B and C together can complete the work in (60/13) days. In how many days C alone can complete the work?

Statement I: B and C together can complete the work in (60/7) days.

Statement II: A is 50% more efficient than B.

(a) The data in statement II alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.

(b) The data either in statement I alone or in statement II alone are sufficient to answer the question.

(c) The data given in both statements I and II together are not sufficient to answer the question.

(d) The data in both statements I and II together are necessary to answer the question.

16. There are four friends namely Raj, Giri, Yadav and Adani in a hostel room having different ages. Find the age of Yadav, fourteen years hence from now.

Statement I: Ratio of present ages of Adani and Yadav is 5:3, respectively. Age of Raj, five years hence from now will be 4 years more than the age of Adani, seven years hence from now.

Statement II: Yadav is 10 years younger than the Raj.

(a) The data in statement II alone are sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.

(b) The data either in statement I alone or in statement II alone is sufficient to answer the question.

(c) The data given in both statements I and II together is not sufficient to answer the question.

(d) The data in both statements I and II together is necessary to answer the question.

17. A bag contains a total of 15 balls of three different colors- red, yellow, and blue. What is the probability of drawing a red ball from the bag?

I. The probability of drawing a yellow ball is 3/5.

II. There are 2 blue balls in the bag.

(a) If the data in Statement I alone is sufficient to answer the question.

(b) If the data in Statement II alone is sufficient to answer the question.

(c) If the data even in both Statements I and II together are not sufficient.

(d) If the data in both Statements I and II together are necessary.

18. Each play kit is packed in a separate box and these small boxes are again packed in the transport boxes.

How many transport boxes of play kits will fit into the cargo area of the lorry?

Fact 1 - Each transport box is cubical in shape and one of its sides is 50 centimetres in length.

Fact 2 - Cargo area of the lorry is thirty square metres.

Select the appropriate option that describes the role of facts 1 and 2 with respect to answering the question asked.

(a) Both facts together are sufficient, but neither fact alone is sufficient.

(b) Each fact alone is sufficient.

(c) Both facts together are not sufficient.

(d) Fact one alone is sufficient, but fact two alone is not sufficient

19. What is the unit digit of A^B, where A and B are positive integers?

Statement (I) B is divisible by 4.

Statement (II) A is an odd multiple of 5.

(a) If the data in Statement I alone is sufficient to answer the question.

(b) If the data in Statement II alone is sufficient to answer the question.

(c) If the data even in both Statements I and II together are not sufficient.

(d) If the data in both Statements I and II together are necessary.

20. Two statements are given below and followed by a Question:

I. AB has exactly 3 factors.

II. The Sum of digits of AB is a prime number.

Question:

What is the value of a two-digit number AB?

Which of the following is correct in respect of the above statements and Question?

(a) If the data in Statement I alone is sufficient to answer the question.

(b) If the data in Statement II alone is sufficient to answer the question.

(c) If the data given in both Statements I and II together are not sufficient.

(d) If the data in both Statements I and II together are necessary.

ANSWERS

1. (*a*)	2. (*d*)	3. (*d*)	4. (*b*)	5. (*a*)	6. (*a*)	7. (*d*)	8. (*c*)	9. (*c*)	10. (*d*)
11. (*c*)	12. (*d*)	13. (*c*)	14. (*d*)	15. (*d*)	16. (*d*)	17. (*d*)	18. (*c*)	19. (*b*)	20. (*c*)

EXPLANATIONS

1. (a)

Explanation:

Statement I: B and C together can complete the whole work in 10 days.

Ratio of efficiency of A:(B + C)

= 1800:(3960 – 1800) = 1800:2160 = 5:6

So, ratio of time taken by *A*:(*B* + *C*) = 6:5

So, required time taken by A alone to complete the work = $\frac{6}{5} \times 10 = 12$ days

So, data in statement I alone is sufficient to answer the question.

Statement II: Data in statement II alone is not sufficient to answer the question.

2. (d)

Explanation:

Statement I: Let the cost price of the article be Rs. '*x*'.

Then, marked price of article = Rs. 1.5x

So, data in statement I alone is not sufficient to answer the question.

Statement II:

Let the cost price and selling price of the article be Rs. 5x and Rs. 6*x*, respectively.

So, profit 6*x* – 5*x* = Rs. *x*

Required profit percentage = $\frac{x}{5x} \times 100 = 20\%$

So, data in statement II alone is sufficient to answer the question.

Statement III: Selling price of article = Rs. 3600

Marked price of article = $\frac{3600}{0.8}$ = Rs. 4500

So, data in statement III alone is not sufficient to answer the question.

Combining statements, I and III:

Selling price of article = Rs. 3600

Marked price of article = $\frac{3600}{0.8}$ = Rs. 4500

So, cost price of article = $\frac{4500}{1.5}$ = Rs. 3000

Required profit percentage = $\frac{600}{3000} \times 100 = 20\%$

So, data in statement I and statement III together are necessary to answer the question.

3. (d)

Explanation:

Statement I: Distance travelled by Raj before meet = $\frac{(720 + 240)}{2}$

= 480 km Therefore, speed of Raj = $\frac{480}{4}$ = 120 km/hour Hence, statement – I alone is sufficient to answer the question.

Statement II:

Nothing can be concluded from statement – II.

Hence, statement – II alone is not sufficient to answer the question.

Statement III:

Let, distance travelled by Raj before meet = x km

So, distance travelled by Anjali before meet

= (720 – x) km

According to question,

$\frac{(720 - x)}{2}$ = speed of Raj

And, speed of Anjali = $\frac{x}{8}$

Hence, statement – III alone is not sufficient to answer the question.

Combining statement – II and Statement – III:

According to question,

$$\frac{\left(\frac{(720-x)}{2}\right)}{\left(\frac{x}{8}\right)} = 2 \text{ or } 1440 - 2x = x$$

$2x + x = 1440$ $x = \frac{1440}{3x} = 480$ km

Therefore, speed of Raj = (720 – 480)/2

= 240/2 = 120 km/hr.

Hence, question can be solved by Combining statement – II and Statement – III

4. (b)

Explanation:

Statement I: Present age of $A = \frac{24}{0.80}$ = 30 years

Present age of B = either 30 – 6 or 30 + 6

= either 24 years or 36 years

Exact age of B can't be determined.

So, statement I alone is not sufficient to answer the question.

Statement II:

Sum of the present ages of A, B and C = 3 × 30 = 90 years

Sum of the present ages of A and B = 2 × 33 = 66 years

Present age of C = 90 – 66 = 24 years

Present age of $A = \frac{24}{0.80}$ = 30 years

Present age of B = 66 – 30 = 36 years

So, statement II alone is sufficient to answer the question.

5. (a)

Explanation:

Let the number of blue balls in the bag be '5x'.

So the number of red balls in the bag = 0.80 × 5x = 4x

Number of green balls in the bag = 0.60 × 5x = 3x

Statement I:

The probability of drawing two blue ball from the bag

$$= \frac{5x_{C_2}}{12x_{C_2}} = \frac{1}{6}$$

$$5x \times \frac{(5x-1)}{12x(12x-1)} = \frac{1}{6}$$

$25x - 5 = 24x - 2 \quad x = 3$

So the total number of balls in the bag = 12 × 3 = 36

So, data in statement I alone is sufficient to answer the question.

Statement II: The probability of drawing a green ball from the bag $= \frac{3x_{C_1}}{12x_{C_1}} = \frac{1}{4}$

$$\frac{3x}{12x} = \frac{1}{4}$$

$$\frac{1}{4} = \frac{1}{4}$$

Here 'x' is eliminated, so the total number of balls can't be determined.

So, data in statement II alone is not sufficient to answer the question.

6. (a)

Explanation:

Statement I: So the probability of drawing a grey ball from the bag $= \frac{x-1_{C_1}}{6x_{C_1}} = \frac{1}{8} \quad \frac{(x-1)}{6x} = \frac{1}{8}$

$8x - 8 = 6x$

$2x = 8$

$x = 4$

So the total number of balls in the bag = 6 × 4 = 24

So statement I alone is sufficient to answer the question.

Statement II: Let the bag contains 3x blue, x grey and 4x black balls

So the probability of drawing a grey ball from the bag

$$= \frac{x_{C_1}}{8x_{C_1}} = \frac{1}{8}$$

$$\frac{x}{8x} = \frac{1}{x}$$

$$\frac{1}{8} = \frac{1}{8}$$

Here x is eliminated, so the total number of balls can't be determined.

So statement II alone is not sufficient to answer the question.

7. (d)

Explanation:

Cost price of book = $\frac{(1500+100)}{2}$ = Rs. 800

Cost price of copy = 1500 – 800 = Rs. 700

Marked price of copy = 1.20 x 700 = Rs. 840

Statement I: Let percentage discount given on copy = percentage by which book is marked above its cost price = x%

So, Selling price of book $= 0.90 \times \left(\frac{1+x}{100}\right) \times 800$

Selling price of copy = (1 – x/100) × 840

So, data in statement I alone is not sufficient to answer the question.

Statement II: As per Data in statement II alone is not sufficient to answer the question.

Combining statement, I and II, we get

$$0.90 \times \left(\frac{1+x}{100}\right) \times 800 - \left(\frac{1-x}{100}\right) \times 840 = 270$$

Or, 720 + 7.2x – 840 + 8.4x = 270

Or, 15.6x = 390

Or, $x = 25$

Selling price of copy = 0.75 × 840 = Rs. 630

So, data in both statements together is necessary to answer the question.

8. (c)

Explanation:

Statement I: Let quantity of milk and water in mixture A is $9x$ ml and $7x$ ml respectively.

Quantity of milk in mixture $B = \frac{5}{16} \times 400 = 125\text{ml}$

Quantity of water in mixture B = 400 – 125 = 275 ml

According to question;

$$\frac{(9x+125)}{(7x+275)} = \frac{23}{25}$$

Or, $225x + 3125 = 161x + 6325$

Or, $64x = 3200$

Or, $x = 50$

Quantity of mixture A = $16x = 16 \times 50 = 800$ ml

So, data in statement I alone is sufficient to answer the question.

Statement II: Let quantity of milk and water in mixture A is $9x$ ml and $7x$ ml respectively.

Let quantity of milk and water in mixture B is $5y$ ml and 11y ml respectively

Quantity of milk in final mixture = $\frac{23}{48} \times 1200 = 575\text{ml}$

Quantity of water in final mixture = 1200 – 575 = 625 ml

According to question;

$9x + 5y = 575$ ...(i)

$7x + 11y = 625$ ... (ii)

Solving equation (i) and (ii), we get

$x = 50$ and $y = 25$

So, quantity of mixture A = $16x = 50 \times 16 = 800$ ml

So, data in statement II alone is sufficient to answer the question.

9. (c)

Explanation:

Let the certain amount of sum is Rs. P.

Let simple interest and compound interest after 2 years is Rs. $30x$ and Rs. $33x$ respectively.

Difference between simple and compound interest for 2 years = $33x - 30x$ = Rs. $3x$

Simple interest for one year = $\frac{30x}{2}$ = Rs. 15x

So, $R = \frac{3x}{15x} \times 100 = 20\%$

Statement I: $P \times \{(1.2)^2 - 1\} = 528$

So, $P = \frac{528}{0.44}$ = Rs. 1200

Desired interest = 1200 × 0.28 × 4 = Rs. 1344

So, data in statement I alone is sufficient to answer the question.

Statement II: $P \times 0.20 \times 2 = 480$

Or, P = Rs. 1200

Desired interest = 1200 × 0.28 × 4 = Rs. 1344

So, data in statement II alone is sufficient to answer the question.

10. (d)

Explanation:

Statement I: From the data in statement I; data in statement I alone is not sufficient to answer the question.

Statement II: Part of work competed by A and B in 20 days $= \frac{20}{30} = \frac{2}{3}$

Remaining part $= 1 - \frac{2}{3} = \frac{1}{3}$

Total time taken by C to complete the whole work = 40 × 3 = 120 days

So, data in statement II alone is not sufficient to answer the question.

Combining statement I and II:

Let total amount of work = 360 units (LCM of 120, 30, 36)

Efficiency of A and C together $= \frac{360}{36} = 10$ units per day

Efficiency of A and B together $= \frac{360}{30} = 12$ units per day

Efficiency of C $= \frac{360}{120} = 3$ units per day

Efficiency of A = 10 – 3 = 7 units per day

Efficiency of B = 12 – 7 = 5 unit per day

Total efficiency of A, B and C together = 5 + 4 + 3 = 15 units per day

So, profit share of B = 5/15 x 26250 = Rs. 8750

11. (c)

Explanation:

Now, from statement I, M is taller than N and P. Q is taller than O, who is taller than M. Neither N nor P is the shortest. So, we have,

Q > O > M > N/P > P/N > R

So, Q is the tallest.

Therefore, statement I alone is sufficient.

Now, from statement II, Number of persons taller than O is equal to number of persons shorter than P. M is the 3rd tallest. R is the shortest. Q is taller than N. So, we have,

$Q > O/P > M > N > P/O > R$

So, Q is the tallest.

Therefore, statement II alone is sufficient.

12. (d)

Explanation:

Let the present ages of 'A' and 'B' be 4x years and 5x years, respectively.

Statement I: According to the question,

Present age of 'C' = (50 – 5x) years

So, data in statement I alone is not sufficient to answer the question.

Statement II: According to the question,

Present age of 'C' = 1.875 × 4x = 7.5x

So, data in statement II alone is not sufficient to answer the question.

Combining statements I and II:

7.5x = 50 – 5x

Or, x = 50/12.5 = 4

Therefore, present age of 'B' = 5x = 20 years

So, data in both statements I and II together are necessary to answer the question.

13. (c)

Explanation:

Statement I: Let cost price of the article is Rs. X

Selling price of the article = 1.8 x 0.6 × x = 1.08x

So, $1.08x - x = 48$

$0.08x = 48$

x = Rs. 600

So, data in statement I alone is sufficient to answer the question.

Statement II: Let cost price of the article is Rs. x

Selling price of the article = Rs. $1.4x$

Profit = $1.4x - x$ = Rs. $0.4x$

New cost price = 1.3x

New selling price = $1.2 \times 1.4x$ = Rs. $1.68x$

Profit now = $1.68x - 1.3x = 0.38x$

So, $0.4x - 0.38x = 12$

0.02x = 12

x = Rs. 600

So, data in statement II alone is sufficient to answer the question.

14. (d)

Explanation:

Let's assume the present ages of Amar, Abi and Aadhi is 'x' years, 'y' years and 'z' years, respectively.

Statement I: As per question;

$z + 5 = y - 8$ Or, $z - y = -3$

Also, present age of Geetha = '$x + 2$' years

So, data in statement I alone is not sufficient to answer the question.

Statement II: $\frac{x}{y} = \frac{5}{11}$

So, data in statement II alone is not sufficient to answer the question.

So, data given in both statements I and II together is not sufficient to answer the question.

15. (d)

Explanation:

Statement I: Let the total work = 60 units

Amount of work done by A, B and C together in one day = 60/(60/13) = 13 units

Amount of work done by B and C together in one day = 60/(60/7) = 7 units

Amount of work done by A alone in one day

= 13 – 7= 6 units

Here we can't find the efficiency of C.

So, data in statement I alone is not sufficient to answer the question.

Statement II: Efficiencies of A and B are given but the efficiency of C is not known.

So, data in statement II alone is not sufficient to answer the question.

Combining statement I and statement II:

Let the total work = 60 units

Amount of work done by A, B and C together in one day = 60/(60/13) = 13 units

Amount of work done by B and C together in one day = 60/(60/7) = 7 units

Amount of work done by A alone in one day

= 13 – 7 = 6 units

Amount of work done by B alone in one day

= 6/1.5 = 4 units

Amount of work done by C alone in one day

= 7 – 4 = 3 units

Time taken by C alone to complete the work

= 60/3 = 20 days

So, data in both the statements I and II together are necessary to answer the questions.

16. (d)

Explanation:

Statement I:

Let assume present ages of Adani and Yadav is '5*x*' and '3*x*' years, respectively.

Age of Adani 7 years hence from now

= (5x + 7) years

Present age of Raj = 5*x* + 7 + 4 – 5= (5*x* + 6) years

So, data in statement I alone is not sufficient to answer the question.

Statement II: Yadav is 10 years younger than Raj

So, data in statement II alone is not sufficient to answer the question.

Combining statements I and II, together

5*x* + 6 – 3*x* = 10

Or, 2*x* = 4

Or, *x* = 2

Therefore, age of Yadav, 14 years hence from now = 3*x* + 14 = 6 + 14 = 20 years

So, data in both statements I and II together is necessary to answer the question.

17. (d)

Explanation:

Statement I: Given, probability of drawing a yellow ball

$$=\frac{3}{5}=\frac{9}{15}$$

So, we can say there are 9 yellow balls in the bag. But we cannot find the number of red balls from statement (I) alone. Hence, (I) alone is not sufficient to answer.

Statement II: Given, there are 2 blue balls in the bag. But Statement (II) alone does not provide any information about the number of red balls in the bag. Hence (II) alone is not sufficient to answer.

Combining both the statements:

We get, number of yellow balls = 9

Number of blue balls = 2

Number of red balls = 15 – (9 + 2) = 4

So, probability of drawing a red ball = 4/15

Therefore, the question can be answered by combining both statements.

18. (c)

Explanation:

Fact – 1 alone provides information about volume of box but it didn't provides us any information about Cargo area.

Thus, Fact – 1 alone not sufficient

Fact – 2 alone provides information about cargo area but it didn't provides us any specific information related to length and breadth of cargo area.

Thus, Fact – 2 alone not sufficient

Even if we combine Fact – 1 and Fact – 2 together still we cannot conclude anything specific about how many transport boxes of play let kit will fill into cargo area of lorry.

Therefore, both facts together are not sufficient.

19. (b)

Explanation:

Statement I: Given B is a multiple of 4, but nothing is known about A. Hence, we cannot conclude about the unit digit of A^B with the help of statement (I) alone.

Statement II: Given A is an odd multiple of 5, i.e. A = {5, 15, 25, 35……}

It is evident that the unit digit of A is always 5. Therefore, the unit digit of A^B would also be 5 (B is a positive integer).

Also, we can observe:

$5^1 = 5$

$5^2 = 25$

$5^3 = 125$

$5^4 = 625$…..

So, statement (II) alone is sufficient to answer the question.

20. (c)

Explanation:

Concept: Square of a prime number always has 3 factors or if a number has exactly 3 factors, it is the square of a prime number.

Statement I:

AB has 3 factors, and it is a two – digit number. So, there are two possibilities either it is $5^2 = 25$ or $7^2 = 49$. But one cannot determine the unique value of AB from the statement I.

Statement II:

From statement II alone, one cannot get any unique value of AB as there are many two digits numbers whose sum is a prime number. For example: 11, 12, 16, 25, 47 etc.

Combine statement I and II together,

We get two values of AB, i.e., 25 & 49, which satisfy both the statements.

Therefore, we cannot get a unique value of AB from both statements together.